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ENGINEERING CONSTRUCTION

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IN

IRON, STEEL, AND TIMBER

BY

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PREFACE

THE primary object which the author had in view in writing this book was to prepare a text-book for students attending the first portion of his lectures on Materials and Structures; but he considers that the work may be found useful not only to engineering students in Technical Colleges and Universities, but also to those engaged in the design of constructional iron and steel work.

The modern methods of determining the safe intensity of working stresses in structures have been considered in the first chapter, and have been employed more or less throughout the work.

The subjects treated in the various chapters have been considered as briefly as possible, and the numerous examples given are relied upon for more complete explanation.

The special feature of the work lies in the various examples which illustrate the design of the most important classes of structures in iron, steel, and timber; these have all been selected from existing works.

The author's experience in teaching has convinced him of the necessity of thoroughly illustrating the various principles underlying the theory and practice of construction, as a student is never certain whether he understands these principles or not until he has attempted to apply them.

The author has freely availed himself of the works of others, which he has endeavoured to acknowledge as far as possible throughout the work: but he is especially indebted to the *Proceedings* of the Institution of Civil Engineers, the American Society of Civil Engineers, the Specifications of Mr. Theodore Cooper, M.Amer.Soc.C.E.; the works of Professors Dubois, Burr, and Waddell; also to Mr. Hickson, M.Inst.C.E., Commissioner and Engineer-in-chief for Roads and Bridges, New South Wales, for drawings of the continuous-girder road bridge, the swing bridge, and the hinged-arch bridge.

Although the subject of foundations is at least as important as that of superstructures, it has only been treated briefly for pile trestle viaducts and cylinder piers.

The author considers that the subject of foundations could be more conveniently dealt with in connection with a book on Engineering Construction in Brickwork, Masonry, and Concrete, which he hopes to write as soon as time will permit.

In spite of the care which has been exercised in preparing this work, it is possible that in a first edition errors may have escaped detection; the author will therefore be thankful for any information as to errors in diagrams or calculations.

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LIST OF FOLDING PLATES

ERRATA.

Page 3, line 9, *for* $\frac{3}{1200}$ *read* $\frac{2}{1200}$.

" 25, line 3, *for* r *read* $\frac{l}{r}$

" 28, line 2, *for* Chapter IV. *read* Chapters IV. and V.

" 31, line 3 from bottom, insert a semicolon after non-volatile.

" 33, line 3, *for* P^1x_1 *read* P_1x_1.

" 34, line 10, *for* Fig. 1 *read* Fig. 10.

" 38, line 24, *for* 11,274 *read* 11,278.

" 39, line 17, *for* 1083 *read* 10·83.

" 39, line 21, *for* 1623 *read* 1624.

" 39, line 22, *for* 5270 *read* 5273.

" 40, line 8, *for* 6·16 *read* −6·16.

" 40, line 22, *for* 43 *read* 42.

" 43, line 21, *for* tension *read* tensile.

" 45, line 22, *for* W *read* w.

" 48, line 15, *for* b *read* l.

" 62, line 2 from bottom, *for* $\frac{l + al}{R}$ *read* $\frac{l + al}{l}$

" 65, line 28, *for* tensile strength *read* modulus of rupture.

" 66, line 17, *for* Fig. 79 *read* Fig. 80.

" 67, line 6, *for* Fig. 80 *read* Fig. 79.

" 82, line 11, *for* cd *read* cb.

" 83, line 12, *for* $(y_1^2 - y_2)$ *read* $(y_1^2 - y^2)$.

" 83, line 15, *for* $\frac{Sy_1^2}{I}$ *read* $\frac{Sy_1^2}{2I}$

" 93, Fig. 114, line showing bottom of stringer and top of corbel is omitted.

" 95, line 19, *for* $\frac{2 \times 12 \times 12 \times f}{6}$ *read* $\frac{2 \times 12 \times 12 \times 12 \times f}{6}$

" 95, line 23, *for* $\frac{31680}{600}$ *read* $\frac{3168{\cdot}0}{600}$

" 99, line 4 from bottom, *for* pp. 37, 38, and 39 *read* pp. 27, 28, 29.

" 109, line 11, *for* Trontwine *read* Trantwine.

" 110, line 1, *for* Trontwine *read* Trantwine.

" 111, Table XXVI., line 9, *read* AZ | +11,274 | +11,216 | +11,247

" 114, Table XXX., line 5, *for* −36,464 *read* −26,464.

" 114, Table XXX., line 9, *for* +22,782 *read* +22,784.

" 115, line 5, *for* RZ *read* YZ.

" 121, table, line 2, *for* 6,314, *read* 63,140.

" 121, table, line 3, *for* 54,820 *read* 54,920.

" 123, line 12, *for* 46 *read* 56.

" 124, line 14, *for* JH *read* HI.

" 124, line 15, *for* $-0{\cdot}5W \sec \theta$ *read* $+0{\cdot}5W \sec \theta$.

" 125, Fig. 150, the loads applied at top apices should be W_1, not W.

" 126, Fig. 151, *for* I *read* L.

" 126, line 10, *for* 47W *read* 4W.

Page 129, replace line 4 by $VG = \frac{-60 \times 40 + 15(30 + 20 + 10)}{10} = -180.$

„ 130, line 5 from bottom, *for* LK = − 33·33 tons *read* LK = − 28·33 and − 1·66.

„ 130, line 4 from bottom, *for* −15 *read* −21·66.

„ 134, Fig. 170, Dead-load stresses are wrong.

For 2·29 *read* 2·83.
For 1·14 *read* 1·41.
For 3·43 *read* 4·24.
For 4·58 *read* 5·66.
The total stresses are wrong accordingly.

„ 134, line 19, *for* W_{12} *read* W_2.

„ 134, last line, *for* − 1·77 *read* + 1·77.

„ 138, line 5, *for* 72′ 0″ *read* 12′ 0″.

„ 139, Fig. 174, Dimensions 8·4′ is from dotted line on left to line 32.

„ 141, line 3, *for* − 2·5BX × + 6R = 0 *read* − 2·5BX + 6R = 0.

„ 144, line 7, *for* + 1·5 *read* −1·5.

„ 145, line 1, insert $-(140 - \frac{6}{8} \times 100)$.

„ 145, line 2, insert $-(140 - \frac{6}{8} \times 100)$.

„ 145, line 11, *for* $128y_4$ *read* $-128y_4$.

„ 145, line 14, *for* −7·5 *read* −3·28.

„ 149, line 3, *for* $-\frac{Wl^3}{24EI}$ *read* $-\frac{wl^3}{24EI}$

„ 150, last line, *for* l_0 *read* i_0.

„ 152, line 17, *for* $\frac{bd_1}{d_1}$ *read* $\frac{bd_1}{d}$

„ 152, line 20, *for* $f = \frac{6Wx}{b_1h_1^2} = \frac{6Wl}{bh^2}$ *read* $f = \frac{6Wx}{b_1d_1^2} = \frac{6Wl}{bd^2}$

„ 152, last line, *for* h *read* d.

„ 153, line 3, *for* $\frac{dy}{dx}$ *read* $\frac{dv}{dx}$

„ 153, line 15, *for* $\frac{36Pl^3}{5Ed^3b}$ *read* $\frac{36Wl^3}{5Ed^3b}$

„ 154, line 8, *for* $\frac{m(f_t + f_c)l^2}{Ed}$ *read* $\frac{m(f_t + f_c)l^2}{2Ed}$

„ 155, line 17, *for* $\frac{0{\cdot}1426(f_t + f_c)l^2}{Ed}$ *read* $\frac{0{\cdot}1426(f_t + f_c)l^2}{2Ed}$

„ 156, line 27, *for* 5·535 *read* 5·525.

„ 157, line 6, *for* 4·42 *read* 3·42.

„ 162, line 18, *for* $-\frac{fl^2}{32EI}$ *read* $-\frac{fl^2}{32Ey}$

„ 167, line 32, *for* x_4C *read* x_4D.

„ 168, line 22, *for* M_4 *read* M_3.

„ 169, line 1, *for* 89·55 *read* 69·45.

„ 169, line 12, *for* 95·85 *read* 94·95.

„ 169, line 12, *for* 57·5 *read* 57·0.

„ 169, line 13, *for* 55·38 *read* 38·4.

„ 169, Fig. 212, *for* 11·8 *read* 10·8.

„ 170, line 2 from bottom, *for* $-(9{\cdot}1 + 0{\cdot}6)\frac{25281}{60}$ *read* $-(9{\cdot}1 - 0{\cdot}6)\frac{25281}{60}$

„ 173, Fig. 216, in right-hand half of figure interchange W_1' and W_2'.

„ 174, line 14, *for* Ax *read* Ax_1.

„ 174, line 16, *for* x *read* x_1.

„ 175, last line, *for* 324 *read* 322.

„ 179, footnote, *for* add $\int \frac{dy}{dx} dx$ to each side *read* multiply each side by $\int \frac{dy}{dx} dx$.

Page 180, line 6, *for* $\frac{\pi^2}{l^2}$El *read* $\frac{\pi^2}{l^2}$EI.

„ 190, Table XL., *for* 55,150 *read* 35,168.

„ 196, line 11, *for* $\left(\frac{l}{r} - 80\right)$ *read* $\left(\frac{l}{r} - \cdot 80\right)^2$

„ 200, line 6 from bottom, *for* $\frac{1}{4}$ *read* $\frac{1}{16}$.

„ 212, line 14. *for* double *read* single.

„ 213, replace 0·628 by 1·1, and 0·8 by 1·4 thoughout page.

„ 220, line 6, *for* $3f_2 = 3$ *read* $3f_2 = 5$.

„ 220, line 6, *for* $f_2 = 1$ *read* $f_2 = 1{\cdot}66$.

„ 220, line 8, *for* $3 + 1 = 4$ *read* $3 + 1{\cdot}66 = 4{\cdot}66$.

„ 220, line 10, *for* $3 - 1 = 2$ *read* $3 - 1{\cdot}66 = 1{\cdot}34$.

„ 221, line 5 from bottom, *for* P *read* *p*.

„ 222, line 6, *for* P *read* *p*.

„ 231, line 26, *for* 104*l* *read* 140*l*.

„ 241, line 7, *for* as *read* of.

„ 255, Fig. 332, interchange b' and b'''.

„ 257, line 19, *for* x'_n *read* x'.

„ 257, line 22, *for* W_{n1} *read* W_n'.

„ 257, last line, *for* W_n *read* W_n'.

„ 262, line 18, *for* 1·02 *read* 1·25.

„ 262, line 23, *for* 1·625 *read* 5·625.

„ 262, line 28, *for* 5·5 *read* 5·6.

„ 263, line 6, *for* 1·94 *read* 0·194.

„ 263, last line, *for* × *read* =.

„ 266, line 17, *for* 9″ 6″ *read* 9′ 6″.

„ 267, line 1, *for* 5″ 4″ *read* 5′ 4″.

„ 272, Table LVIII., *for* 5·7 *read* 5·3.

„ 272, Table LVIII., *for* 10·3 *read* 10·7.

„ 273, line 17, *for* $\frac{11{\cdot}3}{2 \times \frac{1}{2}}$ *read* $\frac{11{\cdot}3}{12 \times \frac{1}{2}}$

„ 308, line 10, insert y_1 (= etc.).

„ 340, line 12 from bottom, *for* $a_1b_1 + a_{19}b_{19}$ *read* $\frac{a_1b_1 + a_{19}b_{19}}{4}$

„ 345, line 1, *for* A *read* D.

„ 346, line 1, *for* A *read* D.

„ 346, last line, *for* W *read* W_s.

„ 349, line 24, *for* left *read* right.

„ 351, line 20, *for* AB *read* AC.

„ 352, line 25, *for* H*w* *read* H_w.

„ 352, line 28, *for* Hw_1 *read* H_{w1}.

„ 353, line 4, *for* D *read* F.

„ 353, line 4, *for* Fig. 371 *read* Fig. 370.

„ 360, line 7, *for* × 5·6 + 62·5 *read* + 5·6 × 62·5.

„ 364, line 16, *for* 30 *read* 12.

„ 364, line 16, *for* 0·58 *read* 1·6.

WARREN'S "Engineering Construction."

CHAPTER I.

STRENGTH, ELASTICITY, ENDURANCE, AND SAFE WORKING STRESS IN IRON AND STEEL.

THE strength of a structure, such as a roof or a bridge, depends not only upon its form and dimensions, but also upon the material used in its manufacture. The load or loads which a structure is designed to carry produce stresses in the various members, which may be tensile, compressive, shearing, and occasionally torsional; the stresses develop resistances in the material, which are generally accompanied by slight alterations in form, such as an elongation or shortening of the member in question. The elongation or compression, as the case may be, is termed the "strain," which must not be confounded with the stress producing it. The stresses produced in the various parts of a structure depend upon the form and dimensions of the structure and the loads which it carries; but, in arranging the sections of the various members to resist the stresses developed in them, it is necessary to know the physical properties of the materials used, such as the tensile, shearing, and compressive strengths, elasticity, ductility, rate of expansion by heat, etc. In order to design a structure in an economical manner we require to know many other things, such as, in the case of wrought iron and steel, the ordinary and maximum sizes of plates and bars, and their relative cost; how best to connect the various members together, so that they may have the necessary strength and otherwise fulfil their purpose efficiently; what portions are best made of cast iron, and how such castings should be designed. If stone, brick, or concrete is used in the structure, such as for abutments and piers of bridges, it will be necessary to know how to dispose these materials to resist the stresses developed in them, and to limit the pressure on the foundations, etc. It is proposed to deal with the principles which govern the design of structures in iron, steel, and timber, and to work out examples in details of the most common of these, such as the engineer is frequently called upon to construct.

In the first place, the strength and elasticity of iron, steel, and timber will be considered. Our knowledge of the physical properties of materials is derived chiefly from experiments made with the testing-machine, which at the present time, in some form or other, finds a place in every engineering laboratory. The most important testing-machines used in England and Europe are based on the constructive principle first adopted by Mr. David Kirkaldie—that, namely, of applying the load by water-pressure, and measuring it by means of weights and levers. In some of the larger testing-machines in America, and generally in smaller machines, spur-gearing and screws are used to apply the load. It should, however, be mentioned that a large and accurate testing-machine was designed by Mr. A. H. Emery for the Watertown Arsenal, in which the power is applied by means of a hydraulic press, supplied by a set of pumps driven by a steam-engine through an accumulator. The stresses are measured by scale-beams, to which they are transmitted through a set of diaphragms and cells containing a mixture of alcohol and glycerine, and which operate as a frictionless reducing mechanism. The machine, once standardized, is said to be almost absolutely accurate. Its capability of recording accurately large and small stresses was shown when it was first used for breaking a bar of iron 5 inches in diameter, and afterwards a single horsehair. It has been since used for a variety of most valuable tests, which are recorded in the reports published each year by the United States Government.

In using a testing-machine for the determination of the tensile strength, elasticity, and ductility of a specimen of metal, it should be accurately prepared to a suitable form, such as in Figs. 1, 2, and 3; the exact sizes, however, will depend upon the machine and the method of holding the specimen.

If the specimen under consideration be iron, and it be desired to test the elasticity, it will be necessary to attach to it a piece of apparatus for measuring the small extensions produced, such as a screw micrometer or Kennedy's extensometer, which latter consists of a light lever multiplying 100 to 1. On applying the load, it will be at once observed, if the apparatus be sufficiently delicate, that the specimen stretches, and that with a load of 1 ton per square inch the stretch will be about $\frac{1}{12000}$ part of the length of the specimen under test. In Figs. 1, 2, and 3, which have a test length of 10 inches, the stretch would be $\frac{1}{1200}$ of an inch. With a load of 2 tons per square inch, the elongation will be $\frac{3}{1200}$ of an inch; and with 3, 4, 5, 6, 7, 8 tons, the elongations will be 3, 4, 5, 6, 7, and 8 times $\frac{1}{1200}$ of an inch respectively, and generally the elongations are sensibly proportional to the loads producing them. In other words, the specimen is said to be perfectly elastic for these loads; which is shown by releasing the pressure, when

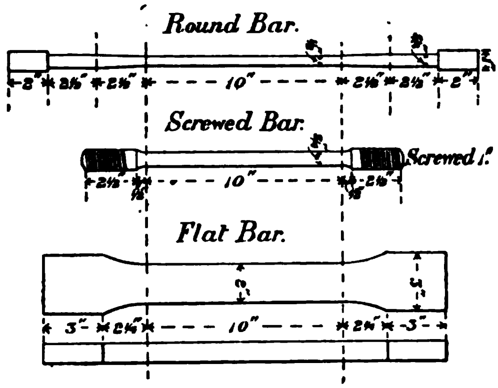


FIG. 1.

FIG. 2.

FIG. 3.

the specimen springs back to its original length of 10 inches. But this so-called elasticity has a limit, which in wrought iron is about 12 tons, and in mild steel about 18 tons, per square inch, after which the elongations increase much more rapidly than the loads producing them, and the material behaves as if it were plastic until the specimen fractures, which generally occurs at from 20 to 24 tons per square inch for iron, and from 24 to 30 tons per square inch for mild steel; the elongations at the point where the material apparently ceases to be elastic being about $\frac{1}{100}$ of an inch, whereas the total elongation at fracture will be from 1 to 1½ inch for ordinary iron, and from 2 to 3 inches for mild steel, measured on a length of 10 inches. The apparatus used for measuring the small elongations is removed after the elastic limit has been determined, otherwise it might be injured when the specimen fractures. The ductility of the specimen is measured by the total percentage of elongation, and the percentage of contraction of the fractured area.

The term "modulus of elasticity" is used to denote the result found by multiplying the stress per square inch by the original length of the specimen, and dividing by the elongation. Thus it has been stated that a stress of 1 ton per square inch will produce an elongation of $\frac{1}{1200}$ of an inch on a length of 10 inches, which gives the modulus of elasticity 26,880,000 lbs. per square inch. The modulus of elasticity may also be defined as the ideal stress which would be capable of stretching a perfectly elastic bar to double its length; it may be calculated from the following formula:—

$$E = \frac{WL}{kl}$$

where E = modulus of elasticity.
W = the load producing the elongation l.
L = the original length of the specimen.
k = the area of the specimen.

This is Young's modulus, or the coefficient of direct elasticity. The coefficient of transverse elasticity is derived from experiments on loaded beams, and will be referred to in connection with the deflection of beams. The modulus of elasticity is an important factor in all calculations where the stress is determined from the strain, and it will be used in connection with the deflection of beams, continuous girders, and arched ribs. For its exact determination, very delicate instruments are necessary;[1] the same remark applies to the determination of the elastic limit, which will now be considered more closely. The method described for testing a piece of iron or steel and determining its so-called elastic limit is that usually adopted in the commercial testing of materials; what is really found is better defined as the "yield-point." It is well known that the yield-point can be raised by mechanical means, that the application of a stress greater than the yield-point raises the yield-point, and that it may be artificially raised almost to the breaking-point. With very delicate instruments a permanent set is observable with stresses well within the yield-point, and the stress fixed upon as the elastic limit depends upon the

[1] See Unwin, "The Testing of the Materials of Construction."

delicacy of the instruments used in determining it. The stretching of a bar within the yield-point consists partly of an elastic extension and partly of a permanent set, and it is this permanent set which makes it so extremely difficult to determine the true elastic limit.

Professor Bauschinger has shown that these artificially raised yield-points are extremely unstable, and may be lowered considerably by hammering the test-bar on the end and reloading it; and that, moreover, the yield-point cannot be raised in tension without at the same time lowering the yield-point in compression. When a bar is subjected to stresses alternating between tension and compression the elastic limit cannot be raised, and the yield-point settles down to the true limit of elasticity. Professor Bauschinger further points out that ordinary materials of construction have their yield-points artificially raised in the process of manufacture, and proves by a most elaborate series of experiments that the true elastic limit—which he still defines as the stress beyond which the strains cease to be proportional to the stresses producing them —can be correctly ascertained by first subjecting the bar to a series of stresses alternating between tension and compression; the limit then decreases to a value not differing appreciably in tension and compression, and below the initial elastic limit or yield-point. The elastic limit found in this way is about 8 tons for wrought iron, and 9½ tons for mild steel. The stretching which occurs at the yield-point for hard and soft steels does not differ materially, from which it is inferred that hard and soft materials may be relied upon to work together in a built-up structure, under ordinary working stresses. The same has also been observed in the case of iron.

It is very desirable, in all important tests of materials, to have a record automatically registered by the machine itself; such an apparatus is called an autographic stress-strain apparatus, because it draws a diagram which shows the strain produced by stresses which vary from nothing to that required to break the bar. Various forms of this apparatus exist, but the one designed by Professor Kennedy produces very perfect diagrams, although it is not the most convenient to handle.

The form of the diagram for a piece of mild steel is shown in the woodcut, Fig. 4, from which it will be seen that the extension produced by a given load is represented as an abscissa, while the load itself is represented as a curved ordinate. The diagram represents the behaviour of the specimen during the test, and shows clearly the limit of elasticity, the maximum load, and the elongation. The curve not only indicates the yield-point, and the amount of extension which occurs at this point, but it is seen by inspection that the local extension which occurs at the breaking-point is measured by drawing ordinates at the commencement and termination of the curve drawn during the time that the specimen is undergoing local extension. Again, the area of the diagram represents the gross mechanical value of the material, as it represents the work done in breaking the bar, which of course depends upon its breaking-strength and ductility.

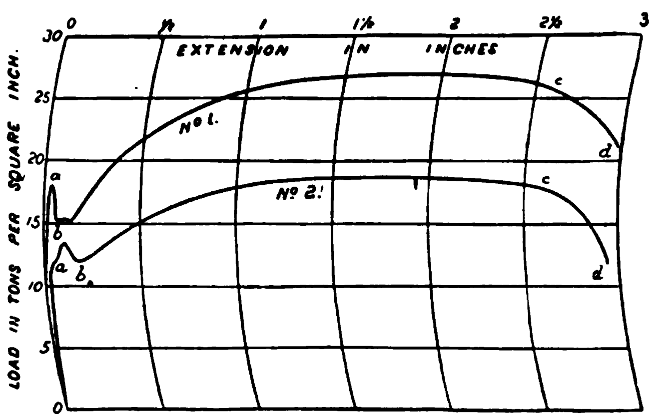


FIG. 4.

The principle of the apparatus is as follows: The test-piece is placed in the machine with a stronger bar, which is called a spring-piece. The material of this bar must be ascertained by previous experiments to be perfectly elastic, so that its extensions are strictly proportioned to the pull on the test-piece; and, moreover, it should be of such an area that its limit of elasticity occurs only at a load greater than that which will break the test-piece. By a simple arrangement a very light pointer is made to swing about an axis through an angle proportional to the pull on the test-bar. The end of this pointer in its motion always touches a piece of smoked glass, to which is given a travel in its own plane proportional to the extension of the test-piece. In this way the diagram is drawn. After the test the glass is varnished to fix the black, and the necessary particulars about the test are written on it with a scriber. The glass is then used as a negative, and copies produced by photography.

Professor Unwin's autographic stress-strain apparatus consists of a revolving drum, whose angular displacement is proportional to the position of the poise-weight which denotes the load on the specimen. The extension produced by the load is recorded by means of a wire passing over pulleys and connected with the test-piece; a pencil attached to this wire draws the diagram.

Mr. Wickstead and others, including the author, have devised autographic apparatus which, like Professor Unwin's, show the limit of elasticity, ultimate strength, and total extension; but the portions of the diagrams recorded by Kennedy's apparatus from *a* to *b* and *c* to *d*, Fig. 4, is much more perfect than in any of the others.

It is well known that the form and dimensions of the test-piece have a very marked effect upon the results obtained in testing (see Hackney, "Forms of Test-pieces," *Proc. Inst. C.E.*, vol. 76). High percentages of elongations may be obtained from short or thick test-pieces; long and thin test-pieces give much lower percentages of elongation for the same material.

The tests intended to govern the quality of the material for a particular purpose will next be considered.

In testing wrought iron and steel intended to be used in engineering construction, it is at least necessary to determine the strength and ductility. The ductility is usually ascertained by measuring the percentage of elongation in the manner already described, or by the percentage of contraction of the fractured area. The contracted area is measured most conveniently by means of micrometer callipers. The strength alone, as first pointed out by Mr. Kirkaldy, is no indication of the quality of the material. "A high breaking-strength may be due to the iron being of a superior quality, dense, fine, and moderately soft, or simply to its being hard and unyielding. A low breaking-strength may be due to looseness and coarseness in the texture, or to extreme softness, although very close and fine in quality. The contraction of area at fracture forms an essential element in estimating the quality of a specimen, and by comparing the breaking-strength with the contraction of area at fracture the respective merits of various specimens can be correctly ascertained."

The contraction of area can generally be measured with sufficient accuracy in round specimens, but in the case of flat specimens, especially very broad, thin strips, it cannot be measured with sufficient accuracy; and when the fracture is oblique, which is often the case, the difficulty is increased. The contraction of area is also largely influenced, as stated by Professor Unwin, by local conditions of hardness and homogeneity at point of fracture.

It is in consequence of the difficulty in measuring accurately the contraction of area at fracture, that many competent authorities have advocated its omission, in specifications of tests of materials, in favour of elongation; but here also a difficulty exists. The elongation consists of two parts—namely, general and local. The general extension in a specimen continues so long as it offers increased resistance to the force producing it, and is proportional to the length of the specimen; but the local extension commences after the general extension has ceased, and is most decided in all ductile materials, such as steel. It is confined to the portion immediately adjacent to the fractured area. The local extension is, in fact, proportional to the contraction of area of the specimen.

Although it is usual to measure the total extension on a specimen, and to express it in percentage of length, the more scientific way, which has been suggested by Professors Unwin, Barba, and Wickstead, is to separate the general extension from the local.

Mr. Wickstead described, at the meeting of the British Association for 1890, a method of doing this from the autographic record, and recommended a column in the test-sheet of "Percentage of General Extension," in addition to the usual columns—namely, "Percentage of Contraction of Area," and "Percentage of Total Extension." The local extension can then be expressed by subtracting the general extension from the total extension.

The local extension is seen by dividing the test-piece before testing, over the length of say 10 inches, into ten equal parts, each 1 inch long; the elongation remeasured after testing will be much greater on the 2 inches or 3 inches which include the fracture than over equal lengths measured on the remaining portion of the 10 inches.

Figs. 5 and 5*a* show a bar 30 inches long, divided into thirty equal parts, with the re-measured lengths after testing. Here the local extension is very decided.

In autographic stress-strain diagrams, such as those produced by means of Professor Kennedy's apparatus, the local extension is easily separated from the general extension; and when this is done it is possible to eliminate the effect of different proportions in regard to the length and area of cross-section, on the percentage of total extension.

The ductility of a specimen can also be ascertained by bending round a bar of given radius. But here, again, the proportions of the test-piece exercise a decided influence on the angle bent through before fracture, which measures the ductility.

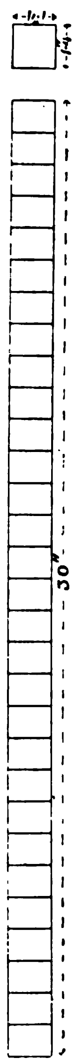


Fig. 5.

Fig. 5a.

In order to secure a suitable material for railway axles, it is usual to specify, in addition to the ordinary tests for tensile strength and ductility made on specimens cut from the axle, that the axle itself should be tested to destruction by a series of blows produced by a falling weight, the axle being reversed after each blow. The results of the drop-test—which this test is called—on axles, when the experiments are carefully conducted, give a fairly close approximation to the endurance which may be expected from similar axles under the conditions existing in ordinary railway practice. The French make use of the drop-test, not only for axles, but for small samples of the materials used in the construction of guns.

In general it may be stated that the conditions under which a given material is tested should conform as nearly as possible with those existing in the structure, machine, rail, tire, axle, or gun of which it forms a part.

Cast Iron.—There is no definite modulus of elasticity for cast iron; its apparent modulus is about the same in tension and compression, and varies from about 5000 to 6000 tons per square inch.

The tensile and compressive strengths depend upon its chemical composition and specific gravity, the best irons containing more combined carbon and manganese, and less graphite, silicon, and phosphorus. Professor Thurston gives the following figures for the tensile strength of good iron :—[1]

TABLE I.

TENACITY OF GOOD CAST IRON.

Kind.	Tenacity in tons per square inch.	Specific gravity.
Good pig-iron	8·9	7·10
Tough cast iron	11·2	7·22
Hard cast iron	13·4	7·28
Good tough gun-iron	13·4	7·25

Professor Unwin considers that the tensile tests give much better indications of the quality of the material for structural purposes than do the compressive tests.[2]

Professor Thurston states that the best castings will have a maximum resistance to compression when the specific gravity is slightly greater than that which gave the highest results in tension. The resistance to compression of ordinary cast iron varies from 38 to 48 tons per square inch. Professor Thurston, however, states that cast iron for ordinary work, if subjected to compression, should have a specific gravity of 7·26 to 7·28, and a compressive strength of 70 tons per square inch.

In order to secure a suitable material for cast-iron girders, it is usual to specify that test-bars 1 inch wide by 2 inches deep should, when supported at points 3 feet apart, bear a central load of from 2500 to 3500 pounds; the mean value, 3000 pounds, will represent the average quality. The deflection at the point of fracture should be not less than $\frac{3}{10}$ of an inch. The resistance of cast iron to shearing is probably about 5 tons per square inch.

Wrought Iron and Steel.—The following table is taken from Professor Unwin's "Elements of Machine Designs," pp. 40 and 41 :—

[1] "Materials of Engineering" (Thurston).
[2] "Materials of Construction" (Unwin).

TABLE II.

THE ULTIMATE AND ELASTIC STRENGTHS OF MATERIALS AND COEFFICIENTS OF ELASTICITY, IN POUNDS PER SQUARE INCH.

Material.	Breaking strength.			Elastic strength.			Coefficient of elasticity.	
	Tension.	Pressure.	Shearing.	Tension.	Pressure.	Shearing.	Direct.	Transverse.
	67,000	—	49,000	—	—	—	31,000,000	—
Wrought-iron bars	57,600	50,000	40,000	30,000	30,000	22,000	29,000,000	10,500,000
	33,500	—	22,400	—	—	—	27,000,000	—
Iron ship plates, ‖	49,000	—	—	—	—	—	—	—
Iron boiler plates, ‖	47,000	—	36,000	24,000	24,000	15,000	26,000,000	14,000,000
„ „ „ ⊥	41,500	—	—	—	—	—	27,000,000	—
Steel plates, ¼ % carbon	65,000	—	50,000	42,000	38,000	—		—
„ „ ½ % „	78,000	—	56,000	47,000	49,000	—	31,000,000	13,000,000
„ „ 1 % „	110,000	—	83,000	67,000	71,000	—		—
Steel boiler plates „	66,000	—	56,000	36,000	—	—	30,000 000	13,500,000
Rivet-steel	65,000	—	55,600	46,000	—	—	30,000,000	13,000,000
	150,000	—	—	—	—	—	—	—
Cast steel untempered	120,000	—	—	80,000	80,000	64,000	30,000,000	12,000,000
	84,000	—	—	—	—	—	—	—
Steel castings from	63,000	—	—	34,000	—	—	30,000,000	—
Steel castings to	34,000	—	—	20,000	—	—	20,000,000	—

The following table is given to show what tests may be specified for wrought iron :—

Table III.

SHOWING STRENGTH AND DUCTILITY OF IRON.

Description of material.	Tensile strength in tons per square in.	Elongation on 10 in. per cent.	Contraction of area at fracture per cent.	Remarks.
Bars—rounds, squares, and flats	22 to 23	9	20	Equivalent to "best" Staffordshire iron, suitable for ordinary girders, tanks, and the cheapest bridge work.
Angles, T's, and channels ...	22	10	18	
Plates $\frac{1}{4}$ to $\frac{5}{8}$ inch thick tested along the fibre	21	10	10	
Plates $\frac{1}{4}$ to $\frac{5}{8}$ inch thick tested across the fibre	18	3	5	
Bars—rounds, squares, and flats up to 4 square inches sectional area	23 to 24	18 to 22	22 to 25	Equivalent to "best best" Staffordshire iron, suitable for railway bridges and roofs.
Angles, T's, and channels ...	22	10 to 12	20	
Plates $\frac{1}{4}$ to $\frac{5}{8}$ inch thick tested along the fibre	22	10	12	
Plates $\frac{1}{4}$ to $\frac{5}{8}$ inch thick tested across the fibre	18	4 to 5	6	
Bars—rounds, squares, and flats up to 4 square inches sectional area	23 to 24	25 to 30	30 to 40	Equivalent to treble-best Staffordshire iron, suitable for railway draw-bars, centre chains, and best smith's work, chain cables.
Angles, T's, channels, and other sections	23	15	30 to 35	
Plates from $\frac{1}{4}$ to $\frac{5}{8}$ inch thick tested along the fibre ...	22	12	15 to 18	
Plates from $\frac{1}{4}$ to $\frac{5}{8}$ inch thick tested across the fibre ...	18 to 19	7	$7\frac{1}{2}$	

The elongations per cent. measured on 8 inches would be slightly greater for reasons already given.

Treble-best plates about 4 feet wide and from 10 to 12 feet long will show the same strength when tested across the fibre for thicknesses varying from $\frac{3}{8}$ to 1 inch ; the ductility, however, as measured by the contraction of area, will vary from 18 to 6 per cent. with the fibre, and from 9 to 3 per cent. across the fibre. Wide plates show greater strength and ductility when tested across the fibre than narrow plates.

The best Yorkshire brands of iron plates, which are superior in uniformity, ductility, and homogeneity to the treble-best Staffordshire, give 22 tons tensile strength and 20 per cent. contraction of area when tested with the fibre, and 20 tons tensile strength and 12 per cent. contraction of area across the fibre, provided that the plates tested are from 6 to 10 feet long, and from 3 to 5 feet wide. Longer and narrower plates give higher results when tested along the fibre, but lower results when tested across the fibre. Yorkshire iron is largely used for manufacture of rivets, railway axles, and superior smith's work; it gives excellent results when subjected to cold bending or to the drop-test.

The British Admiralty cold and hot forge tests for ductility are perhaps the most elaborate. They are as follows: *For plates.* The portion of the plates to be tested in both hot and cold tests to be 18 inches by 10 inches, cut both along and across the fibre, and bent on a cast-iron slab having a fair surface, and an edge at right angles, with the corner rounded off to half an inch radius. The plates should be bent at a distance of from 3 to 6 inches from the edge without fracture through the following angles:—

Hot-forge tests for double-best iron, 120° with the fibre, and 100° across the fibre; for single-best iron, 90° with the fibre, and 60° across the fibre.

The plates must bend cold through the angles given in the following table, according to the thickness:—

TABLE IV.

COLD-FORGE TESTS.

Thickness of plate in inches.	First class, or double-best.		Second class, or single-best.	
	With the fibre.	Across the fibre.	With the fibre.	Across the fibre.
	Through an angle of		Through an angle of	
	Degrees.	Degrees.	Degrees.	Degrees.
1	15	5	10	—
$\frac{15}{16}$	15	5	10	—
$\frac{7}{8}$	20	$7\frac{1}{2}$	15	—
$\frac{13}{16}$	20	$7\frac{1}{2}$	15	—
$\frac{3}{4}$	$22\frac{1}{2}$	10	$17\frac{1}{2}$	5
$\frac{11}{16}$	25	10	20	5
$\frac{5}{8}$	$27\frac{1}{2}$	$12\frac{1}{2}$	$22\frac{1}{2}$	$7\frac{1}{2}$
$\frac{9}{16}$	30	$12\frac{1}{2}$	25	$7\frac{1}{2}$
$\frac{1}{2}$	35	15	30	10
$\frac{7}{16}$	$42\frac{1}{2}$	$17\frac{1}{2}$	$37\frac{1}{2}$	$12\frac{1}{2}$
$\frac{3}{8}$	50	20	45	15
$\frac{5}{16}$	60	25	55	$17\frac{1}{2}$
$\frac{1}{4}$	70	30	65	20
$\frac{3}{16}$	90	40	75	30

The Admiralty standard for best and best-best is about equivalent to the Staffordshire.

The hot-forge test for angle-iron requires that it should be bent thus: 

and flattened thus:  and the end bent over thus: 

The cold-forge test requires that it should be notched and broken across cold to show the quality of the iron, and that one flange should be cut off and bent cold to show the quality of the fibre.

T-iron should be tested hot by being bent thus:

The cold-forge tests are the same as for angle-iron.

For bulb-iron the hot-forge test requires that the bulb should be cut off, and the web bent across the grain thus: 

For the cold-forge test, the bulb should be notched on one side and broken cold to show the quality of the fibre.

Angle bulb-iron should be tested hot by cutting off the bulb and testing the remainder in the same manner as angle-iron; and the bulb should be notched on one side and broken cold to show the quality of the fibre.

T bulb-iron should be tested after the bulb has been cut off in the same manner as T-iron, and the bulb should be tested in the same manner as in angle bulb-iron.

Channel-iron should be tested hot by being bent thus:

and one of the flanges should be cut off and bent cold as in the test for angle-iron. A sample should also be notched and broken cold to show the quality of the fibre.

The following are the British Admiralty tests for mild steel plates, beams, angles, bulbs, and bars. Strips cut lengthways or crossways to have an ultimate tensile strength of not less than 26 and not exceeding 32 tons per square inch, with an elongation of 20 per cent. measured on a length of 8 inches; the beam, angle, bulb, and bar steel to withstand such forge tests as may be sufficient to prove soundness and fitness. Strips cut lengthways or crossways, 1½ inch wide, heated uniformly to a low cherry red, and cooled in water at 82° Fahrenheit, must stand bending in a press to a curve of which the inner radius is one and a half times the thickness of the steel tested. The strips for bending should be planed on the edges, and the sharp edges taken off.

The percentage of elongation in the Admiralty test might just as well have been measured on a length of 10 inches. Rivet-steel, being softer and more ductile, may be taken at between 24 and 30 tons per square inch for tensile strength, and 25 per cent. elongation. The steel used in the construction of tension members in the Forth Bridge was specified to have a strength of from 30 to 33 tons, with 20 per cent. elongation; for tubular columns and struts, the steel was specified to have a tensile strength of from 34 to 37 tons per square inch, and an elongation of 17 per cent. The elongations were specified to be measured on 8 inches.

Working-stress and Factor of Safety.—Working-stress signifies the intensity of stress—generally expressed in tons per square inch—to which a piece of material may be subjected without ceasing to fulfil its purpose efficiently under the conditions on which the stresses are applied.

The factor of safety is the ratio of the ultimate strength to the working-load. Sir William Fairburn proved that a riveted girder, loaded to one-third of the load which would have broken it if gradually applied, failed after 313,000 applications of this load.

The experiments of Herr Wöhler and Professor Spangenberg demonstrate the following law, known as "Wöhler's Law:" "Rupture may be caused, not only by a steady load which exceeds the carrying-strength, but also by repeated applications of stresses none of which are equal to this carrying-strength. The difference of these stresses are measures of the disturbance of the continuity, in so far as by their increase the maximum stress which is still necessary for rupture diminishes."

Professor Bauschinger has made a long series of experiments which confirm those made by Herr Wöhler. Professor Bauschinger's experiments are the most valuable on this subject which have ever been made, and the results are summarized in Table V.

In England, Sir B. Baker has made various experiments which also confirm Wöhler's original experiments. For example, he found that when a shaft was loaded with one-half its gradually applied breaking-weight, and set rotating, about five thousand reversals of stress produced fracture. He mentions an experiment with a bar of cast iron loaded with a weight which, according to Fairbairn's experiments, it should have carried for a long series of years, broken in two minutes when set gently rotating; also, a bar of fine tough steel so changed in constitution at fracture after a few months' rotation as to offer no advantage over a new cast-iron bar of the same section. Sir B. Baker has proved, by experimenting on flat bars of steel by repeatedly bending them, and subsequently testing them in direct tension and direct crushing, that the effect of repeated stresses is more prejudicial in tension than in compression.

The change which a piece of material undergoes when subjected to repeated stresses has been termed "fatigue." It is most marked in the case of stresses alternating between tension and an equal compression, as seen in the fracture of railway axles.

The strength of a piece of material when subjected to a gradually applied load, as in a testing-machine, is termed its "statical strength." When subjected to a load which is entirely removed before being reapplied, the load which will ultimately break the piece is termed its "primitive strength." When subjected to stresses which alternate between tension and an equal compression, the ultimate breaking-stress is termed its "vibrating-strength." Approximately, the vibrating-strength is to the primitive is to the statical as 1 is to 2 is to 3.

The values of the strengths for a variety of materials are recorded in Table V.

Several methods have been proposed which have for their object the representation of the results of the experiments made by Wöhler and Bauschinger, and their extension to the various ranges of stress which occur in engineering practice, among which may be noticed Gerber's parabola. If the ranges of stress given in Table V. are plotted as ordinates, and the

Table V.

Bauschinger's Endurance Tests.

Stresses requiring fifteen million repetitions to cause fracture. Tons per square inch.

Material.	Opposite stresses.		One stress zero.		Similar stresses.		Range zero. Ultimate statical strength.
	Least.	Greatest.	Least.	Greatest.	Least.	Greatest.	
Wrought-iron plate	− 7·15	+ 7·15	0	13·1	11·4	19·2	22·8
Bar iron	− 7·85	+ 7·85	0	14·4	13·3	22·02	26·6
Bar iron	− 8·65	+ 8·65	0	15·75	13·2	21·92	26·4
Bessemer mild-steel plate	− 8·55	+ 8·55	0	15·7	14·3	23·8	28·6
Steel axle	−10·5	+10·5	0	19·7	20·0	32·1	40·0
Steel rail	− 9·7	+ 9·7	0	18·4	19·5	30·85	39·0
Mild-steel boiler plate	− 8·65	+ 8·65	6	15·8	13·3	22·55	26·6

Limits of Stress from Wöhler's Endurance-Test.

Stresses in tons per square inch for which fracture occurs only after an indefinitely large number of repetitions.

Material.	Opposite Least.	Opposite Greatest.	One stress zero Least.	One stress zero Greatest.	Similar Least.	Similar Greatest.	Range zero. Ultimate statical strength.
Wrought-iron plate ...	− 8·6	+ 8·6	0	15·25	12·0	20·5	22·8
Krupp's axle steel ...	−14·05	+14·05	0	26·5	17·5	37·75	52·0
Untempered spring steel	−13·38	+13·38	0	25·5	12·5	34·75	57·5

minimum stress as abscissæ, the points fall on a parabolic curve, which Professor Unwin expresses thus:

Let f max. and f min. denote the limits of stress, and Δ the range of stress; then—

$$\Delta = f \text{ max.} \mp f \text{ min.}$$

The upper sign is to be taken when the stresses are of like kind, and the lower sign when they are of opposite kind, as in alternating stresses. Let f denote the statical breaking-strength; then the equation to Gerber's parabola is—

$$\left(f \text{ min.} + \frac{\Delta}{2}\right)^2 + k\Delta = f^2$$

If the statical strength f be known, and the value of f min. and f max. for any range of stress at which the bar stands a practically unlimited number of repetitions before breaking, then k can be determined, and the limits of stress for all conditions of loading can be calculated. The parabolas are drawn from the above equation, using the results recorded in Table V.

Soon after Wöhler's results were published, Professor Launhardt published a formula which applies to the cases in which the stresses are either tensile or compressive, which may be represented as follows :—

Let b denote the breaking-strength.
s ,, ,, statical strength.
p ,, ,, primitive strength.

$$\text{Then } b = p + (s - p)\frac{f\text{ min.}}{f\text{ max.}}$$

If, in the end lattice-bar of a bridge, the stress produced by

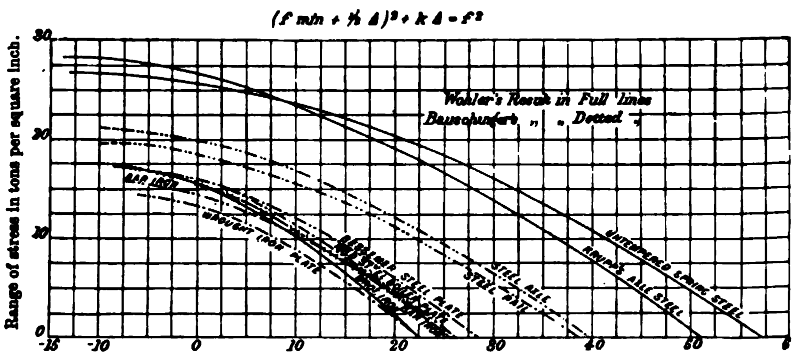


FIG. 6.—Wöhler's and Bauschinger's endurance tests.

the live load were nine times the stress produced by the dead load, then—

$$\frac{f\text{ min.}}{f\text{ max.}} = \frac{1}{10}$$

and, using the results given in Table V. for wrought iron, we have—

$$b = 13{\cdot}10 + (22{\cdot}8 - 13{\cdot}1)\tfrac{1}{10} = 14{\cdot}07 \text{ tons}$$

so that 14·07 tons is the ultimate breaking-strength of the bar; and, if the working-stress be taken at 4·69 tons per square inch, the factor of safety is—

$$\frac{14{\cdot}07}{4{\cdot}69} = 3 \text{ (not } \frac{22{\cdot}8}{4{\cdot}69} = 4{\cdot}6)$$

In order to meet the cases which include stresses alternating between tension and compression, Professor Weyrauch proposed the following formula, in which the vibrating-strength is denoted by v—

$$b = p - (p - v)\frac{\text{max. B}}{\text{max. B}_1}$$

If the greatest tension on a bar be 5 tons, and the greatest compression 10 tons, then—

$$\frac{\text{max. B}}{\text{max. B}_1} = \tfrac{5}{10} = \tfrac{1}{2}$$

and, using the same material as before, we have—

$$b = 13{\cdot}10 - (13{\cdot}10 - 7{\cdot}15)\tfrac{1}{2} = 10{\cdot}125 \text{ tons}$$

and the factor of safety, with a working-stress of 3·375 tons per square inch, is—

$$\frac{10{\cdot}125}{3{\cdot}375} = 3 \text{ (not } \frac{22{\cdot}8}{3{\cdot}375} = 6{\cdot}4)$$

The effect of "fatigue" is considered to be purely local, as it is not possible to discover any change in strength, elasticity, or ductility in material which has been fractured in this way, by retesting it in the ordinary way by means of the testing-machine.

There is no difference in the results obtained from testing specimens cut from an axle broken in ordinary use, or by means of the drop-test, than would be obtained from testing specimens cut from the axle when new. Retesting specimens cut from old structures does not show any measurable change in the material. It is known, however, that the fracture of specimens by repeated stresses in Wöhler's machines and that of railway axles from the fatigue and impact occurring in ordinary work are short and crystalline, showing no sign of ductility; hence the reason given by Professor Unwin, that fatigue is primarily a loss of power of yielding in the particles near the plane of weakness at which fracture occurs. There is, however, no evidence to show that materials which were originally fibrous and ductile may become granular and crystalline when subjected to such conditions as exist in a railway axle, excepting in the plane of the fracture itself.

As stated by Kirkaldie, the crystalline appearance appears to be due to the fibres being broken at right angles to their length, whereas in a gradual fracture time is given for the fibres to stretch.

The question naturally arises whether the phenomena observed in the Wöhler-Bauschinger tests are due to fatigue or impact, or to both these causes combined. We are certainly not able to assert that the impact of the load did not contribute to some extent in causing the bars to fail with a reduced load.

It is known that the sudden application of a load to a beam produces, for an instant only, twice the deflection as the same load applied gradually. The deflections of railway bridges with live loads passing over them are about 20 per cent. greater than with the same load standing on the bridge. The bridge appears to recover from the first effect of the live load as it advances and covers the whole bridge, but the dynamic effect is, however, most probably much greater than 20 per cent., as this only includes the average of the vertical movements, some of which may exceed this amount, while others may be less, while the horizontal vibrations are not included at all.

If we apply the Wöhler-Bauschinger results to railway bridges, we may imagine trains to pass over them in rapid succession without appreciable impact or friction, and, if the results are to form the basis of the determination of working-stresses, without a separate allowance for impact, we must assume that the millions of repetitions in these experiments are about equivalent to the effect of impact and the slower repetitions of stresses which occur in railway bridges.

The rule which still exists in the regulations of the British Board of Trade for railway bridges is simply to limit the working-stresses in iron to 5 tons per square inch in tension and 4 tons in compression. For steel the limiting stress is 6½ tons. No account is taken of the variable range of stress or of impact. The rule obviously gives excessive strength for those parts of a bridge where the range of stress is small, as in plate web box-girders of 150 feet span, while it would be dangerous if applied to those parts of structures where both the range of stress and the impact are large, as in the longitudinals, floor beams, suspenders, and other members of bridges liable to sudden loading.

In America and Europe, and to some extent in England, the practice is to make the working-stress depend upon the range of stress, or upon the impact, or both these causes combined. Thus in the New York Elevated Railroad the flanges of the girders were designed for a stress of 3·6 tons per square inch, the web bracing for 3·4 tons per square inch, and for members subjected to alternating stresses a stress of only 2 tons per square inch was allowed.

In the numerous elaborate specifications which have been written by American engineers to govern the design of important railway bridges,[1] rules are given for limiting the intensity of the working-stresses in various parts of the structure more or less in accordance with the impact which they are likely to sustain and the results given in Table V. The following table was prepared by Mr. J. A. Macdonald, engineer for bridges, New South Wales, for general office use in connection with the design of highway bridges; it gives the ultimate breaking-strength for various ranges of stress as given by Launhardt's and Weyrauch's formulæ.

TABLE VI.

STRENGTH OF WROUGHT IRON UNDER VARYING STRESS FOR IRON HAVING A STATICAL STRENGTH OF 21 TONS.

When $\frac{\text{min.}}{\text{max.}}$	= 1·0,	ultimate strength =	20·1	tons per square inch.
"	0·9	"	19·4	"
"	0·8	"	18·7	"
"	0·7	"	18·1	"
"	0·6	"	17·4	"
"	0·5	"	16·7	"
"	0·4	"	16·1	"
"	0·3	"	15·4	"
"	0·2	"	14·7	"
"	0·1	"	14·1	"
"	0·0	"	13·4	"
"	− 0·1	"	12·7	"
"	− 0·2	"	12·0	"
"	− 0·3	"	11·4	"
"	− 0·4	"	10·7	"
"	− 0·5	"	10·0	"
"	− 0·6	"	9·4	"
"	− 0·7	"	8·7	"
"	− 0·8	"	8·0	"
"	− 0·9	"	7·4	"
"	− 1·0	"	6·7	"

NOTE.—Ordinary factor of safety = ⅓.

Similar tables could be prepared, if thought desirable, for mild steel.

The working-stress for cast iron may be taken as follows, if the statical strength is taken as 10 tons per square inch:—

[1] Mr. Theodore Cooper's specifications, for example.

For a dead load, 2·50 tons per square inch.

For a live load, 1·20 tons per square inch.

For alternating stresses equal in amount, 0·83 tons per square inch.

Intermediate cases may be treated in the same way as for wrought iron and steel.

Launhardt's and Weyrauch's formulæ may be expressed in a more simple manner thus—

$$b = \left(1 + \tfrac{1}{2}\frac{\text{min.}}{\text{max.}}\right) \text{constant}$$

Some American engineers use this form of the formula for determining the working-stresses in bridges, including both fatigue and impact; thus for wrought-iron eye-bars—

Intensity of working stress in pounds per square inch

$$= 10{,}000\left(1 + \tfrac{1}{2}\frac{\text{min.}}{\text{max.}}\right)$$

Others allow more for the impact of the live load, and use the following formula:—

$$\text{Intensity} = 7500\left(1 + \frac{\text{min.}}{\text{max.}}\right)$$

In the new regulations[1] issued by the French Government for ensuring the safety of bridges, the following rules are given for determining the admissible stresses in the various members.

The maximum allowable stress for wrought iron in tons per square inch is—

$$\text{Intensity} = 3{\cdot}81 + 1{\cdot}9\frac{\text{min.}}{\text{max.}}$$

This may be written—

$$b = 3{\cdot}81\left(1 + \tfrac{1}{2}\frac{\text{min.}}{\text{max.}}\right)$$

For steel—

$$\text{Intensity} = 5{\cdot}08 + 2{\cdot}54\frac{\text{min.}}{\text{max.}} = 5{\cdot}08\left(1 + \tfrac{1}{2}\frac{\text{min.}}{\text{max.}}\right)$$

By taking account of the change of sign when the stresses change from tension to compression, the above rules will, according to the new regulations, be applicable to members subject to alternating stresses.

[1] *Engineering*, January 5, 1892.

CHAPTER II.

STRENGTH AND ELASTICITY OF TIMBER.

The strength and elasticity of timber is largely influenced by a variety of circumstances, such as the climate, kind of soil, whether grown on mountain ridges or on low-lying ground, the time when the tree was felled, the age of the tree, the seasoning, etc. A much greater variation is observed in the results of testing timbers than occurs in the case of iron and steel, and timbers of the same name differ widely in their physical properties. In testing timber it is well known that specimens of large scantling give much lower results than specimens of small scantling, which latter, however useful they may be in showing the relative values of different timbers, are of very little value in furnishing data for application to large-sized scantlings.

In testing timber in tension it is very difficult to get trustworthy results with soft woods, and the shoulders of the specimen held in the shackles of the testing-machine must be, according to Professor Lanza, at least five times as long as the length under test in order to prevent shearing along the fibre, even when the stress is applied axially, and then considerable lateral pressure must be applied to the portions held in the shackles. The sectional areas of the pieces tested by Lanza were about one square inch. Tensile tests of timber are of very little practical value, as the timber more frequently fails by transverse, compressive, or shearing stress, as they are less than that required in tension. The author has tested nearly all the varieties of Australian timbers in tension, but these are hard woods, and may be tested by preparing them to a similar form to that generally used for testing round specimens of wrought iron and steel. The average result of testing red pine and spruce in tension from specimens cut from the circumference and heart of the tree is about 10,000 lbs. per square inch, while most of the Australian timbers exceeded 20,000 lbs. per square inch. There is no defined modulus of direct elasticity for timber, the slight stretching which occurs being exceedingly irregular.

Compressive Strength and Elasticity. — The compressive strength of timber is useful not only in designing timber compression members in a structure, but, according to Professor Bauschinger, in determining the quality of the timber, and the influence of seasoning, of the time of felling, and other circumstances of its growth, as the tests are easily made and the results trustworthy. He recommends that prisms one and a half times the length of the side should be properly cut from the various parts of the tree to give a true average, and then tested at a standard dryness, which he fixes as 15 per cent. of moisture.

Professor Lanza[1] has tested a large number of American timbers of large scantling in 12-feet and 2-feet lengths, of circular and rectangular sections, with the results that the columns all gave way by direct crushing, the strength of the columns being found, in the proportions experimented upon, by multiplying the sectional area by the crushing strength, the lateral deflections observed being too small to exert any appreciable effect. The averages of these experiments are as follows :—

TABLE VII.

Name of timber.	Sectional area in square inches.	Ultimate strength in pounds per square inch.	Modulus of elasticity in pounds per square inch.
Yellow pine	42 to 102	4544	1,996,351
White oak	32 to 93	3470	1,398,908
Old and seasoned white oak	23 to 87	3957	1,817,539

Another series of tests made at the Watertown Arsenal, and the averages plotted by Mr. Edward F. Ely, gave the following rules for the breaking-strength of columns of white and yellow pine with flat ends, the load being evenly distributed over the ends :—

[1] Lanza, "App. Mech.," p. 669.

Let A = area of section in square inches.
f = constant whose value is given in the table following.
r = the ratio of length to least side of rectangle, all the tests having been made on rectangular sections.

Then—

Breaking strength $= f\text{A}$

where f has the following values :—

TABLE VIII.

White pine.		Yellow pine.	
$\frac{l}{r}$	f	$\frac{l}{r}$	f
0 to 10	2500	0 to 15	4000
10 to 35	2000	15 to 30	3500
35 to 45	1500	30 to 40	3000
45 to 60	1000	40 to 45	2500
		45 to 50	2000
		50 to 60	1500

The following table has been compiled from experiments made by Mr. D. Kirkaldy on the crushing-strength and modulus of direct elasticity of timber :—

TABLE IX.

Name of timber.	Sectional area in square inches.	$\frac{l}{r}$	f	Modulus of elasticity in pounds per square inch.
English oak	25	9	3,542	1,786,000
" "	54·5	5 to 22	3,501	—
" "	83 to 144	8 to 10	3,431	—
Pitch pine	80	11	4,339	—
" "	75	5	4,569	} 1,539,000
" "	25	9	4,934	
White Riga	169	19	1,960	—
Red Dantzig	178	18	1,742	—
Dantzig fir	83	11	2,588	—
" "	100	5	2,914	—
" "	25	9	4,704	2,630,000
Oregon pine	25	9	6,586	1,923,000
Norway spruce	25	9	5,376	2,040,800
Baltic red	75	5	2,445	1,163,000
English ash	25	9	3,025	1,613,000
Demerara greenheart	25	1·8 to 18	10,229	4,301,800
" "	50	1·2 to 18	8,430	3,744,513

The following results on the compression of long and short columns of a few of the best Australian timbers have been selected from the author's experiments. They have been treated in a similar manner to the white and yellow pine, but are not strictly comparable, as the sectional area of the specimens of Australian timbers were only 9 square inches (3″ × 3″). The results are very high, as the timbers were well seasoned, but certainly not more than 25 per cent. higher than would be obtained by testing logs 12 inches by 12 inches of sound well-seasoned timber. Australian timber is liable to decay at the heart and form a pipe, the area of which must be deducted from the area of the log in calculating the strength by the foregoing rules.

TABLE X.

Timber.	Values of f when $\frac{l}{r}$ equals				
	0 to 4.	4 to 8.	8 to 16.	16 to 24.	24 to 36.
Red, grey, and white ironbark of N.S. Wales; 70 specimens tested. These timbers also grow in Queensland, but are inferior to those grown in N.S. Wales ...	11,000	10,000	9,500	7,500	7,500
Tallow-wood of N.S. Wales and Queensland; 38 specimens tested	8,500	6,900	6,900	5,200	4,500
Spotted-gum of N.S. Wales; 32 specimens tested	8,400	6,000	6,000	6,000	5,200
Spotted gum of Queensland; 8 specimens tested	10,800	9,800	9,800	6,500	5,300
Blackbutt of N.S. Wales and Queensland; 48 specimens tested	8,500	6,800	6,800	4,700	4,700
Red, grey, and flooded gum of N.S. Wales, and red gum of Victoria; 72 specimens tested	8,600	5,600	5,600	4,600	4,300
Turpentine of N.S. Wales and Queensland; 32 specimens tested	9,600	7,700	7,700	5,600	5,000
Jarrah of Western Australia ...	6,800	5,800	5,800	13,900	3,900

The modulus of compressive elasticity was obtained from experiments on test-pieces 12 inches long, which did not deflect laterally. In the iron bark it varied from 1,500,000 to 2,300,000 pounds per square inch; and in the gums, from 1,300,000 to 2,300,000 pounds per square inch.

Shearing along the Fibres.—When timber is used for keys and in joints and connections of timber roofs and bridges, the material is frequently required to resist stresses tending to shear it along the plane of the fibres. The resistance to shearing along the fibres has been determined for the following American timbers at the Watertown Arsenal, and for the Australian timbers by the author.

TABLE XI.

SHEARING-STRENGTH OF AMERICAN TIMBER.

Name of timber.	Shearing-strength in pounds per square inch.	
	Minimum.	Maximum.
Ash	458	700
Yellow birch	563	815
White maple	367	647
Red oak	726	999
White oak	752	966
White pine	267	366
Yellow pine	286	415
Spruce	253	374
Whitewood	382	406

TABLE XII.

SHEARING-STRENGTH OF AUSTRALIAN TIMBERS.

Name of timber.	Shearing-strength in pounds per square inch.	
	Minimum.	Maximum.
Red, grey, and white ironbark	1900	2400
Tallow-wood	1100	2000
Spotted gum	1500	2300
Blackbutt	1400	1800
Red, grey, and flooded gum...	1000	2100
Turpentine	1400	1700
Jarrah	1700	1970
Blackwood	2000	2700

Transverse Strength and Elasticity.—The transverse strength of a beam is proportional to its breadth, the square of its depth, and inversely proportional to the span. If W denote the load, no matter how distributed, l = the span, b = breadth, d = depth, then—

$$W \propto \frac{bd^2}{l}$$

If f denote the coefficient of bending-strength of the material, or modulus of rupture, and the load is applied in the centre, it will be shown (Chapter IV.) that—

$$W = \frac{2fbd^2}{3l}; \text{ and } f = \frac{3Wl}{2bd^2}$$

The stiffness of a beam is inversely proportional to its deflection, to the breadth, and to the cube of its depth, and directly proportional to its load and the cube of the span. It will be shown, Chapter X., that for a beam supported at both ends and loaded in the centre—

$$E = \frac{Wl^3}{4vbd^3}; \text{ and } v = \frac{Wl^3}{4Ebd^3}$$

where E denotes the modulus of transverse elasticity, and v the deflection in the centre.

The following table gives the summary of Professor Lanza's bending-tests :—[1]

TABLE XIII.

Name of timber.	Breadth and depth in inches.	Span in feet and inches.	Modulus of rupture, f, in pounds per square inch.	Modulus of transverse elasticity E, in pounds per square inch.
Yellow pine beams—				
Maximum	from 3 × 12	6 ft. 5 in.	11,360	2,286,000
Minimum	to 9 × 13$\frac{3}{4}$	to 24 ft.	5352	1,169,298
Average	—	—	7486	1,757,900
Spruce beams— ...				
Maximum	from 1$\frac{3}{4}$ × 6$\frac{3}{4}$	6 ft. 8 in.	8120	1,899,800
Minimum	to 6 × 12	to 20 ft.	2828	897,961
Average	—	—	5046	1,332,451
White pine beams—				
Maximum	from 2$\frac{7}{8}$ to 9$\frac{3}{4}$	14 ft.	7251	1,565,000
Minimum	to 6$\frac{1}{16}$ × 12$\frac{1}{16}$	to 20 ft.	2456	727,000
Average	—	—	4451	1,122,000
White oak beams—				
Maximum	from 2 × 9	9 ft. 6 in.	8850	1,777,500
Minimum	to 6 × 12	to 20 ft.	3535	672,724
Average	—	—	5863	1,131,100

In consequence of the resistance to shearing being small, failure occurred frequently by shearing horizontally along the fibres ; this will be again referred to in Chapter V.

Bauschinger has made experiments on the transverse strength of the more important coniferous woods from different districts

[1] Lanza's "App. Mech.," p. 678.

of Bavaria, which give somewhat higher results than those obtained by Professor Lanza for American timbers of the same name. A summary of these tests is recorded in the following table :—

TABLE XIV.

Name of timber.	Average percentage of moisture.	Breadth and depth in inches, approximate.	Span in feet.	Modulus of rupture in pounds per sq. inch.	Modulus of elasticity in pounds per square inch.
Pine (summer felled)...	22	7 × 7	8 ft. 2½ in.	6770	1,539,639
„ (winter felled) ...	33	7 × 7	„	6418	1,468,524
Spruce (summer felled)	28	{ 7 × 7 to 10½ × 10½ }	„	5327	1,412,818
„ (winter felled)	28	{ 7 × 7 to 11 × 11 }	„	5470	1,402,502

The following table has been compiled from experiments made by Mr. D. Kirkaldy :—

TABLE XV.

Description of timber.	Breadth and depth in inches.	Span in feet.	Modulus of rupture in pounds per square inch.	Modulus of transverse elasticity in pounds per square inch.
Pitch pine ...	{ from 11·18 × 11·30 to 13·10 × 13·10 }	12	7626	—
Dantzig fir ...	{ from 10·00 × 12·00 to 13·25 × 14·38 }	from 8 to 12	4581	—
Ditto	{ from 2·50 × 10·10 to 3·00 × 10·10 }	10	3726	571,760
Baltic oak ...	6·40 × 16·00	10	7686	—
Baltic red ...	{ from 11·72 × 11·82 to 11·77 × 11·86 }	12	4890	—
English oak ...	{ from 4·55 × 12·00 to 4·58 × 12·00 }	10	9762	—
St. Petersburg	3·09 × 11·07 3·08 × 11·02	13	8187	2,446,400
St. Petersburg 1st yellow ...	{ from 2·75 × 8·75 to 3·00 × 8·75 }	10	8556	1,677,500
St. Petersburg 2nd yellow ...	{ from 2·87 × 8·75 to 2·99 × 8·75 }	10	6918	1,396,700
Archangel ...	3·00 × 11·06 3·09 × 11·02	13	6738	2,014,300
Archangel deal, 2A	3·00 × 3·00	10	6252	2,043,400
Swedish ...	{ from 3·08 × 11·07 to 4·10 × 9·10 }	10 to 13	5663	1,838,300
Swedish SS ...	{ from 3·00 × 9·10 to 3·15 × 9·10 }	10	6258	1,149,600
Swedish DDD	{ from 2·93 × 8·75 to 2·95 × 8·75 }	10	6978	1,528,700

The following tests made by the author were on specimens 6 inches wide by 4 inches deep, on a span of 4 feet:—

TABLE XVI.

Name of timber.	Weight in pounds per cubic foot.	Modulus of rupture in pounds per square inch.	Modulus of elasticity in pounds per square inch.	Number of specimens tested.
Ironbark, N.S. Wales	73	18,000	2,600,000	24
Tallow wood, "	71	14,000	1,980,000	9
Spotted gum, "	58	16,000	2,400,000	12
" " Queensland	71	18,000	2,500,000	3
Tallow-wood "	71	10,000	1,500,000	3
Red gum, N.S. Wales	68	12,000	1,800,000	12
" Victoria	65	11,000	1,300,000	3
" South Australia ...	59	6,600	890,000	3
" West Australia	66	10,000	1,700,000	3
Grey and flooded gum, N.S. Wales	66	14,900	2,560,000	15
Blackbutt, N.S. Wales	65	14,000	2,000,000	15
Turpentine, "	66	13,700	1,900,000	12
Jarrah, West Australia	62	12,040	1,390,000	7

Lanza recommends that the modulus of rupture which may be safely assumed in practice should, for spruce and white-pine beams, be 3000 lbs. per square inch, but that for carefully selected timber we may take 4000 lbs. per square inch. For yellow pine he recommends 5000 lbs. per square inch, and that the factor of safety should be four, in building construction.

Lanza also found, by loading beams for a considerable time, that the long-continued application of a load may produce two or more times the deflection produced by the load when first applied, and that the modulus of elasticity that may be safely assumed for a beam that may have to carry its load for a considerable time should be from one-half to two-thirds the modulus obtained from quick tests: thus for spruce, he recommends that the modulus of elasticity should be assumed at from 666,300 to 888,300, and for yellow pine from 878,950 to 1,171,930 lbs. per square inch. He further considers that, although weakness in timber may be due to sap-woods, season-cracks, and decay, by far the most frequent cause of weakness is the presence of knots, which in most cases determine the position of fracture; again, that it is not safe to rely on any extra strength due to seasoning.

The author recommends that for well-seasoned Australian timber the modulus of rupture used for 12″ × 12″ beams should be 25 per cent. less than that recorded in the table—this recommendation is based on a few experiments on beams of this size of ironbark, red gum, and box—and that the factor of safety be taken as four for ordinary building construction, but that for bridges it should be eight for the live and four for the dead load. In view of Lanza's time-tests, the modulus of elasticity used in computing the deflections of beams should be taken at one-half that recorded in the tables. The strength and durability of Australian timber are greater for winter than for summer felled timber, and the strength is greater in seasoned than in unseasoned timber; but experiments prove that natural seasoning is better than either kiln-drying or steaming. The average life of timber bridges in Australia is about 25 years.

Generally the best timber is that which has grown slowly upon a soil rather dry than moist, and it is compact and heavy, the annual rings being narrow and uniform. Timber should show a hard, clear surface when cut, and should be free from clefts, radial cracks, cup-shakes, or cracks between the annual rings. The trees should be felled either in mid-summer or in mid-winter, when the sap is quiet; the latter is preferable. Timber beams should not be built into walls, or otherwise subjected to imperfect ventilation, and should be kept dry, although some timbers stand well when kept constantly wet; but most timbers decay rapidly when exposed so as to be wet and dry alternately.

Several different methods are in use for the preservation of timbers. Kyan's process consists of injecting corrosive sublimate (perchloride of mercury) into the pores. In Burnett's process chloride of zinc is used. In Boucherie's method sulphate of copper, while the Bethell process consists of saturating the timber with creosote. In these operations the air is exhausted from the tank in which the timber is placed, the sap drawn out from the pores, and the solution forced in under pressure.

A perfect antiseptic should be insoluble in water, and non-volatile creosote appears to be the best, but enormous pressures would be necessary to force it into the hard woods of Australia. It is largely used for sleepers of soft timber.

CHAPTER III.

RESULTANT OF ANY NUMBER OF FORCES—GRAPHICAL REPRESENTATION OF MOMENTS OF FORCES—THE METHODS OF DETERMINING THE STRESSES IN STRUCTURES.

THE determination of the stresses in structures such as trusses or girders is based upon the two following principles: (*a*) the principle of the resolution of forces; (*b*) the principle of moments. The former may be stated thus: If any number of forces in the same plane act at a point, or at different points, of a rigid body, and are in equilibrium, the algebraic sum of all their components in any direction is zero. That is to say, the tendency to move the body in one direction is exactly balanced by an equal tendency in the opposite. The moment of a force about a given point may be defined as the product of the force and the perpendicular distance from the point to the line of action of the force. The second principle may then be stated thus: If any number of forces in the same plane act at a point, or at different points, of a rigid body, and are in equilibrium, the algebraic sum of the moments of these forces, taken with reference to any point in their plane, is zero. That is to say, the tendency to produce rotation of the body in one direction is exactly balanced by an equal tendency to produce rotation in the opposite direction. These two fundamental principles give rise to two methods of calculation, and each may be applied analytically or graphically, but it will generally happen in any particular problem that one method offers advantages in simplicity over the others. In order to illustrate these principles, let P_1, P_2, P_3, P_4 represent forces acting in the same vertical plane, θ_1, θ_2, θ_3, θ_4 denote the angles they make with the horizon; let x_1, x_2, x_3, x_4 denote the perpendicular distances from any particular point in the plane of the forces to the lines of action of the forces—that is to say, the moment of a force P_1 about a point O situated at a perpendicular distance x_1, from it is P^1x_1.

Fig. 7 represents four forces in equilibrium, and the lines

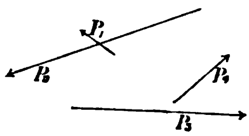


FIG. 7.

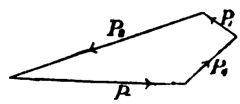


FIG. 8.

are assumed to represent the forces in magnitude and direction, and the arrow-heads indicate the sense of each particular force. Now, the first principle may be stated thus—

$$P_1 \sin \theta_1 + P_2 \sin \theta_2 + P_3 \sin \theta_3 + P_4 \sin \theta_4 = 0, \text{ or } \Sigma Y = 0$$
$$P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3 + P_4 \cos \theta_4 = 0, \text{ or } \Sigma X = 0$$

The second principle may be stated thus—

$$P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 = 0, \text{ or } \Sigma Px = 0$$

The first principle is applied graphically by drawing a polygon, termed the "force polygon," the sides of which are parallel to the forces and equal in length to the magnitude of the forces, as shown in Fig. 8.

This polygon will close, as the forces are in equilibrium. If the polygon does not close, the system of forces is not in equilibrium, and must have a resultant; thus Fig. 9 shows

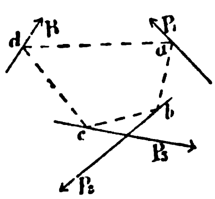


FIG. 9.

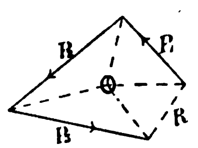


FIG. 10.

a system of forces not in equilibrium, and Fig. 10 the incomplete polygon.

The closing line, shown dotted, represents the resultant force in magnitude, and is parallel to it in direction; but it does not show its line of action, and, in order to discover this, we proceed as follows:—

Take any point O in Fig. 10 in the plane of the forces, which we will call the pole, and join it to the angular points of the polygon as shown in the dotted lines; draw lines in Fig. 9

D

parallel to the dotted lines radiating from O in Fig. 10, commencing at any point *a* on the line of action of the force P_1; thus *a b*, Fig. 9, is parallel to the line from O to the join of P_1 and P_2, Fig. 10; *bc* is parallel to the line from O to the join of P_2 and P_3, the points *b* and *c* lying on the forces P_2 and P_3 respectively. To find the point of application of the resultant R, we draw *ad* and *cd* parallel to the remaining lines in Fig. 10, viz. from O to the join of P_3 and R, and from O to the join of P_1 and R. The figure *abcd*, Fig. 9, is termed the "funicular polygon." As the pole O, Fig. 1, may be selected anywhere in the plane, we have an infinite number of funicular polygons all passing through the point *a*, Fig. 9. Again, *a* may be taken anywhere in the line of action of the force P_1, thus producing an infinite number of funicular polygons for the infinite number of positions of *a*, while the pole O, Fig. 10, remains unchanged. But whatever the position of the pole or the starting-point *a*, the intersection *d*, giving the position of the resultant, always falls on the line of action of the resultant. If the system of forces is in equilibrium, the funicular as well as the force polygon must close. The second principle is therefore briefly expressed by stating that for forces in equilibrium the funicular polygon must close.

Culman's Principle.—Suppose we have a single force P acting at A, Fig. 11. The force polygon in this case is represented by the straight line *m*H*n*, where *mn* represents the magnitude of

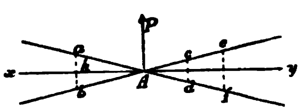


FIG. 11.

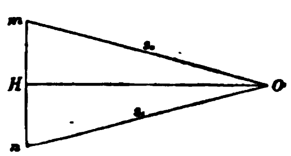


FIG. 12.

the force to scale, Fig. 12. Select a pole O, and join it with *m* and *n*, which is equivalent to resolving the force P in two directions. Draw through the point A, Fig. 11, two lines parallel to s_1, and s_0, Fig. 12, and draw a line OH perpendicular to *mn*; the distance OH is called the polar distance. Draw *xy* perpendicular to the direction of P, Fig. 11, and draw lines *ab*, *cd*, *ef*, parallel to P. Then the moment of the force P with reference to any point *h* is $P \times Ah$. But in Fig. 12 we have by similar triangles—

$$P : HO :: ab : Ah$$
$$\therefore P \times Ah = HO \times ab$$

That is, the moment of the force P, with reference to any point, is equal to the ordinate drawn through this point parallel to P, included by the two components into which P is resolved, multiplied by the polar distance in the force polygon.

The same principle may be applied to represent the moments of any number of forces. Let P_1, P_2, P_3, and P_4 represent four forces acting as in Fig. 13. Draw the force polygon *abcde*, Fig. 14, and select a pole O and draw the lines O*a*, O*b*, O*c*, O*d*, and O*e*. Draw 1 2, Fig. 13, parallel to O*b*, 2 3 parallel to O*c*, 3 4

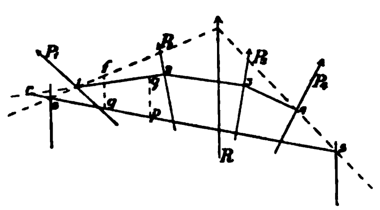


FIG. 13.

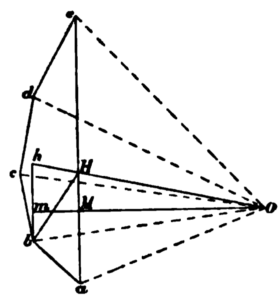


FIG. 14.

parallel to O*d*. Draw a line 6 5 anywhere as a closing line, and OH parallel to it (Fig. 14). Then OH will represent to scale the stress in the closing line 6 5, and *a*H and H*e* will be the forces at 6 and 5 parallel to the resultant R. Draw OM perpendicular to the resultant R. Let the force *x* be resolved along 6 1 and 6 5; these are represented in the force polygon by *oa* and *o*H. By Culman's principle the moment of *x* for any point *g* is OM × *gf*, and since OM is constant, the moment is proportional to *gf*, included between 6 5 and 6 1. The pole distance must be measured to the same scale as the force polygon, the ordinate *gf* to the scale of distance assumed in Fig. 13.

The ordinate included between 6 1 and 6 5 is proportional to the moment of *x* for any point through which the ordinate passes.

To find the combined moment of *x* at any point *p* upon the closing line of all the forces to the left or right of this point, produce 1 2 and 6 5 to meet in *r*; then the moments of the stresses in 1 2 and 6 5 about the point *r* will be zero. Hence the moments of P_1 and *x* about this point must also be zero, since the forces are in equilibrium; but since the combined moment of any number of forces is equal to the moment of the resultant, the moment of the resultant of P_1 and x about r must be zero, also the resultant of P_1 and x must pass through r. The intersection of any two lines in an equilibrium polygon is the point of application for the resultant of the forces at the apices between these lines. Thus the intersection r of 6 5 and 1 2 is the point of application for the resultant of P_1 and x; this is shown in the force polygon by joining H*b*. If the force acting at r is resolved along the closing line 6 5 and parallel to the resultant, these components will be represented in the force polygon by *bh* and *h*H, where *bh* is drawn perpendicular to OM; the moment of H*h* is zero, because it passes through the point *p*. The moment of *b*H is equal to the ordinate *pq* multiplied by the pole distance O upon *bm* by Culman's principle, *i.e.* $pq \times om$, or the projection of *ob* in a direction perpendicular to the resultant, or the distance *om*. We have, therefore, the following properties of an equilibrium polygon—

(*a*) The intersection of any two of its lines gives a point upon the resultant of the intermediate forces.

(*b*) The ordinate parallel to the resultant of all the forces multiplied by the projection, on a line perpendicular to this resultant, of the line in the force polygon corresponding to the line in the equilibrium polygon which is cut by this ordinate, gives the combined moment of all the outer forces, right or left, with respect to that point on the closing line through which the ordinate passes.

The ordinates must in all cases be measured to the scale of length, and the projection in the force polygon to the scale of force.

In the case of a beam supported at each end and loaded with any number of loads P_1, P_2, P_3, etc., Fig. 15, draw the force polygon, which is here a straight line, making *ab* equal to P_1, *bc* to P_2, *cd* to P_3, and select a pole O and join O*a*, O*b*, O*c*, and O*d*, Fig. 16. In Fig. 15 draw 5 1, 1 2, 2 3, and 3 4 parallel to O*a*, O*b*, etc., in the force polygon and between the directions of reactions V, V_1, and the forces P_1, P_2, and P_3 forming the equilibrium polygon. Join 5 4 and draw OH parallel to it; then *a*H and H*d* represent the reactions V and V_1 respectively. Draw OM perpendicular to the direction of the forces or loads, *i.e.* horizontal; then the bending moment at any point *p* in the beam of all the forces to the right or left of it is equal to OM × *pq*.

The bending moment may be defined as the bending couple tending to produce rotation of one part of the beam upon the other considered relatively fixed, as at *p*, Fig. 15. It is

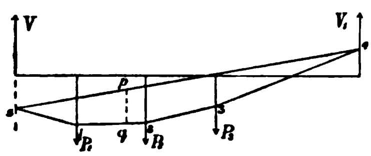


FIG. 15.

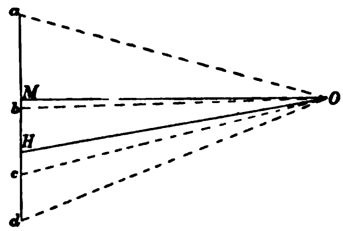


FIG. 16.

the resultant moment of the external forces acting on the part of the beam to one side of the section, these moments being taken about a horizontal axis in the section.

The Determination of Stresses in Braced Structures.—The action of vertical loads in producing transverse stresses in beams will be considered in the next chapter, but the principle of moments may also be applied to find the stresses in braced structures. As stated by Rankine, "If a frame be acted upon by any system of external forces, and if that frame be conceived to be completely divided into two parts by an ideal surface, the stresses along the bars which are intersected by that surface balance the external forces which act on each of the two parts of the frame. "This theorem furnishes in some cases the most convenient method of determining the stresses along the pieces of a frame." This method may be illustrated with reference to a common roof-truss shown in Fig. 125. The system of lettering adopted is that known as Bow's, which consists in assigning a letter to each enclosed area, and also to each space, enclosed or not, around or bounding the truss, and attaching the same letter to the angle, or point of concourse of lines, which represents the area in the diagram of forces. The angular points are denoted by the numbers 1, 2, 3, . . . 7.

The loads written on Fig. 125, acting vertically downwards, as represented by the arrows, are produced by the dead load of the roof-truss, including its own weight and that of the roof-covering, purlins, ridge, etc., included between two principals, amounting to a total load of 6496 lbs., which is taken as concentrated at the joints 1, 2, 3, . . . 7 in the manner shown in Fig. 125. The resultant upward reaction is therefore 2436 lbs. at each support. Divide the truss by the ideal surface or vertical section *aa*, Fig. 17. This section cuts the two bars 1 2 and 1 3, which, according to our notation, we call AH and

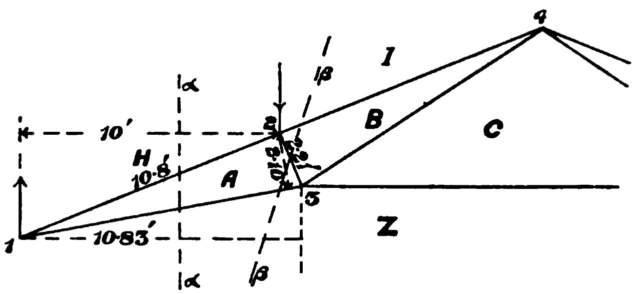


FIG. 17.

AZ respectively. We shall denote the stress in AH or AZ as AH_1 or AZ_1 for shortness. Now, since, before the truss was divided by the section *aa*, the stresses in the two bars cut by it balanced the external forces, it follows that if we apply forces equal to these stresses they also will balance the external forces, and the system will be in equilibrium. The direction in which the arrows point will always be taken away from the portion of the truss which remains after the section is made, and moments which tend to cause rotation in the direction of the hands of a watch will be denoted by the sign + (plus), and moments tending to produce rotation in the opposite direction by the sign − (minus). To find the stress in the bar AZ, we take moments about any point in the bar AH, excepting where it intersects AZ. Select the point 2, and measure the lever arms of the stress AZ_1 and the reaction 2436, which are 2·16 and 10 respectively; these are written on Fig. 17, which represents the forces in equilibrium. We have, therefore—

$$2436 \times 10 - 2{\cdot}16AZ_1 = 0$$
$$\therefore AZ_1 = +\ 11{,}274 \text{ lbs.}$$

The plus sign shows that the bar is in tension, so that we find both the magnitude and the character of the stress by this method.

To find the stress AH_1, take moments about any point in AZ excepting the point 1; select the point 3 and measure the lever arms; then—

$$2436 \times 10{\cdot}83 + 2{\cdot}21AH_1 = 0, \text{ and } AH_1 = -\ 11{,}938 \text{ lbs.}$$

the minus sign showing that the bar is in compression. In like manner we may proceed to determine the stresses in every member of the truss; thus for the stress AB we take a section $\beta\beta$, which cuts two bars in addition to the bar AB, and in such a case it is most convenient to take moments about the point where these two bars meet, viz. the point 1.

In Fig. 13 we saw that the moments of the stresses in 1 2 and 6 5 were zero, because they were measured about the point r where the lines 1 2 and 6 5 intersect. For the same reason the moments of the stress AZ_1 and AH_1 are zero; the moment of the reaction is also zero (Fig. 17);

$$\therefore\ 10{\cdot}8AB_1 + 10 \times 1624 = 0, \text{ and } AB_1 = -\ 1504$$

which is compression.

To find the stress in BI, take the section $\delta\delta$ and moments about the point 3, Fig. 18.

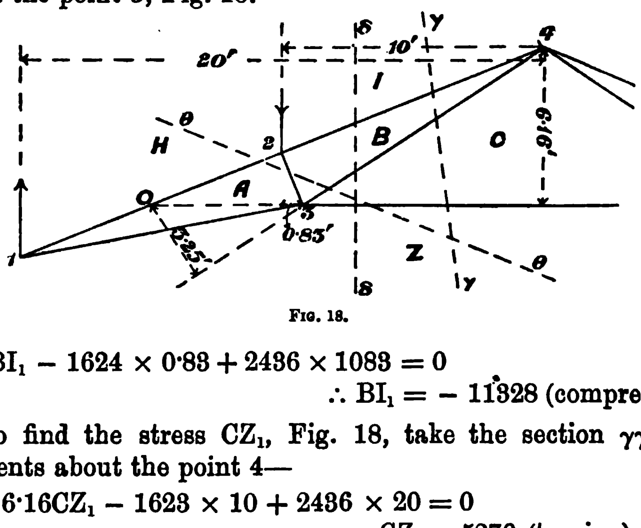


FIG. 18.

$$2{\cdot}21BI_1 - 1624 \times 0{\cdot}83 + 2436 \times 1083 = 0$$

$$\therefore\ BI_1 = -\ 11328 \text{ (compression)}$$

To find the stress CZ_1, Fig. 18, take the section $\gamma\gamma$ and moments about the point 4—

$$6{\cdot}16CZ_1 - 1623 \times 10 + 2436 \times 20 = 0$$

$$\therefore\ CZ_1 = 5270 \text{ (tension)}$$

To find the stress BC_1, take moments about the point O, Fig. 18—

$$-\ 3{\cdot}25BC_1 + 1624 \times 5 + 2436 \times 5 = 0$$

$$\therefore\ BC_1 = 6246 \text{ (tension)}$$

The right-hand members of the truss will give results corresponding with similar bars on the left, which the student may easily prove. It will be observed that the stress upon any member of the truss is found independently of the stresses in other members. The stress on a particular member can always be found if the section taken does not cut more than three bars. If three or more bars are cut, the stress on any bar can be found if the stress on one or more of the bars cut is known; thus in Fig. 18 the section *θθ* cuts four bars, and we wish, for example, to find the stress CZ_1. The stress in the bar AB has been found to be − 1504.

$$\therefore 6{\cdot}16CZ_1 + 2436 \times 20 + (-1504 \times 10{\cdot}8) = 0$$

$$\text{and } CZ_1 = 5270 \text{ (tension) as before}$$

The method of sections would be applied in the same manner to obtain the stresses in more complicated roof and bridge trusses; thus, if Fig. 19 represents a portion of a bridge truss, the stresses in the three bars cut may obviously be found by taking moments about suitable points; but if the points 3 4 be joined the section cuts four bars, and the method fails. The structure in this case is said to be redundant, as it is indeformable without the line 3 4. If both the bars 3 4 and 2 5 are omitted the structure is deformable, and may alter its shape without altering the length of the members 23, 35, 54, and 43. Generally, if n denote the number of sides of a closed figure, such as 2 3 4 5, and v the number of vertices or corners, then it may be proved that—

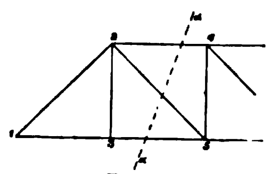


FIG. 19.

$$\text{For a deformable figure, } n < 2v - 3$$

$$\text{For an indeformable non-redundant figure, } n = 2v - 3$$

$$\text{For a redundant figure, } n > 2v - 3$$

Every bridge or roof-truss should be divided up into triangular spaces, so that for every enclosed space bounded by lines, $n = 2v - 3$.

The stresses in braced structures, such as the roof-truss illustrated in Fig. 125, may be found graphically by means of a reciprocal figure. Two figures are said to be reciprocal if for every side of the one there is a corresponding side in the other. Corresponding sides are parallel, perpendicular, or inclined at some constant angle to each other. To every system of lines meeting at a point in the one figure there is a corresponding closed polygon in the other. A reciprocal figure can always be drawn if the original figure is indeformable and non-redundant.

Take, for example, the roof-truss illustrated in Fig. 125. Draw the force polygon to scale; in this case it becomes a straight line. The convenience of Bow's notation will now be more apparent. The reciprocal figure, Fig. 126, is drawn by commencing at K in the force polygon, and drawing the line KE parallel to the line KE in the roof-truss, Fig. 125, and drawing ZE parallel to ZE in Fig. 125 to meet it in E. Then, since KZ, Fig. 125, represents the reaction at the support 7, which is in equilibrium with the stresses KE_1 and EZ_1, the triangle KEZ, Fig. 126, is the triangle of forces, and the length of the line KE, measured to the same scale as the force polygon, represents the stress KE_1; also the length of the line ZE, Fig. 126, represents the stress ZE_1, and the stress in every other member of the truss is found by completing the reciprocal figure in the manner shown, and measuring the length of each line in Fig. 126 to find the stress in the corresponding member of the roof-truss, Fig. 125. Every line in Fig. 126 is parallel to the corresponding line in Fig. 125, and, completing the figure from D by drawing DE parallel to DE, Fig. 125, and so on, we prove the accuracy of the work by the closing line AH, Fig. 126, being parallel to AH, Fig. 125. It will be noticed that for every space in Fig. 125 there are three lines diverging from a point in Fig. 126 having the same letter against it as the space; thus the space C, Fig. 125, bounded by the lines CB, CD, CZ is represented in Fig. 126 by the three diverging lines radiating from C. The determination of the character of the stresses is second only in importance to the determination of the stresses themselves. In Fig. 126 the thick lines denote the compressive stresses, and the thin lines the tensile, which are determined as follows :—

The forces 2436, KE, and EZ, Fig. 125, are in equilibrium, and are represented by the closed polygon KEZ, Fig. 126. Follow round the polygon, Fig. 126, starting with the known upward reaction ZK = 2436, and indicate the direction of the forces by arrows as shown; put arrows on the corresponding stresses in Fig. 125, then it is seen that KE acts towards the point 7, and is therefore compressive; EZ acts away from it, and is tensile. If EK is produced to the right, it is seen that the stress must be tensile. Hence the following rule: Take any apex of the frame as a system of forces in equilibrium. Follow round the polygon formed by these forces in the direction indicated by those forces whose direction is already known, and transfer the directions thus obtained from the forces to the apex under consideration. If the stress acts towards the apex, the piece is in compression; if away from the apex, the piece is in tension. The results found by measuring the lines to the proper scale in Fig. 126 may now be compared with those obtained by the method of sections. The principle of the resolution of forces may be applied algebraically by using the general equations of equilibrium, and the stresses determined by means of the method of sections and also by means of the reciprocal figure might have been determined by means of these equations. The objection to the algebraic method in such cases is the necessity of measuring all the angles which the bars make with their horizontal and vertical components. The method is too tedious when applied to a single roof or other truss, but may be useful when applied to a number of such trusses of similar angles, but with bars of different length.

CHAPTER IV.

BENDING MOMENTS AND SHEARING STRESSES.

THE action of transverse loads in producing stresses in beams may be conveniently studied with reference to a beam fixed at one end and loaded at the other.

Fig. 20 shows such a beam fixed by building it into a wall, and loaded at the extremity with a load W. The deflection or bending produced by the load is exaggerated for the sake of clearness, and it may be observed that the fibres on the convex side of the beam are lengthened, while those on the concave side are shortened. The upper part of the beam is consequently in tension, and the lower part is in compression, while, somewhere between, tension must change to compression. The layer of fibres at which the change from tension to compression occurs is called the neutral layer, as it is neither lengthened nor shortened by the bending of the beam, and is therefore not subjected to either tension or compressive stresses.

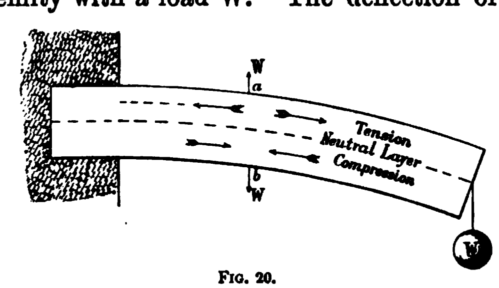


FIG. 20.

If the beam is supported at both ends and loaded in any manner by vertical loads between the points of support, the tensile and compressive stresses developed in the beam will be as indicated in Fig. 21.

Besides the direct tensile and compressive stresses which are developed in the beam in consequence of the bending, there are other stresses which may be realized by clamping together a number of planks, such as would be obtained by making horizontal sections of the beam shown in Fig. 21, and loading in a similar manner. It will then be observed that each section slides upon the other, and overlaps in the manner indicated in Fig. 22;

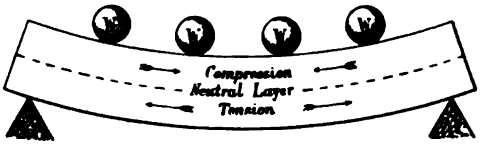


FIG. 21.

hence we conclude that there are horizontal stresses developed in a beam when loaded which tend to make the various layers slide upon each other. Again, if, in the beam shown in Fig. 20, we imagine two forces applied at any section *ab*, each equal to W, and acting in opposite directions so as to neutralize each other, it is clear that the three forces are equivalent to a couple tending to produce rotation of the beam to the right of the section *ab*, and an unbalanced force W. Hence we conclude that there are vertical stresses developed in a loaded beam which tend to make the layers slide upon each other in vertical planes, as indicated in Figs. 23 and 24. These horizontal and vertical stresses are termed shearing stresses.

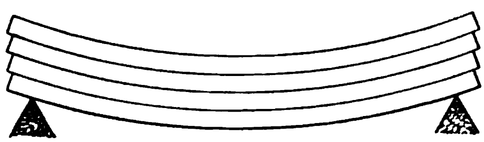


FIG. 22.

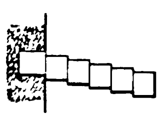


FIG. 23.

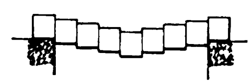


FIG. 24.

The stresses produced in beams subjected to transverse loads may be summarized as follows:—

(*a*) Direct tensile and compressive stresses. (*b*) Horizontal and vertical shearing stresses.

The term "bending moment" has been defined in Chapter III. The bending moments and shearing stresses for the various cases which occur frequently in connection with the construction of ordinary beams and girders will now be considered.

Case I., Figs. 25, 26, and 27.—A beam fixed at one end, termed a cantilever, is loaded at the other with a load denoted by W. This and the following problems may be solved by applying Culman's principle, but it is proposed to treat them algebraically as being rather more convenient for this class of problem. Let x denote the distance from the extremity to the section at which the bending moment is required; let c denote the length of the cantilever. Let M denote the bending moment, and S the shearing stress. From considerations in connection with Fig. 17, it follows that—

$$M = Wx, \text{ and } S = W$$

M will have its greatest value when $x = c$, and S is constant throughout the length of the beam. Hence the bending moments are represented graphically in Fig. 26, where the maximum bending moment Wc is drawn to scale, and the bending moment at any point may be found by measurement. In a similar manner Fig. 27 represents the constant shearing stress. If W = 10 lbs., and c = 8 inches, then Wc = 80 inch-pounds, so that the bending moments are expressed as inch-pounds or foot-tons, according to the units of load and distance used. The shearing force is 10 lbs.

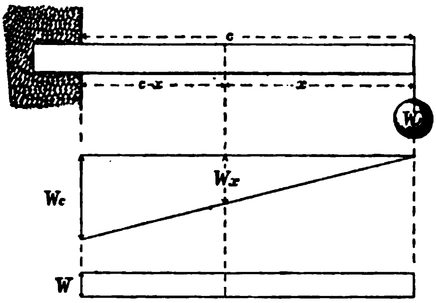


FIG. 25.

FIG. 26.

FIG. 27.

Case II., Figs. 28, 29, and 30.—A cantilever loaded with a uniformly distributed load of W per unit of length over its whole length. Adopting the same notation as before, and remembering that the uniform load to the right of the section may be considered as concentrated at its centre of gravity, and acting with a leverage equal to the distance of the centre of gravity from the section under consideration, viz. $\frac{x}{2}$, we have—

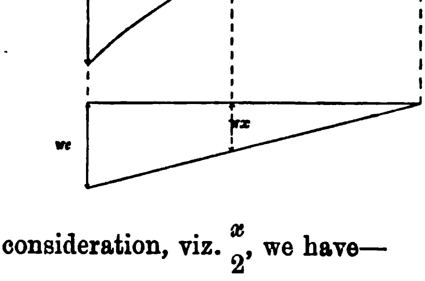


FIG. 28.

FIG. 29.

FIG. 30.

$$M = wx \times \frac{x}{2} = \frac{wx^2}{2}$$

which is a maximum when $x = c$, viz. $M = \frac{wc^2}{2}$, at the point of fixing. Again, the shearing stress is equal to the load on the right of the section, which increases uniformly to the point of fixing; hence Fig. 29 represents the bending moments, and Fig. 30 the shearing stresses on the same principles as explained in connection with Case I. It may be observed that in Case I. the equation of bending moments is that of a straight line, while in Case II. the equation $M = \frac{wx^2}{2}$ is that of a parabola, the origin being at the extremity; hence we may determine points in the curve by solving the equation for M after substituting for x its value in feet or inches, and for w its value in tons or pounds; or the curve may be drawn by any of the geometrical methods, having first calculated the maximum ordinate $\frac{Wc^2}{2}$.

Let $w = \frac{1}{2}$ a ton per foot run, and let $c = 6$ feet; then—

if $x = 1$; $M = 0{\cdot}25$ foot-tons.
$x = 2$; $M = 1{\cdot}00$ „ „
$x = 3$; $M = 2{\cdot}25$ „ „
$x = 4$; $M = 4{\cdot}00$ „ „
$x = 5$; $M = 6{\cdot}25$ „ „
$x = 6$; $M = 9{\cdot}00$ „ „

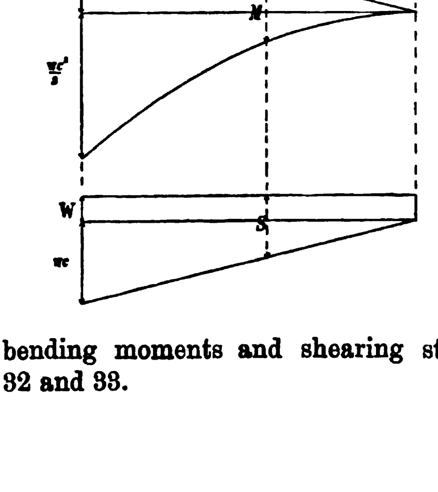


FIG. 31.

FIG. 32.

FIG. 33.

Case III., Figs. 31, 32, and 33.—A cantilever loaded at the extremity with a load W, and with a uniformly distributed load of w per unit of length. This case is a combination of Cases I. and II., and the equations may be written down at once, thus—

$$M = Wx + \frac{wx^2}{2}$$

$$S = W + wx$$

The diagrams of bending moments and shearing stresses are shown in Figs. 32 and 33.

Case IV., Figs. 34, 35, and 36.—A cantilever loaded with any number of concentrated loads.

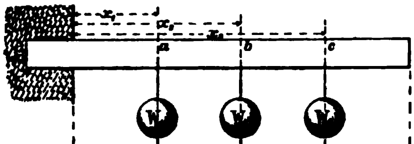


FIG. 34.

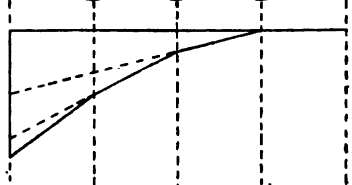


FIG. 35.

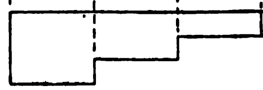


FIG. 36.

Let W_1, W_2, W_3, etc., denote the loads, and x_1, x_2, x_3, etc., the distances from the fixed end respectively. Then the moment at b is $M_b = W_3(x_3 - x_2)$; in a similar manner $M_a = W_3(x_3 - x_1) + W_2(x_2 - x_1)$. The moment about the fixed point is—

$$M = W_3x_3 + W_2x_2 + W_1x_1 + \dots$$

The shearing stress is zero from the extremity to c; from c to b it is W_3; from b to a it is $W_3 + W_2$; from a to the point of fixing it is $W_3 + W_2 + W_1 + \dots$

Hence the diagrams Figs. 35 and 36 represent the bending moments and shearing stresses respectively.

Case V., Figs. 37, 38, and 39.—A cantilever partially loaded with a uniformly distributed load of w per unit of length (Fig. 37). Let x denote the distance of the section of the beam at which the bending moment is required from the point of fixing; then, considering the loads to the right of the section, we have: Load $= w(a + b - x)$ acting with a leverage $= \frac{a + b - x}{2}$; therefore the bending moment is—

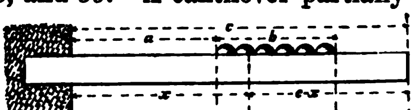


FIG. 37.

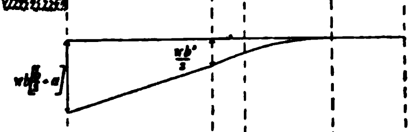


FIG. 38.

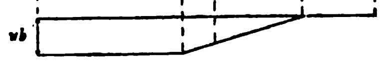


FIG. 39.

$$M = \frac{w}{2}(a + b - x)^2$$

The bending moment at a section situated at a distance a from the point of fixing is found by making $x = a$ in the foregoing equation, thus—

$$M = \frac{w}{2}b^2$$

To find the bending moment between this last section and the point of fixing, we observe that a is greater than x; thus—

$$M = wb\left(\frac{b}{2} + a - x\right)$$

The maximum bending moment occurs at the point of fixing, where $x = o$; then—

$$M = wb\left(\frac{b}{2} + a\right)$$

We observe that the diagram of bending moments, Fig. 38, is a parabola immediately under the load, and a straight line (viz. the tangent at the extremity of the parabola) from the left extremity of the load to the point of fixing. The diagram of shearing stresses, Fig. 39, needs no comment.

Case VI., Figs. 40, 41, and 42.—A beam supported at both ends and loaded in the centre. Let b = the distance between the points of support; c = the half distance; x = the distance of the section at which the bending moment is required from the centre of the beam. Then the pressures on the points of support will be equal to one another, and to half the load W, and there will be a corresponding reaction acting upwards of the same amount. We may consider the beam fixed at the point where the bending moment is required, and consider the forces on one side of it as acting on a beam fixed at one end. Considering the forces to the right of the section, and denoting forces acting upwards by the sign $-$, and those acting downwards by the sign $+$, we have—

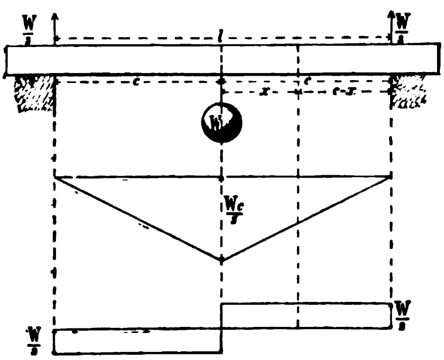


FIG. 40.

FIG. 41.

FIG. 42.

$$M = -\frac{W}{2}(c - x)$$

which is a maximum, and $= \frac{Wc}{2}$ when $x = o$, *i.e.* at the centre of the beam.

If we had taken the moments of forces to the left of the section, we should have found—

$$M = -\frac{W}{2}(c + x) + Wx = -\frac{W}{2}(c - x)$$

as before.

Since $l = 2c$, we may write—

$$M = -\frac{W}{2}\left(\frac{l}{2} - x\right)$$

and when $x = o$, $M = -\frac{Wl}{4}$.

The shearing stress is constant from the supports to the centre, and equals $\frac{W}{2}$; at the centre the shearing stress is zero. The diagrams of bending moments and shearing stresses are shown in Figs. 41 and 42 respectively.

Case VII., Figs. 43, 44, and 45.—A beam supported at both ends, and loaded at a point between the centre and one of the supports (Fig. 43). Let m and n denote the segments into which the load W divides the beam, and let x denote the distance of any section from the right support.

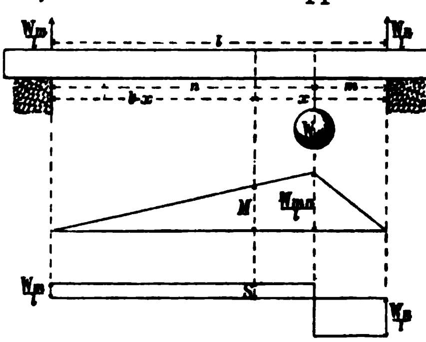


FIG. 43.

FIG. 44.

FIG. 45.

The reaction at the left support is, by the law of the lever, $\frac{Wm}{l}$, and at the right support $\frac{Wn}{l}$. Then—

$$M = -\frac{Wnx}{l} + W(x - m)$$

which equals a maximum when $x = m$, *i.e.* at the point of application of the load—

$$M = \frac{Wmn}{l}$$

The shearing stress is constant from the right abutment to the point of application of the load, and equal to $\frac{Wn}{l}$; it is also constant from this point to the left abutment, and equals $\frac{Wm}{l}$. The diagrams of bending moments and shearing stresses are shown in Figs. 44 and 45 respectively.

Case VIII., Figs. 46, 47, and 48.—A beam supported at both ends, and loaded with a uniformly distributed load of w per unit of length, Fig. 46. The reactions at the supports are both equal to half the load, *i.e.* $\frac{wl}{2}$. Considering the distributed load to the left or right of the section as concentrated at its centre of gravity, and

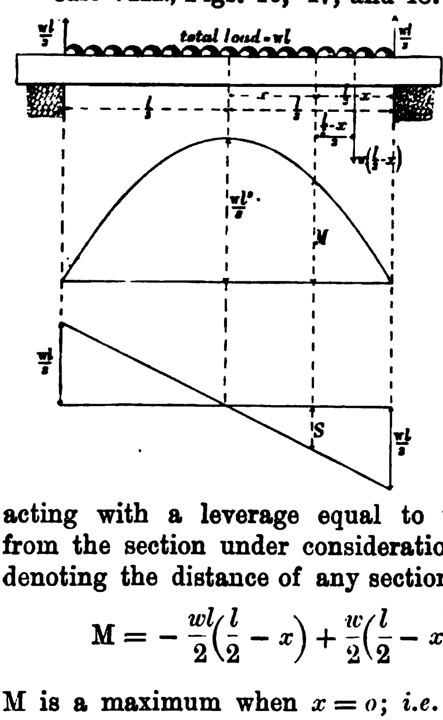


Fig. 46.

Fig. 47.

Fig. 48.

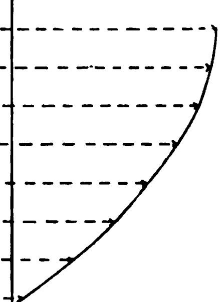
acting with a leverage equal to the distance of this point from the section under consideration as in Case II., we have, denoting the distance of any section from the centre by x—

$$M = -\frac{wl}{2}\left(\frac{l}{2} - x\right) + \frac{w}{2}\left(\frac{l}{2} - x\right)^2 = -\frac{w}{2}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\}$$

M is a maximum when $x = o$; *i.e.* at the centre of the beam we have $M = -\frac{wl^2}{8}$. If x is measured from one of the points of support, we have $M = -\frac{w}{2}(lx - x^2)$. We observe that the curve of bending moments is a parabola (Fig. 47), the central ordinate of which is equal to $\frac{wl^2}{8}$; the bending moment at any other point may be found by drawing the parabola and measuring the particular ordinate to the same scale as $\frac{wl^2}{8}$, or we may make use of the equation of bending moments, substituting for x its distance from the centre of the beam, or from a support.

If the beam or girder is 60 feet between the centres of the supports, and the load per foot run is 1·13 tons, we have the equation of bending moments $M = \frac{1·13}{2}(30^2 - x^2)$, where x is measured from the centre, and we may tabulate the results as in Fig. 49.

The shearing stress at any section in the beam is found by subtracting the load between the section and the nearer support from the reaction at that support. Since the reaction at both supports is half the load, viz. $\frac{wl}{2}$, it follows that the shearing stress must gradually diminish towards the centre of the beam, where it is zero. The shearing-stress diagram is drawn in Fig. 48. If we measure x from the centre of the beam, $S = wx$.



x	$M = 0·565(900 - x^2)$
0	508·50
3	503·40
6	488·16
9	462·73
12	427·14
15	381·37
18	325·44
21	259·53
24	185·06
30	0

FIG. 49.

Case IX.—A beam supported at both ends and loaded with a central load W, also with a uniformly distributed load of w per unit of length. This is obviously a combination of Cases VI. and VIII., and the equations of bending moments and shearing stresses may be written thus—

$$M = -\frac{W}{2}\left(\frac{l}{2} - x\right) - \frac{w}{2}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\}$$

When $x = o$, M is a maximum, and—

$$M = -\frac{Wl}{4} - \frac{wl^2}{8}$$

$$\text{Also } S = \frac{W}{2} + wx$$

which has its maximum value when $x = \frac{l}{2}$, *i.e.*—

$$S = \frac{W}{2} + \frac{wl}{2}$$

Case X., Figs. 50, 51, and 52.—A beam supported at both ends, and loaded with a partially distributed load of w per unit of length (Fig. 50). The total load is wb, and the load to the

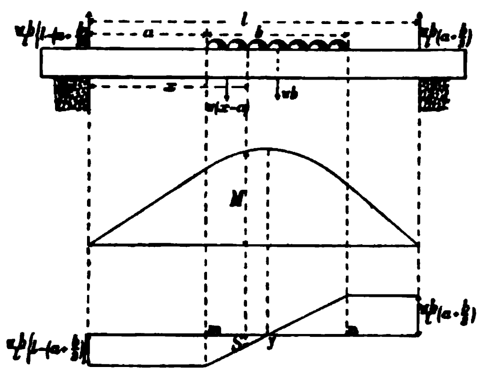


FIG. 50.

FIG. 51.

FIG. 52.

left of the section at a distance x from the left abutment its $w(x-a)$, acting with a leverage of $\frac{x-a}{2}$. The reaction at the right support is $\frac{wb}{l}\left(a+\frac{b}{2}\right)$, and at the left support it is $\frac{wb}{l}\left\{l-\left(a+\frac{b}{2}\right)\right\}$. Taking moments to the right of the section under consideration, we have—

$$M = -\frac{wbx}{l}\left\{l-\left(a+\frac{b}{2}\right)\right\}+\frac{w}{2}(x-a)^2$$

The maximum value of M is found by making $\frac{dM}{dx}=o$, or

$$-\frac{wb}{l}\left\{l-\left(a+\frac{b}{2}\right)\right\}+w(x-a)=o.$$

$$\therefore x = a+b-b\left(\frac{2a+b}{2l}\right)$$

$$\text{If } x = a,\ M = -\frac{wab}{l}\left\{l-\frac{2a+b}{2}\right\}$$

If x falls between the left of the load and the left support—

$$M = -\frac{wbx}{l}\left(l-\frac{2a+b}{2}\right)$$

In a similar manner, if x falls between the right of the load and the right support—

$$M = -\frac{wbx}{l}\left(a + \frac{b}{2}\right)$$

The diagram of bending moments, Fig. 51, is a parabola for the length b immediately under the load, and straight lines tangential to the parabola between its extremities and the supports.

The shearing stress from the right support to the right extremity of the load is equal to the reaction; thus—

$$S = -\frac{wb}{l}\left(a + \frac{b}{2}\right)$$

In a similar manner, the shearing stress from the left support to the extreme left of the load is—

$$S = -\frac{wb}{l}\left\{l - \left(a + \frac{b}{2}\right)\right\}$$

At any section intersecting the load the shearing stress is—

$$S = -\frac{wb}{l}\left\{l - \left(a + \frac{b}{2}\right)\right\} + w(x - a)$$

It equals zero for some section intersecting the partial load, which section may be found by making the above expression for S equal to zero. The maximum value occurs when $x = a$, or $x = a + b$, in which cases S is the reaction at the left and right supports respectively. The diagram of shearing stresses is illustrated in Fig. 52, and the point y denotes the position of the section where the shearing stress vanishes, and the bending moment is a maximum.

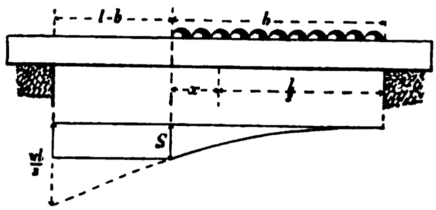


FIG. 53.

FIG. 54.

If the partial load extends from the right abutment to some point in the beam, then (Fig. 53) $a = l - b$, and the equation of shearing stress becomes—

$$S = -\frac{wb}{l}\left\{l - \left(l - b + \frac{b}{2}\right)\right\} + w\{x - (l - b)\}$$

$$= -\frac{wb^2}{2l} + w\{x - (l - b)\}$$

which has its maximum value when $x = l - b$, or at the extremity of the load, thus—

$$S = -\frac{wb^2}{2l}$$

This is the equation of a parabola (Fig. 54). It is convenient to denote the distance to which the load extends beyond the centre of the beam, measuring from the right, by x; then $x + \frac{l}{2} = b$, and the equation of maximum shearing stresses becomes—

$$S = -\frac{w}{2l}\left(x + \frac{l}{2}\right)^2$$

This is in a convenient form for calculating the maximum shearing stress produced by a live load extending from the right support. A similar parabola would indicate the maximum shearing stresses if the live load came on from the left support, the maximum ordinate would, however, in this case appear at the right support. It is clearly only necessary to determine the values of S for the left half of the beam. As an example, let $l = 60$ feet, and the live load $w = 0{\cdot}75$ tons per foot run. Let the dead load per foot run be 0·38 tons; then the total shearing stress is—

x	Value of ·38 x	Value of [illegible]	Total value of S in tons
0	0	5·62	5·62
6	2·28	8·10	10·38
12	4·56	11·02	15·58
18	6·84	14·40	21·24
24	9·12	18·22	27·34
30	11·40	22·50	33·90

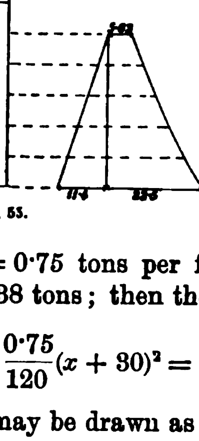


Fig. 55.

$$S = 0{\cdot}38x + \frac{0{\cdot}75}{120}(x + 30)^2 = 0{\cdot}38x + 0{\cdot}00625(x + 30)^2$$

The diagram may be drawn as in Fig. 55.

Case XI., Fig. 56, 57, and 58.—A beam supported at both ends and loaded with a series of concentrated loads, W_1, W_2, W_3, etc. (Fig. 56). Let R_1 and R_2 denote the reactions at the supports; let a_1, a_2, a_3, etc., denote the distances of the points of application of W_1, W_2, W_3, etc., respectively measured from the left support. Then $R_2 = \frac{a_1}{l}W_1 + \frac{a_2}{l}W_2 + \frac{a_3}{l}W_3$, and $R_1 + R_2 = W_1 + W_2 + W_3$; hence $R_1 = W_1 + W_2 + W_3 - R_2$. The reactions at the supports having been determined, the bending moments at the points of application of the loads may be expressed thus—

At the point of application of W_1, $M = -R_1a_1$

,, ,, ,, W_2, $M = -R_1a_2 + W_1(a_2 - a_1)$

,, ,, ,, W_3, $M = -R_1a_3 + W_2(a_3 - a_2) + W_1(a_3 - a_1)$

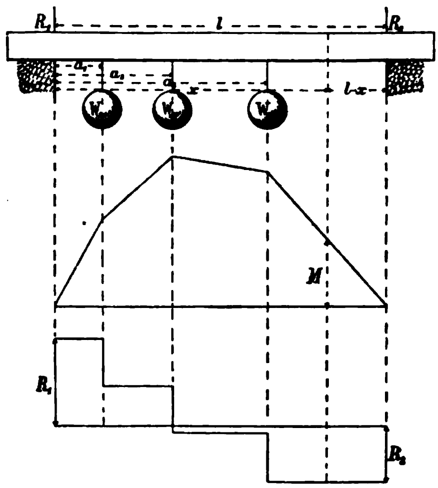


FIG. 56.

FIG. 57.

FIG. 58.

At a section distant x from the left support—

$$M = -R_1x + W_1(x - a_1) + W_2(x - a_2) + W_3(x - a_3)$$

The diagram of bending moments, Fig. 57, is an irregular polygon.

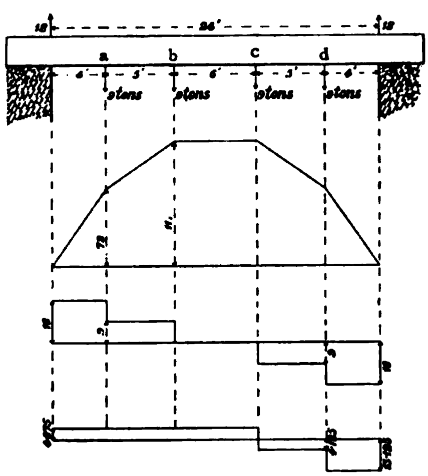


FIG. 59.

FIG. 60.

FIG. 61.

FIG. 62.

The shearing stress from the left support to the point of application of W_1 is R_1; from the point of application of W_1 to that of W_2 it is $(R_1 - W_1)$.

$$\text{From } W_2 \text{ to } W_3,\ S = R_1 - W_1 - W_2$$
$$\text{From } W_3 \text{ to the right support, } S = R_1 - W_1 - W_2 - W_3 = R_2$$

The diagram of shearing stresses is therefore a stepped figure (see Fig. 58).

As an example of this case, take a beam of 24 feet span supported at each end, and loaded, as shown in Fig. 59, with four equal weights of 9 tons each. This example corresponds with that of a cross-girder in a railway bridge carrying a double line of way, and receiving its maximum load from the weights concentrated by the driving-wheels of the locomotive.

Since the loads are symmetrically arranged with regard to the centre of the girder, the reactions at the supports are each equal to 18 tons. The bending moment at *a* and *d* is—

$$M = 18 \times 4 = 72 \text{ foot-tons}$$
$$\text{At } b \text{ and } c,\ M = -18 \times 9 + 9 \times 5 = 117 \text{ foot-tons}$$

The diagram of bending moments is shown in Fig. 60. The shearing-stress diagram, Fig. 61, shows that there is no shearing stress from *b* to *c* in the centre of the girder. If the shearing-stress diagram is drawn for only two loads, as in Fig. 62, the shearing stress in the centre is 4·875 tons, showing that the maximum shearing stress in the centre, *i.e.* from *b* to *c*, Figs. 57 and 60, of a cross-girder carrying a double line of railway occurs when one line only is traversed by the live load. The maximum shearing stress at other portions of the cross-girder occurs when it is fully loaded on both pairs of rails.

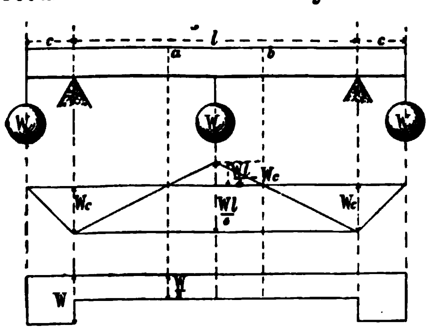


FIG. 63.

FIG. 64.

FIG. 65.

Case XII., Figs. 63, 64, and 65.—A beam supported at two points situated at a distance *c* within the extremities of the beam, with equal overhanging portions, and loaded at the centre and ends with concentrated loads each equal to W.

The bending moment at each support is Wc, as in an ordinary cantilever. This downward bending moment produces an upward bending moment between the supports, which is uniform throughout the length l, and also equal to Wc. The central load produces a downward bending moment, which, in the centre of the beam, is $\frac{Wl}{4}$, so that the actual bending moment in the centre is—

$$M = -Wc + \frac{Wl}{4}$$

The shearing stress is uniform throughout the length l, and is equal to $\frac{W}{2}$; and throughout each of the overhanging portions, as in the ordinary cantilever, the shearing stress is W; hence the pressure on each support is $W + \frac{W}{2} = \frac{3W}{2}$.

The diagrams of bending moments and shearing stresses are therefore as shown in Figs. 64 and 65, from which it may be observed that at the points a and b there is no bending moment.

Case XIII., Fig. 66, 67, and 68.—A beam supported in a

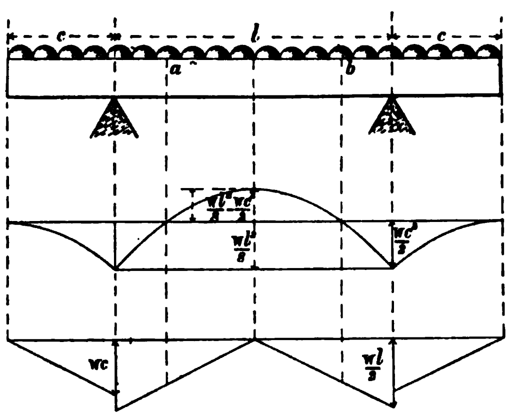


FIG. 66.

FIG. 67.

FIG. 68.

similar manner to that described in Case XII., but loaded with a uniformly distributed load of w per unit of length (Fig. 66). The bending moment at each support is $\frac{wc^2}{2}$, as in an ordinary cantilever loaded uniformly; this downward bending moment produces a uniform upward bending moment between the supports, which also equals $\frac{wc^2}{2}$.

The load between the supports produces a downward bending moment, which at the centre of the beam is $\frac{wl^2}{8}$, so that the bending moment in the centre of the girder is—

$$M = -\frac{wc^2}{2} + \frac{wl}{8}$$

The shearing stress for the cantilevers is wc at supports, and for the length l is $\frac{wl}{2}$, therefore the pressure on each support is—

$$wc + \frac{wl}{2}$$

The diagrams of bending moments and shearing stresses are as shown in Figs. 67 and 68. It may be observed here, as in Case XII., that at a and b there is no bending moment; these points are known as points of inflection or contra-flexure.

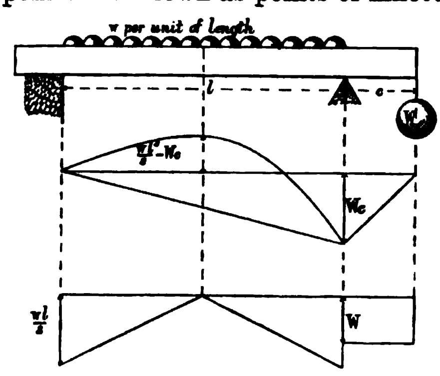


FIG. 69.

FIG. 70.

FIG. 71.

Case XIV., Fig. 69, 70, and 71.—Beam supported at two points situated at a distance l from each other, with an overhanging portion. The beam is loaded with a uniformly distributed load of w per unit of length over the distance l, and with a load W concentrated at the extremity of the overhanging portion.

The bending moment at the end support is zero, and at the other support Wc. The latter bending moment produces an upward bending, and diminishes uniformly from the support, where it is a maximum, to the end, where it is zero. The uniform load produces a downward bending moment, which is

$\frac{wl^2}{8}$ in the centre of the portion of length l. The resultant bending moment in the centre of this portion is—

$$\frac{wl^2}{8} - \frac{Wc}{2}$$

The shearing stresses over the supports are each $\frac{wl}{2}$. The shearing stress to the right of the intermediate support is W, and the pressure at the support is $W + \frac{wl}{2}$. The distribution of bending moments and shearing stresses is shown in Figs. 70 and 71.

Relation between the Bending Moments and Shearing Stresses.—If M and M_1 denote the bending moments at two sections of a beam at a distance m from each other, and if S denote the shearing stress at the section where the bending moment is M, then, if m is very small—

$$\frac{M_1 - M}{m} = S$$

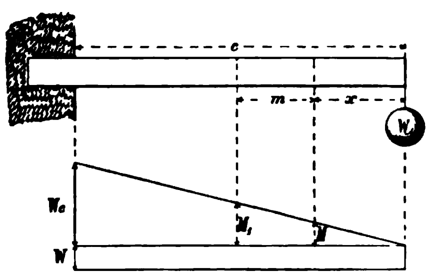


FIG. 72.

FIG. 73.

Consider the case of a cantilever loaded at its extremity (Fig. 72).

$$M = Wx;\ M^1 = W(x + m)$$
$$M_1 - M = Wm$$
$$\frac{M_1 - M}{m} = W = S$$

If the load is distributed uniformly over its length, then—

$$M = \frac{wx^2}{2};\ M_1 = \frac{w}{2}(x + m)^2 = \frac{w}{2}(x^2 + 2xm + m^2)$$

and since m is a very small quantity its square may be neglected, and—

$$\frac{M_1 - M}{m} = wx = S$$

If the load is uniformly distributed over a beam supported at both ends—

$$M = \frac{w}{2}(lx - x^2)$$

Where x is the distance from either abutment—

$$M_1 = \frac{w}{2}\{(lx + m) - (x^2 + 2xm + m^2)\}$$

and neglecting m^2 as before—

$$M_1 = \frac{w}{2}(lx + lm - x^2 - 2xm)$$

$$\therefore \frac{M_1 - M}{m} = \frac{w}{2}(l - 2x) = S$$

If we differentiate the equation of bending moments, we obtain the equation of shearing stress, thus—

$$\frac{dM}{dx} = \frac{w}{2}(l - 2x) = S$$

and generally it may be proved that—

$$\frac{dM}{dx} = S$$

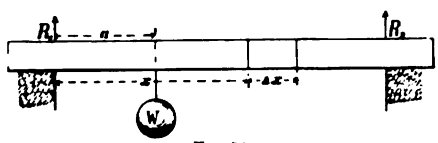


Fig. 74.

Consider the case of a beam supported at each end, and loaded at any point with a load W, Fig. 74.

$$\text{Here } M = R_1 x - \sum_{o}^{x} W(x - a)$$

$$\text{and } M + \Delta M = R_1(x + \Delta x) - \sum_{o}^{x} W(x - a + \Delta x), \text{ nearly}$$

$$\text{Hence } \Delta M = R_1 \Delta x - \sum_{o}^{x} W \Delta x, \text{ nearly}$$

$$\text{and } \frac{\Delta M}{\Delta x} = \frac{dM}{dx} = R_1 - \sum_{o}^{x} W = S$$

If we consider the shearing-stress diagram in Case I., we see (Fig. 27) that the area is Wc, which is equal to the maximum bending moment, Fig. 73; also the area of any portion at a distance x from the end of the cantilever is Wx, which is the corresponding bending moment at the section at the distance x

from the end of the cantilever. In Case II., the area of the whole diagram of shearing stress, Fig. 30, is $wc \times \frac{c}{2} = \frac{wc^2}{2}$, *i.e.* the maximum bending moment; also the area of the portion included between the section at a distance x from the end is $wx \times \frac{x}{2} = \frac{wx^2}{2}$, *i.e.* the bending moment at that section.

In Case VI., the area of the rectangles on either side of the centre is $\frac{w}{2} \times \frac{l}{2} = \frac{wl}{4}$, *i.e.* the bending moment at the centre.

In Case VII. the same relationship exists, and the area of the rectangles on the left of the load is equal to that on the right, viz.—

$$\frac{Wm}{l} \times n = \frac{Wn}{l} \times m$$

i.e. the bending moment at the point of application of the weight. In Fig. 59, the area of the shearing to the left of a is $18 \times 4 = 72$, and to the left of b is $18 \times 4 + 9 \times 5 = 117$.

In Cases XII. and XIII., the areas of the diagrams, Figs. 65 and 68, between a and b, equals the bending moment in the centre, and the areas of each of the diagrams from the points of contra-flexure to support is equal to the bending moment at the support.

CHAPTER V.

MOMENT OF RESISTANCE—INTENSITY OF HORIZONTAL AND VERTICAL SHEARING STRESSES.

We have seen that when a beam is bent as in Fig. 75, the neutral layer is unchanged in length, and it may be proved that the layers on either side of it are lengthened or shortened in proportion to the distance from the neutral layer. It is assumed that the axis of the beam when unloaded is horizontal, and the loads producing bending act at right angles to the beam. The effect of the loading is to cause the beam to assume a curved form, and sections of the beam which were parallel in the unloaded beam converge towards the centre of curvature in the loaded beam.

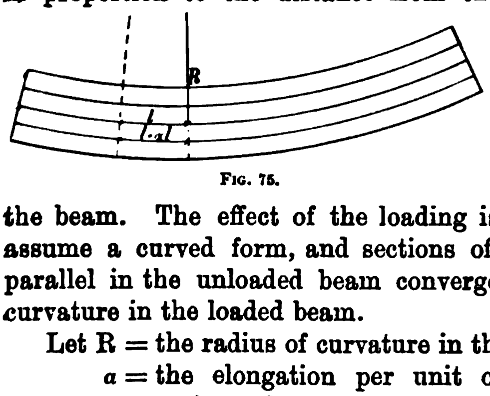


Fig. 75.

Let R = the radius of curvature in the loaded beam.

a = the elongation per unit of length of the fibre situated at a distance y_1 from the neutral layer.

l = the distance between two normal sections of the unloaded beam assumed to be indefinitely near to each other.

Then the fibres at a distance of y from the neutral axis will be extended by an amount al, and the length between two normal sections will be $l + al$, and by similar triangles we have—

$$\frac{R + y_1}{R} = \frac{l + al}{R} \quad \therefore 1 + \frac{y_1}{R} = 1 + a \quad \therefore \frac{y_1}{R} = a$$

and if y_2 denote the distance from the neutral layer to another layer of fibres, and a_2 the extension or strain per unit of length, then—

$$\frac{y_2}{R} = a_2$$

and therefore—

$$\frac{y_1}{y_2} = \frac{a_1}{a_2}$$

i.e. the strain is proportional to the distance from the neutral axis.

We may represent this law graphically as in Fig. 76, where ac equals the strain at the extreme fibres, ab the depth of the beam, and $a'c'$ the strain at the layer of fibres situated at a distance y_1 from the neutral layer or axis. Hence it follows that the resistance which a layer of material will offer to the stresses developed in it will be proportional to the distance of the layer from the neutral axis; also that the neutral axis corresponds with the centre of gravity of the section, provided that the elastic limit of the material is not exceeded.

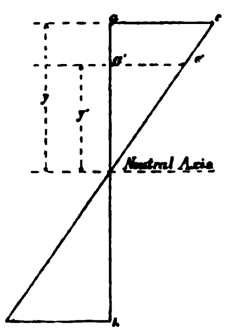


FIG. 76.

If f denote the intensity of stress at a distance y from the neutral axis, and f_1 denote the intensity of stress at a distance y_1 from the neutral axis, then—

$$f_1 : f :: y_1 : y, \text{ and } f_1 = \frac{fy_1}{y}$$

The moment of resistance of a layer is equal to its direct resistance multiplied by its distance from the neutral axis; hence if b equal the breadth of the layer at a distance y_1 from the neutral axis, and dy its thickness, and f_1 its greatest resistance to stress within the elastic limit of the material, then the moment of resistance of the layer is—

$$bdy \times y_1 \times f_1 = bf_1y_1dy = \frac{bfy_1^2dy}{y}$$

and, since this will be true of every section, the moment of resistance of the whole section is, if b is constant—

$$\frac{b}{y}f\Sigma y_1^2 dy, \text{ or } \frac{b}{y}f\int_{-y}^{y} y_1^2 dy$$

$$\text{but } b\Sigma y_1^2 dy \text{ or } b\int_{-y}^{y} y^2 dy = \text{I}$$

where I denotes the moment of inertia of the section with reference to the neutral axis. Therefore the moment of resistance is equal to—

$$\frac{f\text{I}}{y}$$

This formula is true of any section of any material provided the value of f does not exceed the elastic limit of the material. If the beam is loaded till it is at the breaking-point, the formula is no longer true, as the strains produced are not proportional to the stresses producing them, and the neutral axis does not coincide with the centre of gravity of the section.

If M denote the bending moment which produces fracture, then the equation between the bending moment and moment of resistance is generally written thus—

$$\text{M} = \frac{f\text{I}}{y} \qquad (1)$$

from which is obtained—

$$f = \frac{\text{M}y}{\text{I}} \qquad (2)$$

This is an empirical formula, and f is termed the modulus of rupture, or coefficient of transverse strength. The value of f is not the tensile strength of the material, as may easily be proved by breaking rectangular beams of known dimensions in the testing-machine, and calculating f from equation (2).

In the case of wrought iron, cast iron, and steel, the value of f thus obtained will be greater than the tensile strength of the material. Sir B. Baker proved that in the case of steel and wrought-iron beams of rectangular section, the value of f is 70 per cent. in excess of the tensile strength of the material.

If an ordinary rolled iron or steel girder be broken in a similar manner, the value of f will be smaller than in a solid rectangular section, but still greater than the tensile strength of the material. In a well-proportioned wrought-iron or steel-built girder this difference between the value of the modulus of rupture and the tensile resistance of the material is so small that it is generally neglected in calculating the moment of resistance of the section, which is then taken as the moment of resistance of the flanges, the web not being considered with reference to resisting bending. The web is designed to resist the shearing stresses only.

The reason why the modulus of rupture exceeds the tensile strength of the material is probably due to the plastic flow which occurs after the elastic limit has been passed. The extreme fibres in stretching are partially relieved of stress, which is transferred to the layers of fibres nearer to the neutral axis of the section. The neutral axis therefore cannot coincide with the centre of gravity of the section under these circumstances, but must move farther away from the most strained fibres until the point of rupture is reached.

Sir B. Baker has shown, by the results of a large number of experiments on steel rails, that the tensile strength may be deduced from the modulus of rupture thus—

Let A = the area of the section tested, such as a rolled girder, rail, T or bulb iron.

A_1 = the area of a rectangle of the same depth as the section tested, but of a breadth equal to the widest flange.

t = the tensile strength of the material, wrought iron or steel.

Then in the case of solid, rectangular, or square sections—

$$f = t\left(1 + \frac{70}{100}\right) = \text{tensile strength}$$

For other sections—

$$f = t\left(1 + \frac{70}{100} \times \frac{A}{A_1}\right)$$

If, however, the elastic limit of the material is not exceeded, the equation $f = \frac{My}{I}$ is a rational formula, and may be used to determine the intensity of stress at the extreme fibre in any section. The value of $\frac{I}{y}$ has been termed the modulus of the section, and may be found when the moment of inertia and the distance y are known.

F

For rectangular sections the moment of inertia about an axis passing through the centre of gravity of the section may be proved to be—

$$I = \tfrac{1}{12}bd^3,$$ [1]

where b = the breadth and d = the depth of the rectangle.

For a square section it is $\tfrac{1}{12}d^4$, where d = the side of the square.

The moment of inertia about an axis passing through one end of the rectangle is—

$$I = \tfrac{1}{3}bd^3.$$ [1]

The moment of resistance of rectangular beams is therefore—

$$\frac{fI}{y} = \frac{\tfrac{1}{12}bd^3}{\frac{d}{2}}f = \tfrac{1}{6}bd^2f$$

For square beams, $\dfrac{fI}{y} = \tfrac{1}{6}b^3f.$

The moduli of the sections are $\tfrac{1}{6}bd^2$ and $\tfrac{1}{6}b^3$ respectively.

For symmetrical sections the moment of inertia may be found thus.

The section should be divided up into rectangles and dimensioned as indicated in Fig. 79, and, since the moment of inertia of a rectangle about an axis passing through its centre

[1] About an axis passing through the centre of gravity (Fig. 77). Let x denote the distance of an elementary portion of the section from the axis; let dx denote its thickness. Then the area of the element is $b \cdot dx$, and its moment of inertia

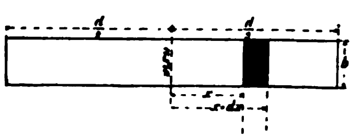


FIG. 77.

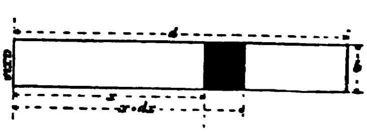


FIG. 78.

$b \cdot dx \times x^2$, since dx is indefinitely small. The moment of inertia of the whole beam is—

$$I = b\int_{\frac{d}{2}}^{\frac{d}{2}} x^2dx = \frac{bd^3}{12}$$

About an axis passing through one end (Fig. 78)—

$$I = b\int_{o}^{d} x^2dx = \frac{bd^3}{3}$$

of gravity is $\frac{1}{12}bd^3$, the moment of inertia of the whole section is—

$$I = \frac{1}{12}\{ac^3 + b(f^3 - e^3) + c(g^3 - f^3) + d(h^3 - g^3)\}$$

For unsymmetrical sections the moment of inertia may be found thus.

The section should be divided up into rectangles and dimen-

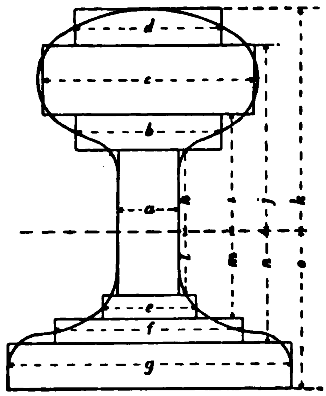


FIG. 79.

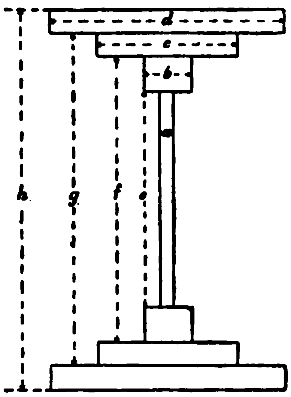


FIG. 80.

sioned as indicated in Fig. 80, and, since the moment of inertia of a rectangle about an axis passing through one end is $\frac{1}{3}bd^3$, the moment of inertia of the whole section is—

$$I = \frac{1}{3}\{a(h^3 + l^3) + b(i^3 - h^3) + c(j^3 - i^3) + d(k^3 - j^3) + e(m^3 - l^3) + f(n^3 - m^3) + g(o^3 - n^3)\}$$

As an example of a symmetrical section, take a rolled iron girder, 12″ × 6″ × ½″, as shown in Fig. 81.

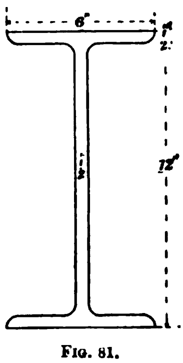


FIG. 81.

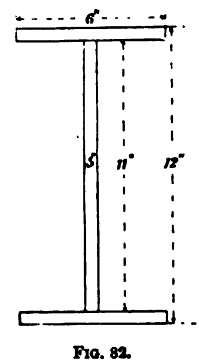


FIG. 82.

The girder should be dimensioned as shown in Fig. 82, then the moment of inertia may be written down as follows :—

$$I = \frac{1}{12}\{\cdot 5(11)^3 + 6(\overline{12}^3 - \overline{11}^3)\} = 254 \text{ nearly}$$

The moment of resistance may be found thus—

$$\text{Moment of resistance} = \frac{fI}{y} = \frac{254}{6} \times f = 42{\cdot}3f$$

If f be taken as 5 tons per square inch—

$$\text{Moment of resistance} = 42{\cdot}3 \times 5 = 211{\cdot}5 \text{ inch-tons}$$

If it is desired to know the greatest load it will safely carry uniformly distributed over a span of 12 feet, we proceed as follows: Let W denote the total distributed load. The bending moment is—

$$\frac{Wl}{8} = \frac{W \times 12 \times 12}{8} = 18W \text{ inch-tons}$$

$$\therefore 18W = 211{\cdot}5, \text{ and } W = 11{\cdot}75 \text{ tons}$$

The maximum shearing stress is $\frac{11{\cdot}75}{2} = 5{\cdot}87$ tons; and the intensity of shearing stress is $\frac{5{\cdot}87}{5{\cdot}5} = 1$ ton (about) per square inch.

If we require to know the maximum intensity of stress produced by a central load of 5 tons, we have—

$$\frac{Wl}{4} = 42{\cdot}3f = \frac{5 \times 144}{4} = 180$$

$$\therefore f = \frac{180}{42{\cdot}3} = 4{\cdot}2 \text{ tons per square inch (nearly)}$$

The maximum intensity of shearing stress is $\frac{2{\cdot}5}{5{\cdot}5} = {\cdot}45$ tons per square inch.

Moment of Resistance of Built Girders.—In plate web girders, consisting of horizontal flange plates united to a thin vertical web-plate by means of double angles top and bottom riveted to both the web and the flanges, the moment of resistance of the web-plate is very small compared with that of the flanges, and may be neglected without causing any serious error. In box girders, in which there are two or more web-plates, the webs should be considered by finding the moment of inertia and using the equation—

$$\text{Moment of resistance} = \frac{fI}{y}$$

In lattice girders the moment of resistance is usually calculated on the assumption that the flanges resist the bending stresses, while the web resists the shearing stresses as in the case of plate web girders. The moment of resistance in such cases may be expressed as follows:—

Let f = the intensity of working stress, generally taken as the working tensile stress.

a = the area of the tension flange after deducting for rivet-holes.

d = the effective depth of the girder.

Then we have—

$$\text{Moment of resistance} = fad$$

The bottom flange is designed by means of the above equation, and the top flange is usually made similar to it in every respect.

The effective depth of the girder is generally taken as the distance between the centres of gravity of the top and bottom flanges whenever the flanges of the girder consist of both horizontal plates and angles; but when there are no horizontal plates, and the angles form the flanges, the effective depth is more correctly measured between the centres of the rivets uniting the web to the flanges.

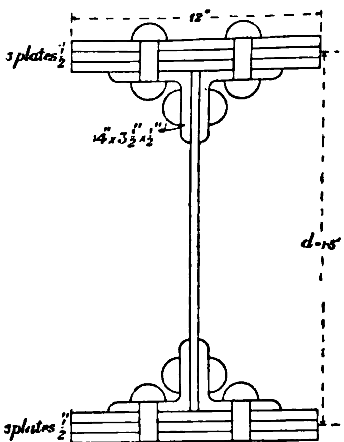


FIG. 83.

Fig. 83 represents a plate web girder, which will be used as an example of the foregoing method of finding the moment of resistance, thus—

The area of the tension flange, after deducting two rivets 1 inch diameter, is—

$$\text{Three plates} = 3(12 - 2 \times 1)\tfrac{1}{2} = 15 \text{ square inches}$$
$$\text{Two angles} = 2(7 - 1)\tfrac{1}{2} = 6 \text{ square inches}$$
$$\therefore \; A = 21 \text{ square inches}$$

The effective depth measured between the centres of gravity of the flanges is 1·5 foot. The working stress may be taken at 5 tons; then—

Moment of resistance $= fad = 5 \times 21 \times 1 \cdot 5 = 157 \cdot 5$ **foot-tons**

The safe distributed load on such a girder on an effective span of 16 feet is found thus—

$$\frac{Wl}{8} = \frac{W \times 16}{8} = 2W = 157 \cdot 5$$

$$\therefore W = 78 \cdot 75 \text{ tons.}$$

This weight may be carried without the intensity of stress exceeding 5 tons on the flanges, provided that the girder is properly designed in other respects.

As an example of an unsymmetrical section, take a cast-iron girder of the form and dimensions shown in Fig. 84.

Let A_1 = **area of top flange** $= 5 \times 1\frac{1}{2} = 7 \cdot 5$ **square inches.**

$A_2 =$ „ **bottom flange** $= 15 \times 2 = 30 \cdot 0$ **square inches.**

$A_3 =$ „ **web** $= 17 \cdot 5 \times 1 \cdot 75 = 30 \cdot 625$ **square inches.**

$\bar{x}$ = **distance of the centre of gravity of the section from the bottom edge of the bottom flange.**

x_1 = **distance of the centre of gravity of the top flange from the same edge.**

x_2 = **distance of the centre of the bottom flange from the same edge.**

x_3 = **distance of the centre of the web from the same edge.**

$$\bar{x}(A^1 + A_2 + A_3) = A_1x_1 + A_2x_2 + A_3x_3$$

$$\bar{x}(68 \cdot 125) = 7 \cdot 5 \times 20 \cdot 25 + 30 \times 1 + 30 \cdot 625 \times 10 \cdot 75$$

$$\therefore \bar{x} = \frac{511 \cdot 094}{68 \cdot 125} = 7 \cdot 5$$

The section of the girder should be dimensioned as indicated in Fig 85, for the purpose of finding the moment of inertia

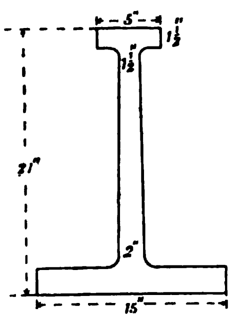


Fig. 84.

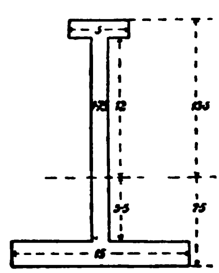


Fig. 85.

with reference to the axis passing through the centre of gravity, which may be now written down—

$$I = \tfrac{1}{3}\{1\cdot75(\overline{12^3} + \overline{5\cdot5^3}) + 5(\overline{13\cdot5^3} - \overline{12^3}) + 15(\overline{7\cdot5} - \overline{5\cdot5^3})\}$$
$$= \tfrac{1}{3}(3315\cdot156 + 3161\cdot875 + 3832\cdot5)$$
$$= 3436\cdot531$$

In this case we have the moment of resistance for the tension flange.

$$\text{Moment of resistance} = \frac{f_t I}{y_t} = \frac{f_t \times 3436\cdot531}{7\cdot5}$$

The moment of resistance of the compressive flange is—

$$\text{Moment of resistance} = \frac{f_c I}{y_c} = \frac{f_c \times 3436\cdot531}{13\cdot5}$$

If we allow $2\frac{1}{2}$ tons per square inch as the maximum intensity of safe tensile stress on the extreme fibre, we have—

$$\text{Moment of resistance} = \frac{2\cdot5 \times 3436\cdot531}{7\cdot5} = 1145\cdot5 \text{ inch-tons}$$

If we allow 7·5 tons per square inch as the maximum intensity of compressive stress, we have—

$$\text{Moment of resistance} = \frac{7\cdot5 \times 3436\cdot531}{13\cdot5} = 1909 \text{ inch-tons}$$

Hence since the moment of resistance of the tension flange is smaller than that of the compression flange, we must equate the former with the bending moment. The strength of cast-iron beams is, however, usually calculated by the following formula, which neglects the area of the web; thus—

Let f = the mean intensity of stress on the bottom or tension flange.
a = the area of the tension flange.
d = the depth measured between the centres of gravity of the flanges.

Then—

$$\text{Moment of resistance} = fad = 30 \times 19\cdot25f = 577\cdot5f$$

Referring to Fig. 85, it will be seen that if 2·5 tons is the maximum intensity of tensile stress at the extreme fibre, the intensity of stress at the inner edge of the bottom flange is—

$$\frac{2\cdot5 \times 5\cdot5}{7\cdot5} = 1\cdot83 \text{ tons per square inch}$$

The mean intensity of stress is therefore—

$$\frac{2 \cdot 5 + 1 \cdot 83}{2} = 2 \cdot 16 \text{ tons per square inch}$$

Hence moment of resistance = 577·5 × 2·16 = 1247·4 inch-tons, which is slightly greater than before.

The ultimate resistance of cast iron such as would be used in girder work is as follows :—

Tension	9 tons per square inch.
Compression	43 " "
Shearing	8½ " "

It follows, therefore, that in cast-iron beams of similar proportions to the one shown in Fig. 84, the tension flange will always be the weaker of the two. If the area of the top flange is diminished, so that the moment of resistance in compression is more nearly equal to that of the tension flange, the form of the section will not be so suitable for casting, as the thinner top flange will cool first, and there will be a danger of a flaw in consequence of the subsequent contraction of the thick bottom flange. For the same reason the web is made tapering, and of a thickness equal to that of the flange it joins. Again, the flanges are tapered so as to draw easily from the sand. The thickness of the web is always greatly in excess of that necessary to resist the shearing stresses, and feathers or stiffeners should be avoided. The top flange, if reduced to the size required by theory, would probably buckle laterally.

If the span of the girder, measured between the centre of its bearings, is 21 feet 6 inches, what is the greatest safe load it will carry uniformly distributed over its length ?

Here—

$$\frac{Wl}{8} = \frac{W \times 21 \cdot 5 \times 12}{8} = 32 \cdot 25W = \text{maximum moment of resistance}$$

If moment of resistance = 1146 inch-tons

$$32 \cdot 25W = 1146$$

$$\therefore W = \frac{1146}{32 \cdot 25} = 35 \cdot 53 \text{ tons}$$

The greatest shearing stress will occur over the supports, and is—

$$\frac{35{\cdot}53}{2} = 17{\cdot}76 \text{ tons}$$

If the beam is of uniform depth and the flanges of uniform width, the maximum intensity of shearing stress is—

$$\frac{17{\cdot}76}{\text{area of web}} = \frac{17{\cdot}76}{30{\cdot}625} = {\cdot}58 \text{ tons per square inch}$$

The beam may be varied in depth by curving the upper flange, or the width of the flanges may be varied, or both. If the safe intensity of working stress in shear is 1 ton per square inch, the depth over the supports may be reduced thus: where d denotes the reduced depth, and $1{\cdot}75d$ the area of the web—

$$\frac{17{\cdot}76}{1{\cdot}75d} = 1$$

$$\therefore d = 10 \text{ inches}$$

The modulus of the section of any material depends upon its form and dimensions. In symmetrical sections the relative resistance to bending per unit of area, in regard to a plane perpendicular to the axis about which the moment of inertia has been calculated (the axis passing through the centre of gravity of the section), may be found by dividing the modulus of the section by the area of the section. In unsymmetrical sections the same applies, if we remember that the distance from the most strained fibre must be that which will give the smaller moment of resistance.

We have seen that the resistance of a layer of fibres is directly proportional to the area of the layer and to the distance from the neutral axis. If the resistance of the fibres at the extreme (or most strained) layer be denoted by unity, the resistance of the other layers may be represented graphically by drawing lines across the section, the lengths of which are reduced in the ratio of their distances from the neutral axis. In the case of the rectangle, Fig. 86, the lines are enclosed by two triangles, which represent the equivalent figures of uniform stress on the assumption that the stress is constant and the section varying, and the modulus of the section is equal to the area of one of the triangles multiplied by the distance between the centres of gravity of the triangles.

If a denote the area of one of the triangles, and d_1 the distance between the centres of gravity, we have the modulus of the section, M, thus—

$$M = ad_1$$

$$\text{Area of triangle} = \frac{bd}{4}\text{; and } d_1 = \frac{2d}{3}$$

$$\therefore\ M = Ad_1 = \frac{bd^2}{6}\text{, as before}$$

If the same method be applied to a circular section, the

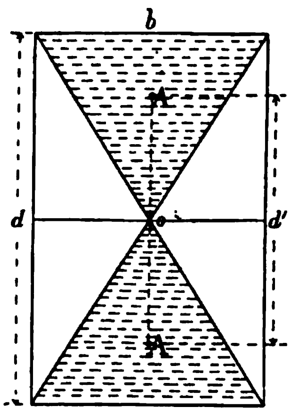


Fig. 86.

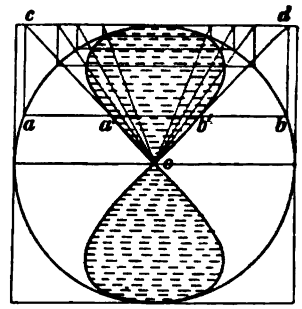


Fig. 87.

figure of uniform stress will be as shown in Fig. 87. The points through which these curves are drawn are found thus: Lines are drawn across the circle, such as *ab*, and perpendiculars let fall from their extremities, such as *bd* and *ac*, on to a horizontal line drawn through the top of the section; *c* and *d* are then joined to *o*, intersecting *ab* in *a'* and *b'*; *a'* and *b'* are points in

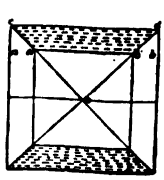


Fig. 88.

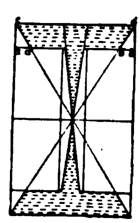


Fig. 89.

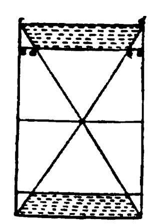


Fig. 90.

the curve of uniform stress. In a similar manner other points in the curves may be determined. Figs. 88 to 96 show the same method applied to other symmetrical sections.

In the case of unsymmetrical sections, such as a rail, Fig. 97, proceed as follows: Find the centre of gravity of the section,

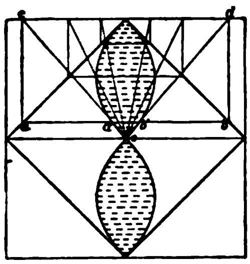


FIG. 91.

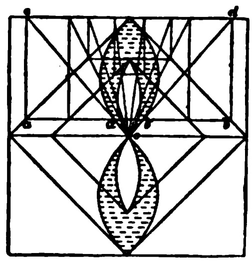


FIG. 92.

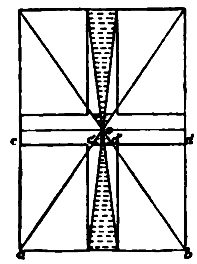


FIG. 93.

o; draw a line *ab* parallel to the line *cd*, and at a distance from *o* equal to *op*. Since *cd* is nearer to *o* than *ef*, it will offer a

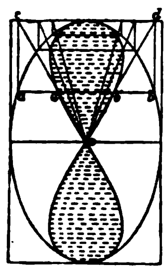


FIG. 94.

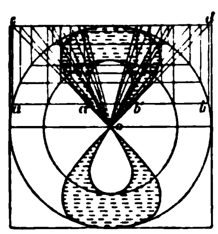


FIG. 95.

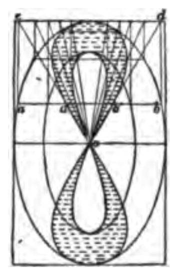


FIG. 96.

correspondingly smaller resistance, which is found by joining *a* and *b* to *o*, and noting the intersections *c'* and *d'* with *cd*. In

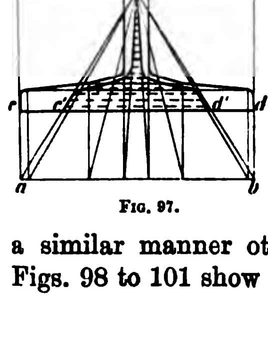


FIG. 97.

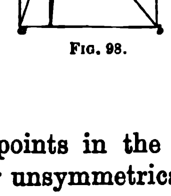


FIG. 98.

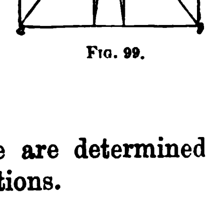


FIG. 99.

a similar manner other points in the figure are determined. Figs. 98 to 101 show other unsymmetrical sections.

Sir Benjamin Baker suggested[1] that the areas of the shaded portions may be obtained by cutting them out from a sheet of some material of uniform thickness and weighing them against strips of the same material one inch wide; thus, if zinc is the material, and a strip 10 inches long by 1 inch wide is used, pieces may be cut off and placed in the scale until an exact

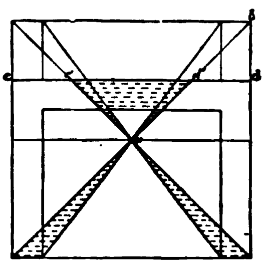


FIG. 100.

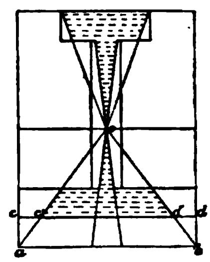


FIG. 101.

balance is obtained; the remainder subtracted from 10 inches gives the required area in square inches. The centres of gravity may be obtained by balancing each shaded area on a knife-edge. The accuracy of the work is proved by the equality of the two shaded areas on each side of the neutral axis, as demonstrated by weighing one against the other.

The following tables give the dimensions, weights per foot, and moduli of section of some wrought-iron and steel girders made by a few well-known manufacturers. The central breaking weight for any span may be found by the following equation as given in the beginning of the chapter :—

$$\frac{f\mathrm{I}}{y} = \frac{\mathrm{W}l}{4}$$

or, for a distributed load—

$$\frac{f\mathrm{I}}{y} = \frac{\mathrm{W}l}{8}$$

where W includes the weight of the girder, l being the span in inches.

The value of f may be taken at 22 tons for iron, and 28 tons for steel.

The strength of the steel girders manufactured by Messrs. Dormon, Long and Co., as given in their circulars, are calculated with 32 tons as the value of f.

[1] *Proc. Inst. C.E.*, vol. lxii. p. 251.

Table XVII.

Description.	Dimension in inches. Depth of girder.	Width of flanges.	Average thickness of flanges.	Minimum thickness of web rolled.	Sectional area in square inches.	Weight per lineal foot in pounds.	Modulus of section $= \frac{I}{y}$
Rolled steel girders manufactured by Carnegie, Phipps & Co., Limited, Pittsburg, Pa.	20·0	7·00	0·900	0·60	23·5	80·0	144·9
	20·0	6·25	0·765	0·50	18·8	64·0	114·6
	15·0	6·31	1·035	0·67	22·1	75·0	101·0
	15·0	6·04	0·900	0·54	17·6	60·0	85·9
	15·0	5·75	0·750	0·45	14·7	50·0	70·6
	15·0	5·50	0·590	0·40	12·0	41·0	56·6
	12·0	5·50	0·690	0·39	11·7	40·0	46·9
	12·0	5·25	0·535	0·35	9·4	32·0	37·0
	10·0	5·00	0·645	0·37	9·7	33·0	32·3
	10·0	4·75	0·485	0·32	7·5	25·5	24·7
	9·0	4·75	0·585	0·31	7·9	27·0	24·6
	9·0	4·50	0·440	0·27	6·2	21·0	18·7
	8·0	4·50	0·510	0·27	6·5	22·0	18·0
	8·0	4·25	0·410	0·25	5·3	18·0	14·4
	7·0	4·25	0·500	0·27	5·9	20·0	14·2
	7·0	4·00	0·390	0·23	4·6	15·5	11·0
	6·0	3·625	0·465	0·26	4·7	16·0	9·54
	6·0	3·50	0·375	0·23	3·8	13·0	7·83
	5·0	3·13	0·435	0·26	3·8	13·0	6·28
	5·0	3·00	0·335	0·22	3·0	10·0	4·96
	4·0	2·75	0·395	0·24	2·9	10·0	3·86
	4·0	2·625	0·290	0·20	2·2	7·5	2·95

TABLE XVIII.

Description.	Dimension in inches.				Sectional area in square inches.	Weight per lineal foot in pounds.	Modulus of section $=\frac{I}{y}$
	Depth of girder.	Width of flanges.	Average thickness of flanges.	Minimum thickness of web rolled.			
Rolled iron girders manufactured by Carnegie, Phipps & Co., Limited, Pittsburg, Pa	15·0	6·08	1·185	0·76	24·0	80·0	108·5
	15·0	5·45	0·970	0·57	18·0	60·0	83·4
	15·0	5·05	0·845	0·49	15·0	50·0	69·7
	12·0	5·16	0·875	0·78	17·0	56·5	58·1
	12·0	4·63	0·780	0·51	12·6	42·0	45·8
	10·5	4·80	0·735	0·55	12·0	40·0	33·4
	10·5	4·53	0·630	0·41	9·5	31·5	31·4
	10·0	4·75	0·895	0·50	12·6	42·0	39·8
	10·0	4·50	0·780	0·44	10·8	36·0	34·1
	10·0	4·31	0·675	0·37	9·0	30·0	29·2
	9·0	4·71	0·875	0·46	11·6	38·5	33·4
	9·0	4·16	0·660	0·40	8·6	28·5	24·5
	9·0	3·96	0·560	0·34	7·1	23·5	20·5
	8·0	4·50	0·775	0·50	10·2	34·0	25·5
	8·0	4·09	0·660	0·41	8·1	27·0	20·6
	8·0	3·71	0·560	0·33	6·5	21·5	16·5
	7·0	3·82	0·580	0·38	6·6	22·0	14·8
	7·0	3·52	0·545	0·26	5·4	18·0	12·6
	6·0	3·44	0·515	0·25	4·8	16·0	9·7
	6·0	3·24	0·435	0·24	4·1	13·5	8·1
	5·0	2·96	0·405	0·28	3·6	12·0	5·8
	5·0	2·85	0·360	0·23	3·0	10·0	5·0
	4·0	2·50	0·300	0·18	2·1	7·0	2·9
	4·0	2·18		0·18	1·8	6·0	2·3
	3·0	2·58	0·345	0·40	2·7	9·0	2·4
	3·0	2·22	0·285	0·16	1·7	5·5	1·7

Table XIX.

Description.	Dimension in inches. Depth of girder.	Width of flanges.	Average thickness of flanges.	Minimum thickness of web rolled.	Sectional area in square inches.	Weight per lineal foot in pounds.	Modulus of section $= \frac{I}{y}$
Rolled iron girders manufactured by the Butterley Iron Company.	20·00	10·00	1·2500	0·8125	39·22	140 to 144	236·8
	19·75	6·25	0·7500	0·6250	20·78	69 „ 70	101·5
	18·00	6·25	0·7500	0·6250	19·68	67 „ 70	89·8
	16·00	6·25	0·7500	0·6250	18·43	63 „ 66	78·1
	16·00	5·50	1·0000	0·6875	20·62	69 „ 72	89·0
	15·00	5·50	0·8750	0·6250	17·90	57 „ 60	78·2
	14·00	6·25	0·7500	0·6250	17·18	59 „ 62	64·6
	12·00	6·25	0·7500	0·6250	15·93	59 „ 62	59·0
	12·00	6·00	1·0000	0·7500	16·50	67 „ 77	70·5
	12·00	5·00	0·8125	0·5000	13·31	46 „ 50	47·6
	10·50	5·50	0·6250	0·5000	11·50	38 „ 41	31·5
	10·00	5·00	0·6250	0·5000	10·62	36 „ 40	29·8
	9·00	5·50	0·7500	0·5000	12·00	42 „ 45	33·8
	9·00	4·50	0·6250	0·5000	9·50	33 „ 37	25·0
	8·50	4·00	0·7500	0·5625	9·93	33 „ 36	24·5
	8·00	4·00	0·8750	0·8125	12·07	40 „ 42	27·3
	6·00	6·00	0·5625	0·5000	9·18	30 „ 32	17·6
	6·00	5·00	0·5625	0·4375	7·75	26 „ 28	14·5
	6·00	4·00	0·5625	0·5000	6·93	23 „ 25	12·7
	3·00	1·125	0·2500	0·15625	0·95	3 „ 4	0·7

TABLE XX.

Description.	Dimensions in Inches. Depth of girder	Width of flanges.	Average thickness of flanges.	Minimum thickness of web rolled.	Sectional area in square inches.	Weight per lineal foot in pounds.	Modulus of section $= \frac{I}{y}$
Rolled steel girders manufactured by Messrs. Dorman, Long & Co.	20·0	8·26	0·97	0·76	29·67	100·0	182·5
	20·0	8·18	0·97	0·68	28·24	95·0	177·2
	20·0	8·11	0·97	0·61	26·78	90·0	172·5
	18·0	6·90	0·94	0·61	22·88	78·0	128·6
	16·0	5·95	0·82	0·54	17·51	59·0	87·0
	15·0	6·01	0·81	0·64	16·96	57·0	82·5
	15·0	5·06	0·80	0·50	14·88	50·0	67·6
	14·0	5·87	0·81	0·50	15·76	53·0	70·4
	12·0	5·99	0·87	0·49	15·47	52·0	61·3
	12·0	4·98	0·65	0·48	11·60	39·0	43·0
	10·0	6·01	0·70	0·51	12·76	43·0	41·9
	10 0	5·03	0·66	0·52	10·26	34·5	34·7
	10·0	4·38	0·66	0·38	8·63	29 0	29·4
	9·0	6·90	0·81	0·66	10·06	54·0	46·7
	9·0	3·63	0·50	0·38	6·70	22·5	18·37
	8·0	5·90	0·62	0·40	10·12	34·0	27·54
	8·0	4·91	0·62	0·35	8·48	28·5	23·02
	8·0	3·91	0·56	0·31	6·55	22·0	18·91
	7·0	3·70	0·46	0·32	5·36	18·0	12·10
	6·25	3·38	0·50	0·26	4·76	16·0	9·96
	6·0	4·94	0·50	0·44	7·14	24·0	14·01
	6·0	2·99	0·50	0·29	4·46	15·0	8·60
	6·0	1·99	0·38	0·31	3·12	10·5	5·23
	5·5	1·99	0·38	0·31	2·97	10·0	4·61
	5·0	4·90	0·5625	0·33	6·82	23·0	11·55
	5·0	4·35	0·58	0·35	6·39	21·5	9·91
	5·0	3·04	0·46	0·30	4·02	13·5	5·78
	4·75	1·72	0·35	0·32	2·53	8·5	2·46
	4·62	3·04	0·40	0·29	3·57	12·0	4·70
	4·0	3·04	0·407	0·30	3·42	11 5	4·01
	4·0	1·76	0·35	0·26	2·08	9 0	2·06
	3·5	2·84	0·30	0·15	2·68	9·0	2·50
	3·5	1·51	0·30	0·20	1·49	5·0	1·33
	3·0	2·81	0·40	0·13	2·58	8·5	2·55
	3·0	1·80	0·25	0·21	1·19	4·0	0·82

Distribution of Shearing Stresses.—We have seen that when a beam is loaded as in Figs. 20 and 21, horizontal and vertical shearing stresses are developed.

If a rectangular prism of area A is subjected to a compressive force W along its axis (see Fig. 102), the intensity of stress, p, on a section normal to the axis is—

$$p = \frac{W}{A}$$

On any oblique section, such as ab, inclined at any angle α, the intensity of stress may be found by resolving W along and perpendicular to ab, and dividing by the area of the section over which the stress is distributed, thus—

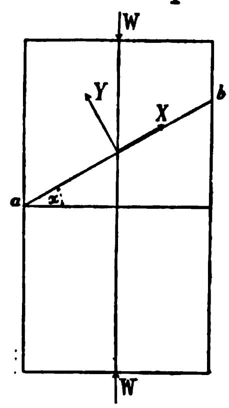


FIG. 102.

$$\text{Along } ab,\ X = W \sin \alpha$$
$$\text{Perpendicular to } ab,\ Y = W \cos \alpha$$
$$\text{Area } ab = A \sec \alpha$$

Therefore the intensity of stress is—

$$\text{Along } ab,\ W \sin \alpha \cos \alpha$$
$$\text{Perpendicular to } ab,\ W \cos^2 \alpha$$

If we consider the intensity of stress on a section at right angles to ab, we have only to substitute $\frac{\pi}{2} - \alpha$ for α in the above equations; thus we have—

$$\text{Along section, } W \sin \alpha \cos \alpha$$
$$\text{Perpendicular to section, } W \sin^2 \alpha$$

Hence the intensity of stress along two sections at right angles to one another is the same.

The greatest shearing stress will occur when $W \sin \alpha \cos \alpha$ is a maximum, *i.e.* when $\alpha = \frac{\pi}{4}$, or—

$$W\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{W}{2}$$

If we consider a cantilever loaded as shown in Fig. 103, the tensile and compressive stresses in the upper and lower flanges acting on a panel $abcd$ will produce equal and opposite reactions at the junction of the web with the flanges in the direction c to d and a to b, forming a couple the moment of which, if H denote the forces and x the depth of the cantilever, is Hx. This couple tends to produce rotation of the panel $abcd$; hence there must be developed a couple tending to produce an equal and opposite rotation, as c to b and a to d. If V denote the forces and y the arm of the couple, the moment is Vy. The intensity of the stresses on the sections ab and cd at right angles to each other will equal the forces divided by the areas over which they are distributed; thus, if h and v denote the intensity of the stresses—

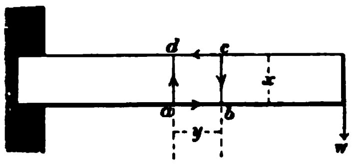


FIG. 103.

$$h = \frac{H}{y}, \text{ and } v = \frac{V}{x}$$

therefore, since $Hx = Vy$, $hyx = vyx$, and $h = v$. Hence the intensities of shearing stress on two planes at right angles to each other are equal, and if one is measurable, the other is known.

Let us consider the distribution of shearing stress on a solid beam of rectangular section, supported at both ends and loaded (Figs. 104 and 105).

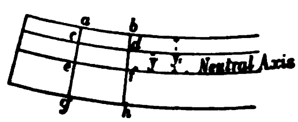


FIG. 104.

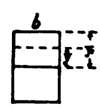


FIG. 105.

Let M = the bending moment of section $aceg$.
M_1 = " " " $bdfh$.
m = the distance between the sections $aceg$ and $bdfh$.
Generally M_1 will be greater than M.
Let y_1 = the distance of the top layer ab from the neutral axis, and y the distance cd.
b = the breadth of the beam.

The mean intensity of stress at a, due to bending moment M, is $\frac{M}{I}y_1$; at c, $\frac{M}{I}y$; at b, $\frac{M_1}{I}y_1$; at d, $\frac{M_1}{I}y$. The mean intensity of stress on bd is $\frac{M_1}{2I}(y_1 + y)$; and on ac is $\frac{M}{2I}(y_1 + y)$.

The total force with which the portion *bd* is pushed towards *ac* is equal to the difference between the mean intensities of stress multiplied by the area over which it is distributed, viz.—

$$\frac{M_1 - M}{2I}(y_1 + y) \times (y_1 - y)b = \frac{M_1 - M}{2I}(y_1^2 - y^2)b$$

The intensity of shearing stress along *cd* is equal to this force divided by the area of the portion *cd*, i.e. *mb*. Therefore the intensity of shearing stress along *cd* is—

$$\frac{M_1 - M}{2Ibm}(y_1^2 - y^2)b = \frac{M_1 - M}{2Im}(y_1^2 - y^2)$$

But if S denote the total shearing stress on the section *bdfh*, it has been shown that—

$$\frac{M_1 - M}{m} = S$$

$$\therefore \text{Intensity of shearing stress} = \frac{S}{2I}(y_1^2 - y_2)$$

The maximum intensity of stress occurs along the neutral axis, where $y = o$, and is—

$$\frac{Sy_1^2}{I}$$

In rectangular beams $y_1 = \frac{d}{2}$, and $I = \frac{bd^3}{12}$, hence the maximum intensity of shearing stress is—

$$\frac{3S}{2bd}$$

Fig. 106 represents the distribution of direct and shearing stresses on a beam of rectangular section.

The shaded area, *abcd*, is a graphical representation of the intensity of shearing stress at a distance y from the neutral axis; it equals—

$$\frac{S}{2I}(y_1^2 - y^2) = \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

$$\text{If } y = \frac{d}{3}, \text{ we have } \frac{6S}{bd^3}\left(\frac{d^2}{4} - \frac{d^2}{9}\right) = \frac{5S}{6bd}$$

$$\text{If } y = \frac{d}{6}, \text{ we have } \frac{6S}{bd^3}\left(\frac{d^2}{4} - \frac{d^2}{36}\right) = \frac{4S}{3bd}$$

The curve representing the distribution of shearing stress is a parabola, the ordinates of which are shown in Fig. 106.

The foregoing investigation shows that the intensity of horizontal shearing stress is greatest along the plane of the neutral axis, and, from the equality of the intensities of shearing stresses on planes at right angles to each other, it follows that the intensity of vertical shearing stress must equal the intensity of horizontal shearing stress at every point in the beam. In timber beams the resistance to shearing along the fibres is so small, especially in spruce and pine timbers, that it is necessary to so proportion the section that resistance to shearing along the neutral axis is at least as great as the resistance to bending. This can always be done when the modulus of rupture and shearing resistance along the grain are known.

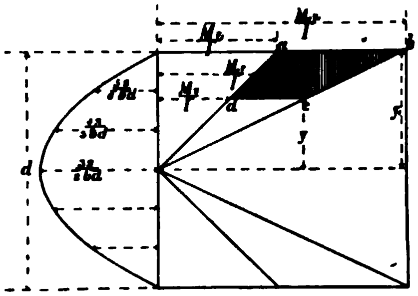


FIG. 106.

If W denote the total load uniformly distributed over the length of the beam; l the span, b the breadth, and d the depth of the beam; f the modulus of rupture; S_h the resistance to shearing along the grain; then—

$$W = \frac{4bd^2f}{3l}, \text{ and } S_h = \frac{3S}{2bd} = \frac{3W}{4bd}$$

$$\therefore W = \frac{4bdS_h}{3}$$

$$\therefore \frac{4bd^2f}{3l} = \frac{4bdS_h}{3}$$

$$\text{and } \frac{d}{l} = \frac{S_h}{f}$$

If the load is concentrated at the centre, as is usually the case when beams are tested—

$$\frac{d}{l} = \frac{2S_h}{f}$$

The values of S and f vary with the size of the specimen tested, being always smaller for large specimens than for small.

Professor Lanza has shown, by experimenting with spruce and yellow pine timber of large scantlings, such as are used in ordinary building construction, that the average values of f are—

For spruce	4451 lbs. per square inch.
For yellow pine	7486 „ „

The values of S_h in a large number of specimens, which failed by shearing along the grain, were—

For spruce	191 lbs. per square inch.
For yellow pine	248 „ „

These values for S_h are less than those obtained by direct experiments on shearing resistance, as in these cases the specimen is compelled to shear at a particular plane, whereas in testing an ordinary beam failure naturally occurs along the plane of least resistance. In the experiments referred to, failure occurred as often by shearing along the grain as by direct tensile stress developed at the extreme fibre. Using the average values given, we have for spruce—

$$\frac{d}{l} = \frac{191}{4451} = \frac{1}{23} \text{ for a distributed load}$$

$$\text{and } \frac{1}{11 \cdot 5} \text{ for a central load}$$

For yellow pine—

$$\frac{d}{l} = \frac{248}{7486} = \frac{1}{30} \text{ for a distributed load}$$

$$\text{and } \frac{1}{15} \text{ for a central load}$$

These experiments are confirmed by those made at the Watertown Arsenal and by Professor Bauschinger. Australian timber of ordinary proportions is not likely to fail by shearing along the fibres near the neutral axis, except from the presence of gum veins.

For ironbark timber—

$$\frac{d}{l} = \frac{2000}{18000} = \frac{1}{9} \text{ for a distributed load}$$

$$\text{and } \frac{1}{4 \cdot 5} \text{ for a central load}$$

If the depth of the beams exceeds the proportion of the span given in the above examples, the beam may fail by longitudinal shearing along the grain; if it is less, there will be no danger of shearing, and the strength of the beam will be proportional to its modulus of rupture.

CHAPTER VI.

BRIDGES AND VIADUCTS OF SMALL SPAN IN TIMBER AND IRON—TIMBER PILE BRIDGES.

IN America, Australia, and New Zealand native timbers are largely used for the construction of highway and railway bridges, as well as in ordinary building construction, and there is reason to believe that they will continue to be used extensively in the future. Timber is also used more or less in other countries.

The following examples have been selected to show how to design beam bridges, and as illustrations of the foregoing principles.

Consider, in the first place, a road bridge over a small stream as shown in Figs. 107, 108, and 109.

The bridge consists of three spans, each 30 feet from centre to centre, and the width of roadway is 15 feet. The abutments and piers are also of timber. Each span is formed with four beams spaced 5 feet centres across the bridge; the two outer beams are 12 inches square, and the two inner beams are 17 inches in diameter, adzed on the upper side and on the lower side at the ends where they rest upon corbels.

The deck consists of timber boards 4 inches thick resting upon the main beams, and there is a timber hand-rail on each side. The strength of the bridge is calculated as follows:—

Let w denote the load per square foot which the bridge will carry safely if distributed uniformly over the deck, then the total load on each inner beam is—

$$5 \times 30 \times w = 150w \text{ lbs.}$$

The bending moment is—

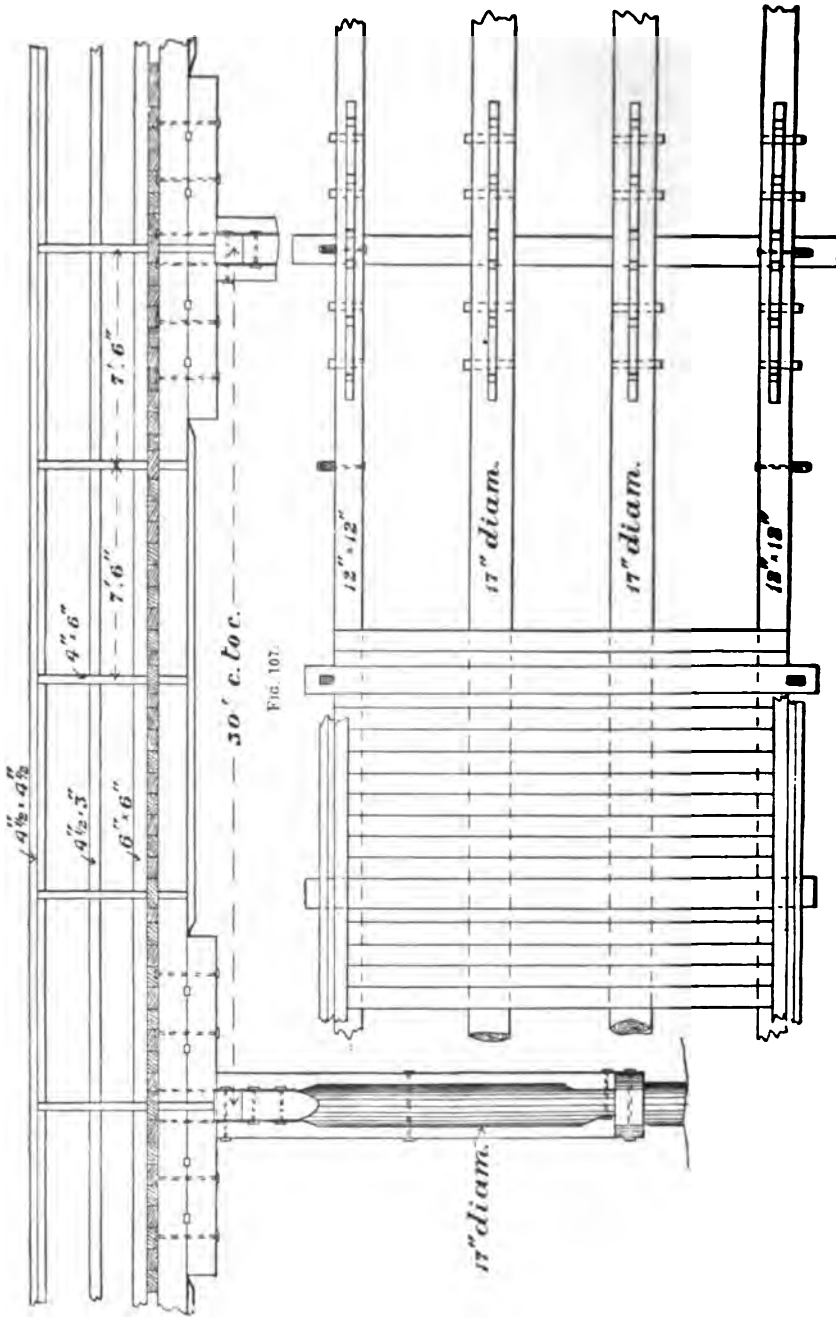


FIG. 107.

FIG. 108.

$$\frac{150w \times 30 \times 12}{8} = 6750w \text{ inch-pounds}$$

The moment of resistance is—

$$\cdot 7854R^3 f$$

If the timber is New South Wales ironbark, the value of f may be taken at 11,000 lbs. per square inch, since the scantling is large and may not consist entirely of sound timber; for other Australian timbers, such as jarra and gums, f may be taken as from 6000 to 8000 lbs. per square inch for such large scantling.

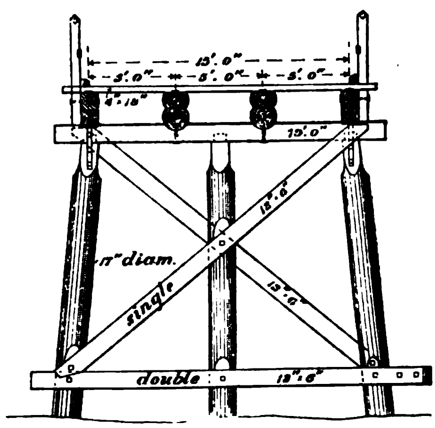


FIG. 109.

The Roads and Bridges Department, New South Wales, usually allow a factor of safety of from 3 to 4 for such bridges, which gives for the best ironbark a maximum working fibre stress of $\frac{11000}{3\cdot 5} = 3142$ lbs. per square inch, say 3000 lbs. per square inch. Then the moment of resistance for a beam 17 inches in diameter is—

$$0\cdot 7854 \times 8\cdot 5^3 \times 3000 = 1{,}447{,}000 \text{ inch-pounds}$$

and the moment of resistance of the 12 × 12 beams is about half this amount, since the outside beams carry about half as much as the inner beams, hence—

$$6750w = 1{,}447{,}000$$
$$\therefore w = 214 \text{ lbs. per square foot}$$

The deck is usually made of tallow-wood, which is an excellent timber for the purpose, and weighs 80 lbs. per cubic foot, or 20 lbs. per square foot 3 inches thick.

The weight of the main beams and hand-railing is about 27 pounds per square foot of deck, leaving 187 lbs. per square foot for the live load.

These bridges, however, are supposed to carry two traction engines, each weighing 16 tons, on two wheels 10 feet 4 inches apart, so that they are not unnecessarily strong, as would at first sight appear.

If such a bridge were constructed with pine timber beams, they would require to be spaced closer together across the bridge, or the span reduced.

The following table shows the sizes of girders used in the Roads and Bridges Department, New South Wales, for bridges, similar to the above:—

TABLE XXI.

Width of span.	Corbels.	Inner girders.	Outer girders.	In single spans.	
				Inner.	Outer.
28′ 0″	One 6′ 0″ × 12″ × 12″	17″ diam.	12″ × 12″	18″ diam.	14″ × 12″
30′ 0″	Two 6′ 0″ × 12″ × 12″	17″ "	12″ × 12″	19″ "	14″ × 13″
35′ 0″	Two 7′ 0″ × 13″ × 12″	18″ "	13″ × 12″	20″ "	15″ × 13″
40′ 0″	Two 8′ 0″ × 14″ × 13″	19″ "	14″ × 13″	20″ "	16″ × 13″
45′ 0″	Two 9′ 0″ × 15″ × 13″	20″ "	15″ × 13″	21″ "	16″ × 13″

As a second example, take a timber railway viaduct for a single line of way constructed over a considerable length of low-lying ground liable to floods near a river, which form the approaches to the main bridge over the river. If the foundations are fairly good, and the height of the rail-level above the

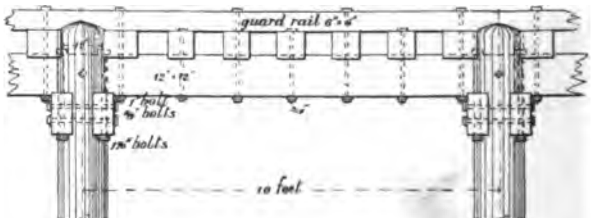


FIG. 110.

ground is about 8 feet, a series of spans of 10-feet centres, such as shown in Figs. 110 and 111, will be found to be most economical.

If the height of the rail-level above the ground be from

15 to 20 feet, it may be found to be cheaper to design for spans of 24-feet centres, such as shown in Figs. 112, 113, and 114.

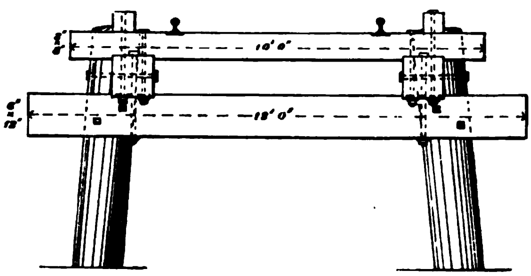


FIG. 111.

In each case an open deck is used, similar to the deck so largely used on American bridges. The sizes of the timber sleepers, if Australian timber is used, must be decided with reference to the most convenient sizes obtainable; thus it is easier to obtain a good durable sleeper 12 inches wide by 7 inches deep from ironbark trees than one 8 inches square.

Fig. 115 shows an American deck with ironbark sleepers suitable for the type of viaduct illustrated, and Fig. 116 shows a similar deck in which a central beam is used. The former would be suitable for an iron or steel bridge in which the plate web-girder stringers take the place of the main timber beams, we will therefore consider it more fully.

The strength of the deck may be calculated as follows for Fig. 115: The maximum load on the driving-wheels of a locomotive does not generally exceed 16 tons per pair of wheels, therefore the weight on each wheel is 8 tons; but since this weight may be increased on one side, and correspondingly reduced on the other, from the oscillation and plunging of the engine produced in various ways, it is assumed that the total maximum effect may reach 10 tons per wheel.

At least three sleepers will take part in carrying the weight brought on any one of them, the sleeper immediately under the driving-wheel taking one half of this weight, and the sleepers on each side taking one quarter each, so that the maximum value for W, Fig. 115, is 5 tons.

The bending moment is 7·5 foot-tons, or 90 inch-tons.

The moment of resistance for ironbark, where $f = 14,000$ lbs. per square inch since the section is comparatively small, is—

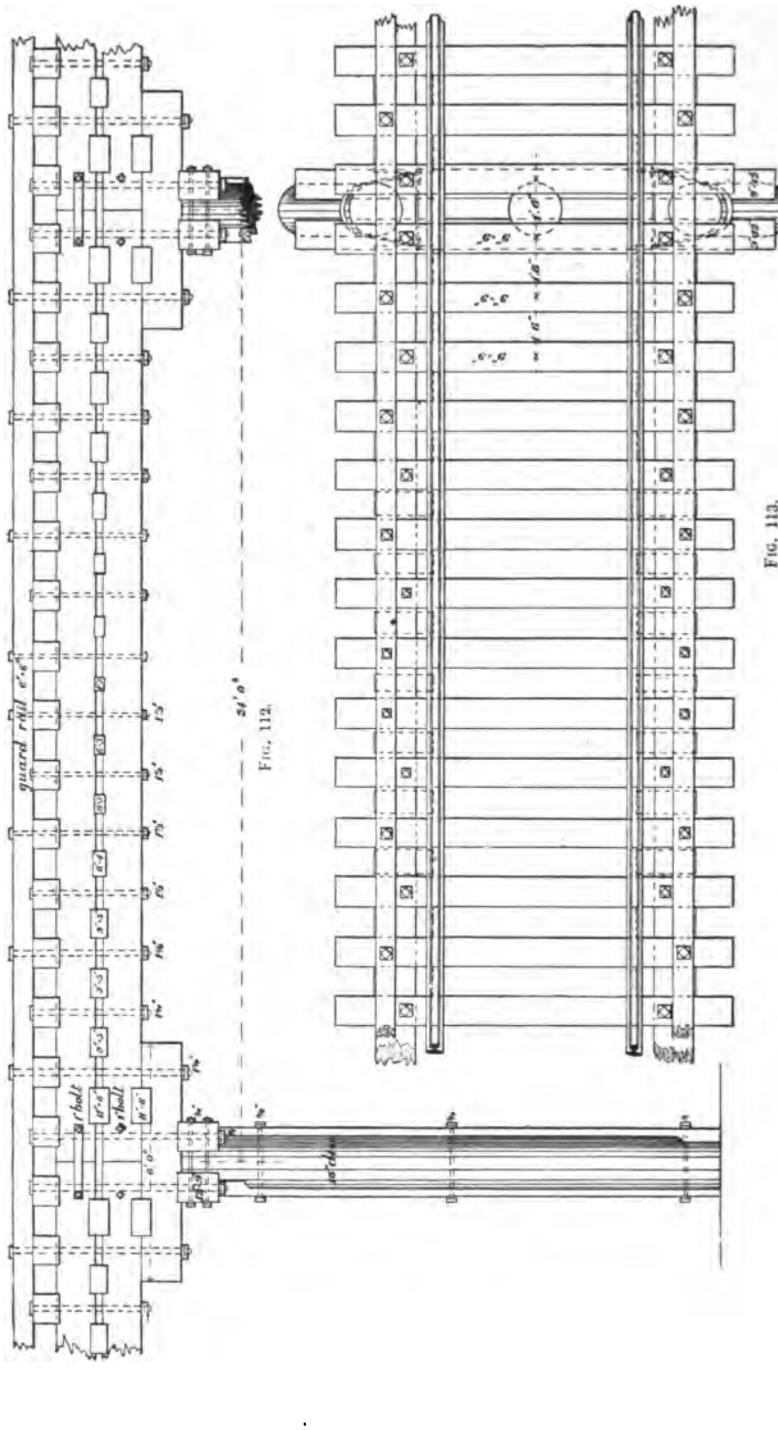


Fig. 112.

Fig. 113.

$$\frac{bd^2}{6}f = \frac{12 \times 49 \times 14000}{6} = 1{,}372{,}000 \text{ inch-pounds}$$
$$= 612 \text{ inch-tons}$$

Therefore the factor of safety is $\frac{612}{90}$ = about 6·8.

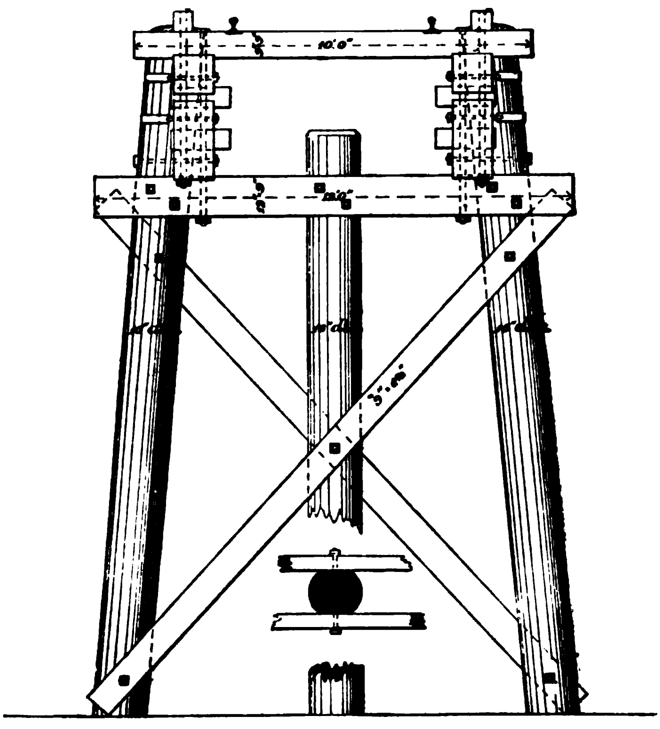


FIG. 114.

If Oregon timber is used, f will be about 7000 lbs. per square inch, and the section must be increased to 10 inches

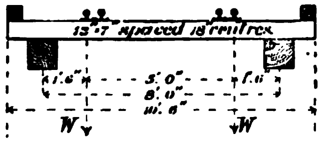


FIG. 115.

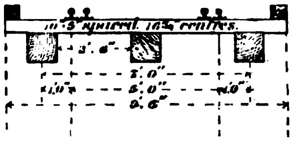


FIG. 116.

square with 10 inches space between sleepers, or 20 inches centre to centre, in which case—

$$\frac{bd^2f}{6} = \frac{10 \times 100 \times 7000}{6} = 1{,}166{,}666 \text{ inch-pounds}$$

$$= 521 \text{ inch-tons}$$

Therefore the factor of safety is $\frac{521}{90} = 5{\cdot}8$ nearly.

The sizes of sleepers have been calculated on the assumption that only three sleepers carry the load on the driving-wheel; this assumption is on the safe side, as the iron guard-rails and the timbers very probably distribute the weight over more than three sleepers.

This kind of deck in America is usually constructed with 8×8 timbers spaced 16 inches centre to centre, thus leaving an 8-inch space. The advantages of this deck over the ordinary close timber deck are as follows: The sleepers are spaced sufficiently close to allow the wheels of a carriage to run without sticking in the case of derailment; and they are exposed on four sides to the air, and therefore dry uniformly and last a much longer time; moreover, any defective sleeper can be readily seen and replaced. The timber guard-rail is a precaution against the effect of derailment, and serves to some extent in assisting to distribute the weight of the driving-wheels. The spacing is large enough to allow hot material from the engine to fall between the sleepers. The number and strength of the sleepers gives considerable lateral strength and stiffness to the structure, enabling it to resist wind pressure and oscillations. The 8-feet spacing of the main beams ensures an elastic road and easy running.

The strength of the main beams in the 10-feet spans of the viaduct will now be considered.

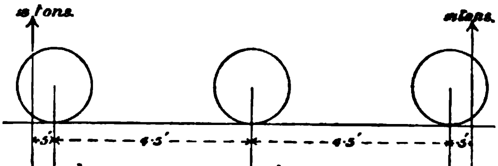


FIG. 117.

Fig. 117 shows the position and loads produced by the three wheels of a heavy consolidation, mogul, or other engine with axle loads of 12 tons. The maximum bending moment occurs at the centre of the span, and is—

$$18 \times 5 - 12 \times 4 \cdot 5 = 36 \text{ foot-tons} = 432 \text{ inch-tons}$$

The dynamic effect of the live load will increase this bending moment by about 30 per cent., so that it may be taken as 562 inch-tons, or 46·8 foot-tons.

The equivalent uniformly distributed load which will produce the same moment in the centre of the beam is found thus—

$$\frac{wl^2}{8} = 46 \cdot 8 \qquad \therefore w = \frac{8 \times 46 \cdot 8}{10 \times 10} = 3 \cdot 75 \text{ tons per foot run}$$

The dead load, consisting of main beams, sleepers, rails, guard-rails, and guard-timbers, is about 0·25 ton per foot run, so that the totally equivalent uniformly distributed load may be taken as 4 tons per lineal foot, and the maximum bending moment is—

$$\frac{4 \times 10 \times 10}{8} = 50 \text{ foot-tons, or } 600 \text{ inch-tons}$$

The moment of resistance of two beams of ironbark timber, each 12 by 12 inches, is—

$$\frac{2 \times 12 \times 12 \times f}{6} = 576f \text{ inch-tons}$$

If f is taken as 5·5 tons per square inch, which may reasonably be expected from good ironbark timber, the moment of resistance is 3,168·0 inch-tons, and the factor of safety—

$$\frac{31680}{600} = 5 \cdot 28$$

The factor of safety is quite sufficient for such a structure of ironbark timber.

Figs. 118, 119, and 120 illustrate the standard pile-trestle on the Toledo, St. Louis, and Kansas City Railroad.[1] The sleepers are 9 feet long × 8 inches wide × 6 inches deep, and the guard-rails 6 inches × 6 inches, notched 1 inch over sleepers.

[1] "Treatise on Wooden Trestle Bridges," by Wolcott C. Foster. Published by J. Wiley and Sons, New York.

The main beams or stringers are arranged in groups of three under each of the rails, and are 7 inches wide × 16 inches deep on a span of 15 feet.

Cast-iron separators 4 inches thick are arranged between

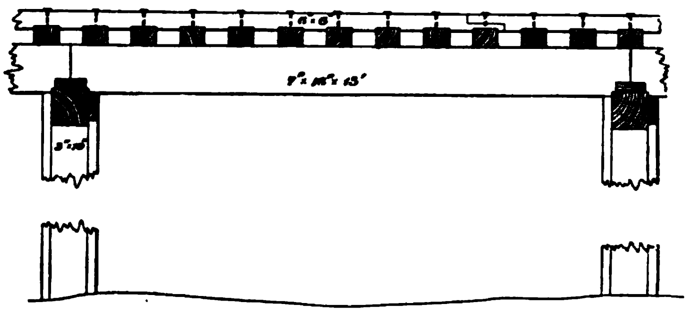


FIG. 118.

the stringers over the caps, and the stringers are protected with sheet iron 30 inches wide, as shown in Fig. 119. The caps are 14 feet long × 12 inches × 12 inches, notched 1 inch

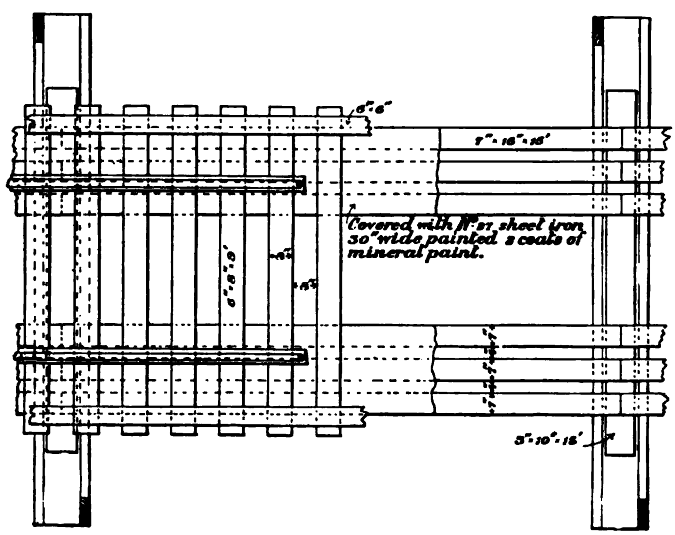


FIG. 119.

over the four piles, and the sway braces are 10 inches × 3 inches. On the Pennsylvania Railroad the sizes of the stringers used are given in the following table :—

TABLE XXII.

Clear span in feet.	Number of pins under each rail.	Width of each piece in inches.	Depth of stringer in inches.
10	2	8	15
12	2	8	16
14	2	10	17
16	3	8	17

The stringers are generally long enough to extend over two trestles, and should break joint, and should be securely fastened to the caps by means of drift-bolts.

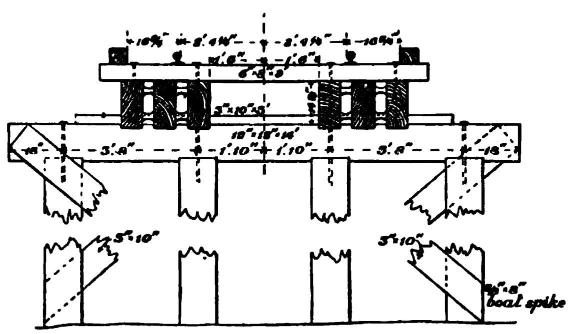


FIG. 120.

Fig. 121 shows the method of connecting the stringers over the caps adopted in the Pennsylvania Railroad, in which the tendency of the joint, when it settles under a weight through the support becoming weakened, is to close at the top and to open at the bottom. This arrangement provides most material where there is the greatest liability to split. The packing strips are notched over the caps as shown. On the Chicago, Milwaukee, and St. Paul Railroad, the standard pile-trestle in 1890 had six stringers in two groups of three, each 16 inches deep × 8 inches wide, on a span of 15 feet 9 inches, and two outside stringers each 16 inches deep × 6 inches wide. The dimensions of the stringers will depend upon the character of the timber available, and the train loads.

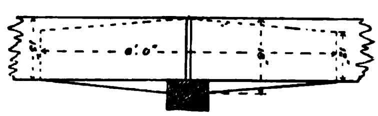


FIG. 121.

With an equivalent uniform load of 3·5 tons per foot run, allowing for the dynamic effect of the live load, and stringers 16 inches × 8 inches on a span of 15 feet, we have—

$$\text{Bending moment} = \frac{3\cdot5 \times 15 \times 15}{8} = 98\cdot43 \text{ foot-tons}$$
$$= 1181 \text{ inch-tons}$$

The moment of resistance of the six beams is—

$$\text{Moment of resistance} = \frac{6 \times 8 \times 16 \times 16}{6} f = 2048 f$$

For white pine or spruce Professor Lanza recommends $f =$ 3000, and the experiments of Mr. Onward Bates on white pine stringers[1] show that this value is about correct; hence—

$$\text{Moment of resistance} = 2048 \times 3000 = 6{,}144{,}000 \text{ inch-pounds}$$
$$= 2743 \text{ inch-tons}$$

Therefore the factor of safety would be—

$$\frac{2743}{1181} = 2\cdot3$$

For yellow pine $f = 5000$, and the factor of safety is 3·83. For Oregon fir $f = 6500$, and the factor of safety is 5·4. The latter value of the factor of safety may be considered sufficient, but it is clear that American engineers do not use large factors of safety in their timber structures.

There are two distinct classes of timber trestle piers—(*a*) framed trestles, (*b*) pile trestles.

Framed trestles consist of square timber framed together; they are usually built upon a foundation of some kind, which may be a pile foundation, masonry, concrete, grillage, crib, solid or loose rock.

Framed trestles are provided with a sill which rests upon the foundations, and, in order that decay may be delayed as long as possible, the sill should be above the ground, and not covered with earth.

Framed trestles are used for almost any height, but they are exclusively used for lofty timber viaducts.

Pile trestles consist of two or more piles surmounted by a cap. When the height is less than 10 feet sway bracing is not used; above this height sway braces are used, bolted to the cap

[1] *Trans. Amer. Soc. C.E.*, November, 1890.

and to the piles at their intersection. Pile trestles rarely exceed 30 feet in height; from 10 to 20 feet the outside piles are driven with a batter of from 1 to 3 inches per foot.

The timber used for piles should be straight and sound, free from wind-shakes, wanes, large loose black or decayed knots, cracks, worm-holes, and all descriptions of decay, and should be stripped of bark; they must be cut off square at the butt, and be properly sharpened.

Piles of Australian hardwood, which is liable to decay at the heart, should be free from large pipes.

The heads and feet of piles should be protected before driving, if they are liable to be injured by the hammer or ram. A wrought-iron ring is generally used at the head, and some form of iron shoe at the foot according to the nature of the material into which the pile is driven. Timber piles for viaducts are usually driven by a drop-hammer or ram, other methods will therefore not be considered. An upright frame is used with a pulley at the top, over which a rope passes which supports the hammer or ram. The frame consists of two uprights called leaders, from 10 to 60 feet long, placed about 2 feet apart, which guide the falling ram. The weight of the ram varies from 500 to 7000 lbs., but is usually about one ton, and it is provided with grooves or guides to fit the guides. The rope is usually wound up and the weight raised by a steam-engine, but horses may be used hitched directly to the end of the rope, or men may be employed working a windlass or pulley directly at the rope. There are two methods of detaching the weight, *i.e.* letting the ram fall (*a*) by means of a nipper, (*b*) by means of a friction clutch.

A pile is considered to be sufficiently driven if the penetration from the last five blows with a ram weighing 2000 lbs. falling 25 feet does not exceed 5 inches. Heavier rams with smaller falls give better results, such as at Brooklyn, where a ram weighing 6720 lbs. was used falling 3 feet.

The bearing power of piles may be considered in the following manner—When a pile rests upon a hard stratum it should be treated as a long column, and its bearing capacity may be inferred by means of the tables, pp. 37, 38, and 39, Chapter II., or it may be calculated by means of formulæ such as those given in Chapter XII. The safe bearing pressure on a pile driven into a yielding stratum may be found in the following manner:—

(*a*) By considering the relationship between the supporting power and the length and size of the pile, the weight of the ram, height of fall, and the distance the pile was moved by the last blow. The pile is assumed to be driven under ordinary conditions; the head, if broomed or battered unreasonably, should be sawn off before striking the test blow; and it is also assumed that the pile has been sinking with a fair degree of regularity under the last few blows, and that the apparent uniformity of set is not deceptive.

(*b*) By applying a load or direct pressure to each of a number of piles, observing the amount each will support, and expressing the result in terms of the depth driven, size of pile, and kind of soil. The former method may be expressed in a rational formula, the latter in an empyrical formula.

The former method has been used more or less in the formulæ of Rankine, Weisbach, Sanders, Professor Baker, Trautwine, Crowell, Wellington, and others.

The energy accumulated in the ram when it strikes the pile head is Wh, where W = weight of ram in pounds, and h = height of fall in feet. This energy is expended in compressing the ram and the head of the pile, in moving the ram as a whole against the resistance of the soil, in overcoming the inertia of the pile and soil. Only Wellington's formula will be here given, as it is a rational formula less complex than the others, and gives quite as good results.

Let L denote the bearing pressure; s, the penetration under the last blow in inches; then—

$$L = \frac{12Wh}{s+1}$$

Wellington recommends a factor of safety of 6, so that the safe working pressure, L′, is—

$$L' = \frac{2Wh}{s+1}$$

Fig. 112 shows a timber viaduct for 24-feet centres, designed to carry a single line of railway. In this example there are two main compound beams, each formed with two beams of iron-bark timber 12 inches square, bolted together, with a space of two inches between them, with wedges inserted in corresponding notches cut in the beams, and resting upon corbels over the piers. An American deck of ironbark timber is shown similar to that of the 10-feet spans. In order that the compound beam should be equal in strength to a solid beam 24 inches in depth, it is necessary that the wedges and bolts should be capable of resisting the maximum horizontal and vertical shearing stresses. Since the bending moment increases as the square of the span, and the moment of resistance of the beam as the square of the depth, it follows that a span of 20 feet would give the same factor of safety as that found for the 10-feet spans, provided the equivalent distributed load remained the same. The equivalent distributed load on a 24-feet span, with consolidation engines having 12 tons on each of the four driving-axles, would be about 3·1 tons per foot run; hence the span may be increased from 20 feet to $20 \times \frac{3{\cdot}75}{3{\cdot}1} = 24$ feet. The total depth of the compound beam is 26 inches, but it will be taken as 24 inches solid throughout. The area is reduced in the centre by the bolt-hole, and to a slight extent by the wedges, but this is more than compensated for by the reduction of span due to the corbels over piers.

The shearing stress over the piers is—

$$\frac{3{\cdot}1 \times 24}{2} = 37{\cdot}2 \text{ tons}$$

or 18·6 tons for each beam, which is distributed over the 24 inches by 12 inches in the manner explained in Chapter V., Fig. 106.

The maximum shearing stress horizontally and vertically is, therefore—

$$S = \frac{3 \times 18{\cdot}6}{2 \times 24} = 1{\cdot}16 \text{ ton per lineal inch of beam}$$

The wedges immediately over the corbels are spaced 15 inches centre to centre, and will have to resist a horizontal shearing stress of $15 \times 1{\cdot}16 = 17{\cdot}4$ tons, and the corresponding bolts will have to resist a vertical shearing stress of the same amount.

Let x denote the width of the wedges, measured along the beam; the area exposed to shearing along the fibre (neglecting the portion of the wedge which projects beyond the beam for driving) is $12x$. The safe intensity of shearing stress may be taken as 450 pounds per square inch in the timbers which are most suitable for this purpose (which require at least 2000 pounds per square inch to shear them along the grain), therefore the resistance of the wedge is—

$$12 \times 450 \times x = 5400x = 17{\cdot}4 \times 2240$$
$$\therefore x = 7{\cdot}22 \text{ inches}$$

The wedges may therefore be made 7½ inches wide by 6 inches deep. The area required in the bolts is $\frac{17{\cdot}4}{6} = 2{\cdot}908$ square inches, or 2 inches in diameter. The working stress on the bolts is taken as 6 tons.

The shearing stress at any other point may be calculated from the formula—

$$S_1 = \frac{w(c + x)^2}{4c}$$

If $x = 0$, $S_1 = 4{\cdot}65$ tons; if $x = 6$, $S_1 = 10{\cdot}5$ tons; and these stresses are distributed over the section of the beam as before. The maximum shearing stress per lineal inch in the centre of the beam is, therefore—

$$\frac{3 \times 4{\cdot}65}{2 \times 24} = 0{\cdot}291 \text{ tons}$$

The bolts and wedges are spaced 18 inches centre to centre, therefore the central bolts may be called upon to resist a stress of—

$$18 \times 0{\cdot}291 = 5{\cdot}24 \text{ tons}$$

The area required is consequently $\frac{5{\cdot}24}{6} = 0{\cdot}874$ square inches, or say 1⅛ inch in diameter.

In a similar manner it may be shown that the bolts 6 feet from the centre of the beam will require to be 1⅝ inch in diameter, and the width of the corresponding wedges—

$$5400x = 11{\cdot}8 \times 2240 \quad \therefore x = 4{\cdot}89 \text{ inches}$$

hence the section of the wedge may be made 5 inches by 4 inches.

The sizes of the remaining bolts and wedges may be determined in a similar manner. The design of the piers and abutments is sufficiently illustrated in Figs. 112, 113, and 114.

In the foregoing calculations, it has been assumed that each wedge and bolt resists the shearing stresses which are allotted to them according to their position in the span, the largest wedges and bolts occurring near the points of support. Some engineers make the wedges and bolts uniform in section throughout the span, while others omit the wedges in the centre third of the span. If heavier engines are used than those considered, as, for example, on the New South Wales railways, where the heaviest consolidation engine would produce a bending moment in the centre of a 24-feet span which is equivalent to a uniform load (allowing for dynamic effect) of 4 tons per foot run, in such a case it is found to be most convenient to arrange three compound beams, so that each carries $\frac{4}{3}$ of a ton per foot run for the live load, and about 0·4 ton per foot run for the dead load. The wedges are made 6 inches wide by 3 inches deep, and the bolts 1 inch in diameter throughout.

We will now consider a compound beam consisting of two beams of spruce timber, each 12 inches by 12 inches, carrying a floor in a building over a span of 30 feet. The two beams are bolted together, and wedges are inserted between the beams. Let it be required to determine the sizes of the wedges and bolts, (*a*) for carrying a distributed load, and (*b*) for carrying a central load. The working stress on the extreme fibres of the spruce beam may be taken at 1000 lbs. per square inch. The shearing resistance of the keys or wedges, which should be made of suitable timber, may be taken at 200 lbs. per square inch. The bolts may be stressed up to 12,000 lbs. per square inch.

(*a*) Distributed load.

$$\text{Moment of resistance} = \frac{12 \times 24 \times 24 \times 1000}{6}$$

$$= 1{,}152000 \text{ inch-pounds}$$

$$\text{Bending moment} = \frac{Wl}{8} = \frac{W \times 30 \times 12}{8} = 45W$$

$$\therefore 45W = 1{,}152{,}000$$

$$\therefore W = 25{,}600 \text{ lbs.}$$

Hence the reactions at supports = 12,800 lbs., and the maximum shearing stress per lineal inch is—

$$\frac{3S}{2d} = \frac{3 \times 12800}{2 \times 24} = 800 \text{ lbs.}$$

Let the wedges be the same length as the width of the beam, and assume that the beams are notched each 3 inches deep, and the depth of the wedges is 6 inches. Assume also that the wedges are spaced 3-feet centres at the ends, and let $x =$ the width of the wedges; then—

$$12x = \text{area of wedge}$$
$$\text{and } 12x \times 200 = 2400x = \text{resistance to shearing}$$
$$\therefore 2400x = 800 \times 36$$
$$\therefore x = 12 \text{ inches}$$

If the wedges are spaced 24 inches centre to centre, they will only require to be 8 inches wide. Adopting this latter pitch for the wedges, the area of the end bolts will be—

$$a = \frac{24 \times 800}{12000} = 1{\cdot}6 \text{ square inches}$$
$$\text{and the diameter} = 1\tfrac{7}{16} \text{ inch}$$

At a distance of 7 feet 6 inches from the supports, only half of the area will be necessary if the same pitch is adopted throughout, or the pitch may be doubled for the middle half of the beam, retaining the same sections in wedges and bolts throughout.

The reduction in the moment of resistance in the centre of the beam may be neglected.

(*b*) If the beam carries a load in the centre—

$$\frac{Wl}{4} = \frac{W \times 30 \times 12}{4} = 90W \text{ inch-pounds}$$
$$\therefore 90W = 1{,}152{,}000$$
$$= 12{,}800 \text{ lbs.}$$

The shearing stress is uniform throughout the beam, excepting at the centre, where it is zero; hence—

$$\frac{3S}{2d} = \frac{3 \times 6400}{2 \times 24} = 400 \text{ lbs. per lineal inch}$$

Hence, if the bolts and wedges are spaced at 4-feet centres, they will have the same dimensions as the largest in the last example, namely, 8 inches wide by 6 inches deep.

In order to test the accuracy of the foregoing theory of the compound beam, the author has made several large-size scale models, which, when tested, gave results in close agreement with the calculations. Compound beam road bridges have been used in Australia up to a span of 42 feet, the main beams being formed with three beams each 12 inches square.

Bolts, Washers, Spikes, etc.—The bolts used in holding stringer pieces together, fastening on the sway-braces, guard-rails, etc., are usually made ¾ inch in diameter. In compound timber beams the bolts may be from 1 inch to 2 inches in diameter, being proportioned to resist the shearing stresses in the manner explained. Cast or wrought iron washers should be placed under the heads and nuts of all the bolts in a timber structure. Sometimes the washers under the nut are provided with slots, which enable the nut to be locked by driving in a nail close to the nut after it has been screwed down tightly.

Cast-iron washers for bolts ¾ inch to 1¼ inch in diameter vary from 3 inches to 4½ inches in diameter by ¾ inch thick. Drift-bolts are largely used in timber structures, such as in fastening caps to posts; they are usually made of iron, ¾ inch square or ¾ inch round. They should always be long enough to penetrate the last timber which it is desired to be held to a depth sufficient to give a good firm hold. Drift-bolts have square or taper heads, and are pointed at the ends; they are really long spikes.

Experiments made by the United States Government show that smooth rods have a greater holding power than ragged ones; that the resistance ten months after being driven is 10 per cent. greater than the resistance immediately after driving; that round drift-bolts are 25 per cent. more efficient per pound of metal than square ones. The holding power of a round bolt 1 inch in diameter in a ¾-inch hole, with white pine and hemlock timber, is about 10,000 lbs. per lineal foot immediately after driving. With Norway pine the resistance is about 9000 lbs. per lineal foot.

The resistance of screw-bolts was found to be about 50 per cent. greater than that of plain round rods. All bolt-holes in pine timbers should be bored ⅛ inch smaller than the bolt. The holding power of drift-bolts or dog-spikes in Australian hardwood timbers is much greater than in pine timbers; but the actual resistance developed has not been determined, as far as the author is aware.

Painting Timber.—Whenever two surfaces of timber touch, they should be painted with white lead or tar; the bolts should also be coated with white lead and linseed oil, hot or cold tar. Mr. Wolcott C. Foster recommends[1] the following specification for creosoted trestles, piles, and other timber:—

"All piles used in creosoted trestles must have the bark peeled off, and be pointed before treatment. None of the sap wood must be hewn from the piles. No notching or cutting of the piles will be allowed after treatment, except the sawing off of the head of the pile to the proper level for the reception of the cap, and the levelling of such part of the head as may project from under the cap.

"The heads of all creosoted piles, after the necessary cutting and trimming has been done to receive the cap, must be saturated with hot creosote oil, and then covered with hot asphaltum before putting the caps in place.

"Timber in creosoted trestles must be cut and framed to the proper dimensions before treatment. No cutting or trimming of any kind will be allowed after treatment, except boring of the necessary bolt-holes. Hot creosote oil must be poured into the bolt-holes before the insertion of the bolts, in such a manner that the entire surface of the holes shall receive a coating of creosote oil. All creosoted timber and piles shall be prepared in accordance with the following process:—

"The timber and piles, after having been cut and trimmed to the proper length, size, and shape, shall be submitted to a contact steaming inside the injection-cylinders which shall last from two to three hours, according to the size of the timbers; then to a heat not to exceed 230° Fahr. in a vacuum of 24 inches of mercury, for a period long enough to thoroughly dry the wood. The creosote-oil, heated to a temperature of about 175°, shall then be let in the injection-cylinder, and forced into the wood under a pressure of 150 pounds per square inch, until not less than 15 lbs. of oil to the cubic feet has been absorbed."

"The oil must contain at least 10 per cent. of carbolic and cresylic acids, and have at least 12 per cent. of naphthalin."

Small Flood Opening.—Figs. 122, 123, and 124 illustrate a small flood opening carrying a single line of way, which is constructed with four rolled iron or steel girders resting on cast-iron bed-plates attached to the coping stone of the brick supports. The span is 16 feet in the clear, and 18 feet effective between centres of bearings. The rolled girders are 18 inches deep, with flanges

[1] "A Treatise on Wooden Trestle Bridges."

6¼ inches by ¾ inch and web ⅝ inch thick, weighing from 67 to 70 lbs. per foot. If the girders are selected from the Butterley

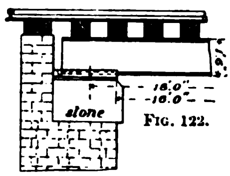


FIG. 122.

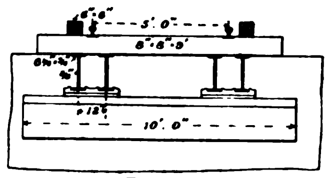


FIG. 124.

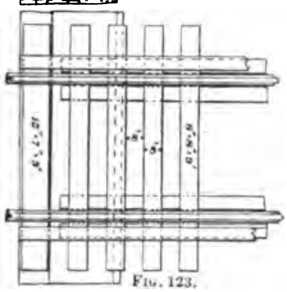


FIG. 123.

Iron Company's list, the central breaking load is 32 tons. The moment of inertia is 932, and the moment of resistance, allowing a working stress of 3 tons per square inch, is—

$$\frac{fI}{y} = \frac{3 \times 932}{9} = 310{\cdot}6 \text{ inch-tons}$$

The bending moment in the centre for the engine shown in Fig. 327 is—

$$\frac{2{\cdot}25 \times 18^2 \times 12}{8} = 1093{\cdot}5 \text{ inch-tons}$$

The dead load is about 0·3 tons per foot run, and the bending moment 145·8 inch-tons. Hence we require—

$$\frac{1239{\cdot}3}{310{\cdot}6} = 4 \text{ girders}$$

The details are sufficiently illustrated in the figures. This is a cheap form of bridge for a short span.

CHAPTER VII.

EXAMPLES OF GRAPHICAL STATICS APPLIED TO THE DETERMINATION OF THE STRESSES IN TRUSSES.

THE graphical method of determining the stresses in structures described in Chapter III. may be applied to any form of truss, and also to the solution of a great variety of problems in connection with structures and machinery. The method will be used in this chapter in connection with a few ordinary trusses such as are used for roofs; but it will generally be advisable in such cases to apply also the method of moments in order to check the work, measuring the lever arms of the forces from the same scale drawing of the truss which is used in connection with the reciprocal diagrams of stresses.

It is a good plan to tabulate the stresses obtained by measuring the lines in the reciprocal diagrams side by side with those obtained by the method of moments, and to use the mean of the two stresses so obtained in determining the total maxima stresses from which the structure is designed.

The amount and distribution of the load upon roofs will be first considered.

Dead Load upon Roofs.—The dead load upon roofs consists of the weight of the roof principals or trusses; the purlins, which are the longitudinal beams carrying the load between the trusses and discharging it on the principals; also the rafters and the roof-covering.

The weight of the roof principal or truss may be estimated from the known weight of similar roofs for the purposes of calculation, and the sectional areas thus determined; afterwards the correct weight may be ascertained, and the truss recalculated with the corrected weight.

The weights of the purlins may be ascertained without much trouble, as they consist of timber beams, angles, T's, or rolled girders. In large roofs the purlins may be lattice girders or truss-beams.

The weight of the roof-covering may be estimated by making use of the following table, which is taken from "Instruction in Construction," by Colonel Wray, R.E.:—

TABLE XXIII.

WEIGHT OF ROOF-COVERING.

Description of material.	Weight in pounds per square foot.
Lead covering, including laps, but not boarding or rolls	5½ to 8½
Zinc covering, including laps 14 to 16 zinc gauge	1½ to 1¾
Corrugated iron, 16 W.G.	3½
" " 18 "	2½
" " 20 "	2
Sheet iron, 16 W.G.	2½
" 20 "	1½
Slating laid with a 3-inch lap, including nails, but not battens or iron laths:—	
Slates, doubles, 13 inches × 9 inches, at 18 cwt. per 1200 ...	8¼
Ladies, 16 inches × 8 inches, at 31·5 cwt. per 1200	8¼
Countesses, 20 inches × 12 inches, at 50 cwt. per 1200... ...	8
Duchesses, 24 inches × 12 inches, at 77 cwt. per 1200... ...	8½
Slate battens, 3½ inches × 1 inch:—	
For doubles	2
For countesses	1¼
Boarding, ¾ inch thick	2¼
" 1 " "	3½
" 1¼ " "	4¼
Wrought-iron laths, angle-irons:—	
For duchess slates	2
For countess slates	1¾

The weight of snow to be allowed for depends entirely upon the locality. In England it is sometimes assumed that a roof of flat pitch may have 6 inches of snow upon it, the weight of which may be 5 lbs. per square foot.

Trontwine, in his handbook, recommends an allowance of 12 lbs. per square foot of area covered; and Stoney, 20 lbs. per square foot. The depth of snow will diminish as the pitch of the roof increases.

The wind pressure is estimated in the manner explained in Chapter XIX.

The following table gives the weights of some well-known roofs[1]:—

[1] "Notes on Building Construction," by Rivington. Part IV.

Table XXIV.

Description of roof.	Clear span in feet.	Distance apart of principals in feet.	Weight in pounds per square foot of area covered. Of purlins.	Of principal.	Total ironwork.	Total with covering.
Common truss	37	5	1·1	3·5	4·6	6·9
	40	12	2·0	3·5	5·5	—
	54	14	6·5	3·0	9·5	—
	55	6½	4·6	7·0	11·6	—
	72	20	4·2	2·8	7·0	—
	84	9	2·6	5·9	8·5	—
	50	10	—	—	3·0	5·2
	100	14	—	—	7·0	9·0
	130	26	0·8	5·6	6·4	8·0
	140	12	—	4·5	—	—
Bowstring roof, Manchester, London Road Station ...	50	11	—	—	9·6	—
Bowstring roof, Lime Street Station, Liverpool	154	26	—	4·9	—	—
Bowstring roof, Birmingham ...	211	24	—	7·3	11·0	—
Arched roofs, corrugated iron ...	40	—	—	—	—	2·5
	60	—	—	—	—	3·5
Strasburg Railway	97	13	—	—	12·0	—
Paris Exhibition	153	26	9·5	5·5	15·0	—
Dublin	41	16	3·4	7·3	10·7	—
Derby	81½	24	10·8	6·0	16·8	—
Sydenham	120	—	7·9	3·9	11·8	—
St. Pancras	240	29½	7·4	17·1	24·5	—

The following table, taken from Trontwine's handbook, may be used in estimating approximately the total loads on roof-trusses up to 75 feet span; the principals or trusses are supposed to be spaced 7 feet apart, centre to centre.

Table XXV.

Table of Approximate Loads on Roofs in Pounds.

Description of roof.	Weight of roof, or dead load.	Wind and snow.	Total loads.
Roof covered with corrugated iron weighing from 1½ to 2 lbs. per square foot, unboarded ...	8	20	28
Ditto, if plastered below rafters	18	20	38
Ditto, corrugated iron on boards	11	20	31
Ditto, if plastered below rafters	21	20	41
Ditto, slate unboarded or on laths	13	20	33
Ditto, „ on boards 1¼ inch thick	16	20	36
Ditto, „ if plastered below rafters	26	20	46
Ditto, „ shingles on lath	10	20	30
Ditto, „ if plastered below rafters or tie-beam	20	20	40

For spans from 75 to 150 feet, it will suffice to add 4 lbs. to each of the above total loads.

The above total loads may be considered to act vertically, and a reciprocal diagram similar to those obtained for the dead load may be drawn to determine the stresses and the dimensions of the various members, from which the weight of the truss or principal may be calculated. The weight thus obtained may be used in estimating the dead loads for the complete calculations.

In Chapter III., two methods were considered for determining the stresses in braced structures, which were illustrated with reference to the common roof-truss shown in Fig. 125. The stresses in this roof will now be more fully considered, and, in the first place, the results obtained in Chapter III. may be tabulated thus—

TABLE XXVI.

DEAD-LOAD STRESSES.

Bar.	Method of moments.	Scale measurement of reciprocal diagram.	Mean of the two methods.
	lbs.	lbs.	lbs.
AB	− 1,504	− 1,596	− 1,550
BC	+ 6,246	+ 6,240	+ 6,243
CD	+ 6,246	+ 6,240	+ 6,243
DE	− 1,504	− 1,596	− 1,550
AH	− 11,938	− 11,944	− 11,941
BI	− 11,328	− 11,296	− 11,312
JD	− 11,328	− 11,296	− 11,312
EK	+ 11,938	+ 11,944	+ 11,941
AZ	+ 11,274	+ 11,216	+ 11,245
CZ	+ 5,270	+ 5,268	+ 5,629
EZ	+ 11,274	+ 11,216	+ 11,245

With regard to the stresses due to wind :—

If a horizontal force of wind of 50 lbs. per square foot act upon one side of the roof, the normal pressure due to this force may be obtained from Table LXIX., p. 286.

The angle of the roof is $22\frac{1}{2}°$, so that the normal pressure is 25 lbs. per square foot.

Since the roof principals are 12 feet apart centre to centre, and the length of the rafters is 21·6 feet, the total wind force on one side acting normally to the roof is—

$$12 \times 21{\cdot}6 \times 25 = 6480 \text{ lbs.}$$

If the roof principal is bolted down at each shoe, so that the ends are fixed, the stresses may be obtained by measuring the reciprocal diagram, Fig. 127. The results are given in the following table, compared with those obtained by the method of moments, thus:—

Table XXVII.

Stresses due to Wind, both ends fixed.

Bar.	Wind on left.			Wind on right.		
	Method of moments.	Scale measurement of reciprocal diagram.	Mean of the two methods.	Method of moments.	Scale measurement of reciprocal diagram.	Mean of the two methods.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
AB	− 3,240	− 3,240	− 3,240	0	0	0
BC	+ 10,377	+ 10,296	+ 10,336	+ 2,090	+ 2,280	+ 2,185
CD	+ 2,090	+ 2,280	+ 2,185	+ 10,377	+ 10,296	+ 10,336
DE	0	0	0	− 3,240	− 3,240	− 3,240
AH	− 14,574	− 14,472	− 14,523	− 7,860	− 7,974	− 7,917
BI	− 14,574	− 14,472	− 14,523	− 7,860	− 7,974	− 7,917
JD	− 7,860	− 7,974	− 7,917	− 14,574	− 14,472	− 14,523
EK	− 7,860	− 7,974	− 7,917	− 14,574	− 14,472	− 14,523
AZ	+ 14,864	+ 14,784	+ 14,824	+ 6,668	+ 6,764	+ 6,718
CZ	+ 4,775	+ 4,764	+ 4,770	+ 4,775	+ 4,764	+ 4,770
EZ	+ 6,668	+ 6,764	+ 6,718	+ 14,864	+ 14,784	+ 14,824

We see that if the wind is on the left, the bar DE is unstressed; if the wind is on the right, AB is unstressed.

The direction of the reactions R_1 and R_2 is normal to that of the principal rafter, and the values of these reactions may be determined by means of the polar and funicular polygons shown in dotted lines, Figs. 127 and 125. The point z is determined by drawing the line oz parallel to the closing line in the funicular polygon.

If one end of the principal is fixed, and the other end is free to move on expansion rollers, the principal will tend to expand or contract when the temperature changes, motion taking place as soon as the forces due to changes of temperature are sufficient to overcome the frictional resistance of the expansion rollers. When the frictional resistances just balance the forces due to temperature, the whole of the horizontal component of the wind pressure must be resisted by the fixed end, since the roller end can only supply a vertical reaction.

The vertical components of the normal reactions at the fixed and free ends remain the same as in the case last considered. Two cases require to be considered, namely, the wind on the left and the wind on the right (Figs. 128 and 129); wind on the left with the left end free to move (Fig. 128).

The diagram obtained for this case differs slightly from Fig. 127, as the horizontal component of the reactions at both ends

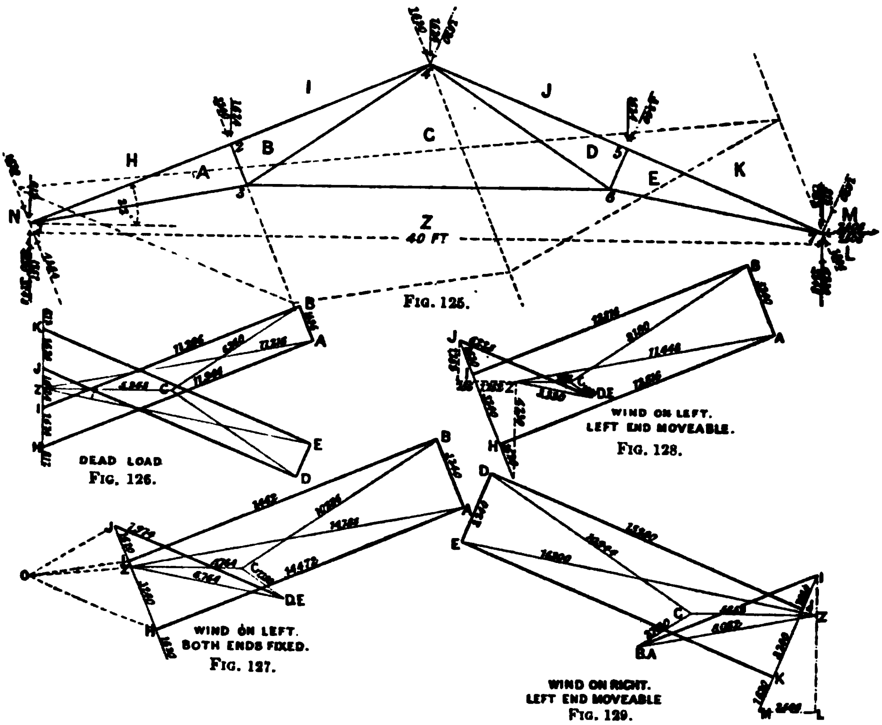


FIG. 125. FIG. 126. FIG. 127. FIG. 128. FIG. 129.

must be entirely supplied by the fixed end. The stresses are given in the following table:—

TABLE XXVIII.

STRESSES DUE TO WIND ON LEFT, LEFT END MOVABLE.

Bar.	Method of moments.	Scale measurement of reciprocal diagram, Fig. 128.	Mean of the two methods.
	lbs.	lbs.	lbs.
AB	− 3,240	− 3,240	− 3,240
BC	+ 9,158	+ 9,180	+ 9,169
CD	+ ,947	+ 1,098	+ 1,022
DE	0	0	0
AH	− 12,964	− 12,816	− 12,890
BI	− 12,964	− 12,816	− 12,890
JD	− 6,137	− 6,228	− 6,182
EK	− 6,137	− 6,228	− 6,182
AZ	+ 11,627	+ 11,448	+ 11,537
CZ	+ 2,473	+ 2,376	+ 2,424
EZ	+ 3,171	+ 3,330	+ 3,250

Wind on the right, with the left end movable. The stresses due to this case may be tabulated in a similar manner to the foregoing, thus—

Table XXIX.

Stresses due to Wind on right, Left End movable.

Bar.	Method of moments.	Scale measurement of reciprocal diagram, Fig. 129.	Mean of the two methods.
	lbs.	lbs.	lbs.
AB	0	0	0
BC	+ 2,692	+ 2,700	+ 2,696
CD	+ 10,917	+ 10,944	+ 10,930
DE	− 3,240	− 3,240	− 3,240
AH	− 8,523	− 8,572	− 8,547
BI	− 8,523	− 8,572	− 8,547
JD	− 15,360	− 15,280	− 15,320
EK	− 15,360	− 15,280	− 15,320
AZ	+ 8,051	+ 8,082	+ 8,066
CZ	+ 5,646	− 5,652	+ 5,649
EZ	+ 16,506	+ 16,300	+ 16,403

The details of the calculations by the method of moments is left for the student to verify.

The maximum stresses on any member of the truss due to wind may be obtained from an inspection of the foregoing tables. The resultant stress is found by combining the maximum wind stress with the dead-load stress, thus—

Table XXX.

Table of Maximum Stresses due to Dead Load and Wind Pressure.

Bar.	Dead-load stresses in pounds.	Wind stresses in pounds.	Resultant stress due to dead load and wind in pounds.
AB	− 1,550	− 3,240	− 4,790
BC	+ 6,243	+ 10,336	+ 16,579
CD	+ 6,243	+ 10,930	+ 17,173
DE	− 1,550	− 3,240	− 4,790
AH	− 11,941	− 14,523	− 36,464
BI	− 11,312	− 14,523	− 25,835
JD	− 11,312	− 15,320	− 26,630
EK	− 11,941	− 15,320	− 27,261
AZ	+ 11,245	+ 11,537	+ 22,782
CZ	+ 5,269	+ 5,649	+ 10,918
EZ	+ 11,245	+ 16,403	+ 27,648

The resultant stresses should be used in designing the truss.

The method of obtaining the stresses in a framed structure, such as a crane, is illustrated in Figs. 130 and 131.

The force polygon in this case consists simply of the line RZ, representing the load W suspended from the end of the

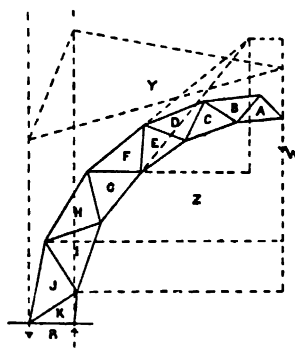


FIG. 130.

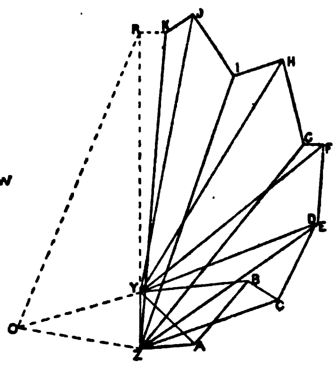


FIG. 131.

cantilever. The magnitude of the RY and RZ, acting as shown in Fig. 130, are found by means of the polar and funicular polygons shown in dotted lines, the point Y being determined by drawing OY parallel to the closing line as before. The magnitude and sense of the stresses in the various members of the cantilever may be determined by measuring the lines of the

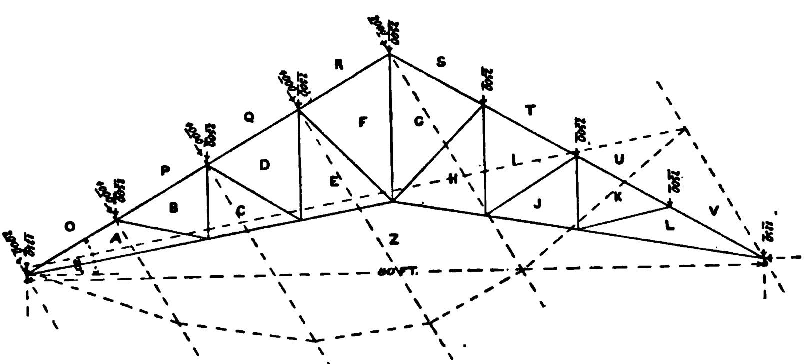


FIG. 132.

reciprocal diagram, Fig. 131, to the proper scale, or by applying the method of moments.

Fig. 132 represents a diagram of a roof-truss, for which the reciprocal diagrams have been drawn for the dead load and wind pressure shown in Figs. 133 to 136.

The following data have been employed in drawing these diagrams :—

Span of roof-truss	80·00	feet.
Distance between principals	10·00	„
Length of principal rafters	46·20	„
Distance between joints measured along principal rafters	11·55	„
Inclination of principal rafters to the horizon	30	degrees.
Rise of tie-rod in the centre	7	feet.
Dead load at each of the seven intermediate joints ...	2500	lbs.
Horizontal wind pressure per square foot	50	„
Normal „ „ „	33	„
Total normal pressure on one side	16,000	„
Normal pressure at the three intermediate joints on one side	4000	„
Normal pressure at the apex and point of support ...	2000	„

Fig. 133 represents the stresses due to the dead load; Fig. 134, the stresses due to the wind acting on the left when both ends are fixed; Fig. 135 represents the stresses due to the wind acting on the left when the left end is free to move; and Fig. 136 represents the stresses with the wind on the right when the left end is free to move.

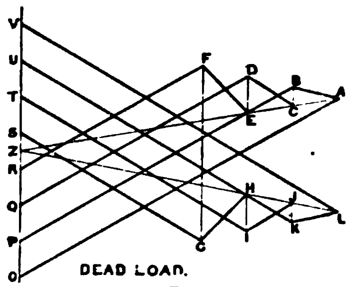


FIG. 133.

The stresses may be tabulated in a similar manner to that illustrated in connection with the roof-truss shown in Fig. 125.

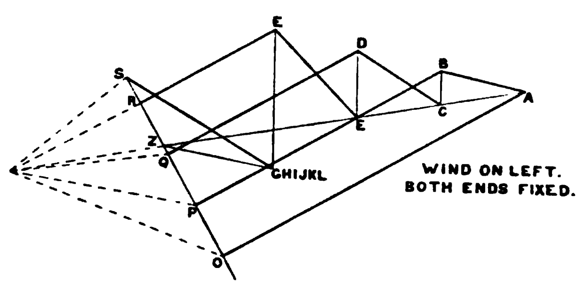


FIG. 134.

Fig. 137 represents another example of a roof-truss, for which reciprocal diagrams have been drawn from the following data—

Span of truss	60 feet.
Distance between trusses	7·5 „
Length of principal rafters	32·5 „
Distance between the joints measured along the principal rafters	8·1 „
Inclination of the principal rafter to the horizon	22½ degrees.
Rise of the main tie-rod in the centre	2 feet.
Dead load at each of the seven intermediate upper joints ...	1053 lbs.
„ „ „ „ at supports	526·5 „
Horizontal wind pressure per square foot	50 „
Normal „ „ „	33 „
Total normal pressure on one side	8100 „
Normal pressure at the three intermediate joints on one side ...	2025 „
Normal pressure at apex and support	1012·5 „

Fig. 138 represents the stresses due to the dead load; Fig. 139, the stresses due to the wind acting on the left when both ends are fixed; Fig. 140, the stresses due to wind acting on the left when the left end is free to move; and Fig. 141, the stresses with the wind acting on the right when the left end is free to move.

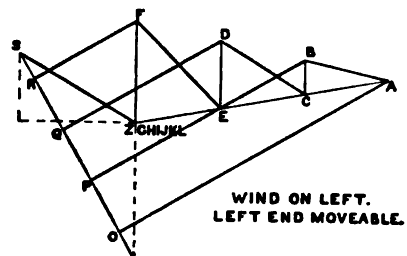


FIG. 135.

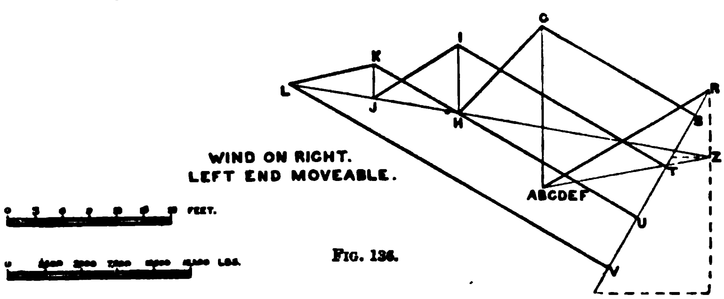


FIG. 136.

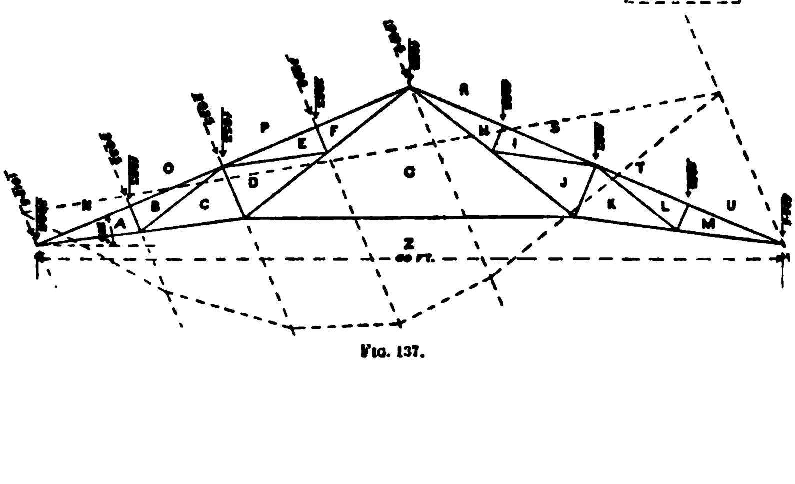
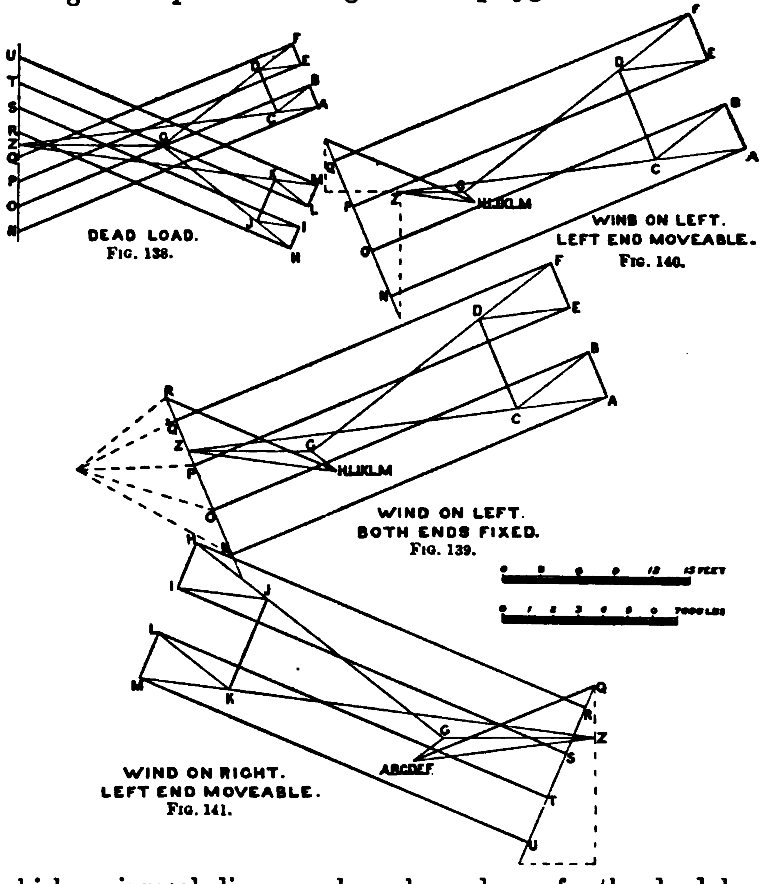


FIG. 137.

The stresses in this and the foregoing example may be tabulated in the manner fully explained in the first example, Figs. 125 to 129.

Fig. 142 represents a diagram of a polygonal roof-truss, for



DEAD LOAD.
FIG. 138.

WIND ON LEFT.
LEFT END MOVEABLE.
FIG. 140.

WIND ON LEFT.
BOTH ENDS FIXED.
FIG. 139.

WIND ON RIGHT.
LEFT END MOVEABLE.
FIG. 141.

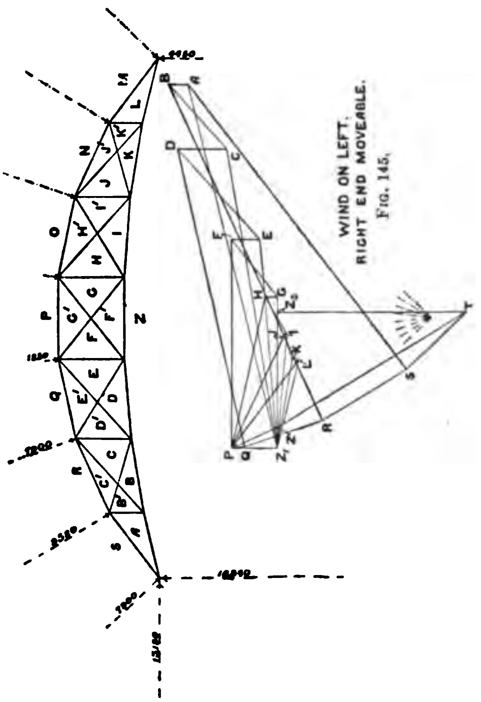
which reciprocal diagrams have been drawn for the dead load and wind pressures, Fig. 143 to 146.

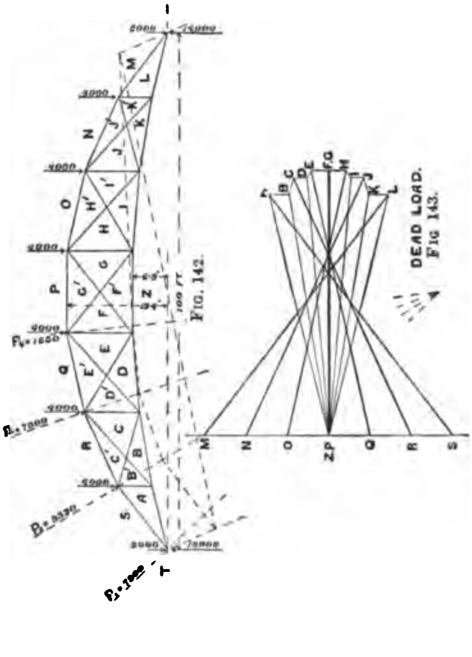
The following data have been employed:—

Span of roof-truss	100·0 feet.
Distance between principals	20·0 „
Dead load per square foot of area covered	14 lbs.
Horizontal wind pressure per square foot	50 „
Dead load at joints 2, 3, 4, 5, 6, and 7	4000 „
„ „ „ 1 and 8	2000 „
Wind pressure at joints 1 and 5	7900 „
„ „ „ 2 „ 7	9520 „
„ „ „ 3 „ 6	7900 „
„ „ „ 4 „ 5	1250 „

The figure is redundant, but one set of diagonals is supposed to be omitted, otherwise the reciprocal stress diagrams could not be drawn.

Fig. 143 represents the stresses due to the dead load. Fig. 144 represents the stresses due to wind acting on the left side when both ends of the principal are fixed in position. The point Z is determined by means of the polar and funicular polygons shown in dotted lines. The force polygon for the loads P_2, P_3, and P_4, which produce reactions at both supports, is shown in Fig. 144, viz. PQRSZP, in which SR, RQ, and PQ are drawn parallel to P_2, P_3, and P_4 (Fig. 142). PS is the resultant, and equals the sum of the reactions at the supports due to the three loads under consideration. The force polygon for the four forces P_1, P_2, P_3, and P_4 is PQRSTZP, in which ST is drawn parallel to P_1, and ZT joined.





The remainder of Fig. 144 will be readily understood by the student, as Bow's notation has been employed throughout.

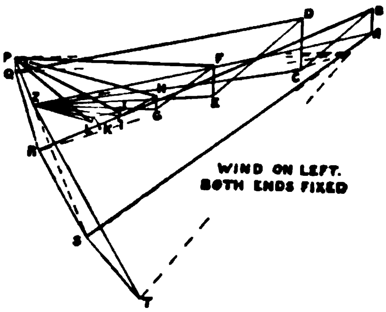


FIG. 144.

Fig. 145 represents the stresses due to wind acting on the left-hand side, but with the left end fixed and the right on the point of moving; under these circumstances the left support must supply the horizontal component of the inclined reactions at both supports. The point Z in Fig. 144 becomes Z_1 in Fig. 145; PZ_1 represents the vertical reaction at the right support, and TZ_2 represents the vertical reaction at the left support.

The horizontal reaction at the left support is represented by Z_1, Z_2; the stresses in the various members of the truss are determined by completing the figure as before.

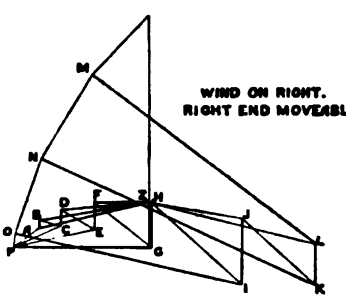


FIG. 146.

Fig. 146 represents the stresses due to wind acting on the right with the right end movable as before.

The stresses due to the various cases of loading obtained by measuring the reciprocal diagrams, Figs. 143 to 146, and also the total maxima stresses, are given in the following table:—

TABLE XXXI.

Bar.	Dead load stresses	Both ends of principal fixed to the support. Wind on the right.	Wind on the left.	Right end movable, left end fixed. Wind on the right.	Wind on the left.	Total maxima stresses.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
SA	− 29,200	− 9,780	− 33,910	+ 3,110	− 34,010	− 63,210
RC′	− 27,460	− 11,510	− 35,130	+ 5,000	− 35,680	− 6,314
QE′	− 26,240	− 14,230	− 28,350	+ 8,000	− 28,680	− 54,820
PG′	− 25,570	− 19,340	− 19,340	− 13,840	− 20,180	− 45,750
OH′	− 26,240	− 28,350	− 14,230	− 22,460	− 14,900	− 54,590
NJ′	− 27,460	− 35,130	− 11,510	− 29,130	− 12,170	− 62,590
ML	− 29,200	− 33,910	− 9,780	− 27,120	+ 10,780	− 63,110
AZ	+ 24,120	+ 5,890	+ 33,570	− 10,900	+ 35,350	+ 59,470
BZ	+ 23,570	+ 5,780	+ 26,180	− 10,670	+ 29,640	+ 53,210
DZ	+ 24,960	+ 8,450	+ 17,450	− 8,450	+ 20,230	+ 45,190
F′Z	+ 25,570	+ 11,900	+ 11,900	− 5,220	+ 14,560	+ 40,130
IZ	+ 24,960	+ 17,450	+ 8,450	+ 230	+ 11,010	+ 42,410
KZ	+ 23,570	+ 26,180	+ 5,780	+ 9,120	+ 8,670	+ 49,750
LZ	+ 24,120	+ 33,570	+ 5,890	+ 16,450	− 8,840	+ 57,690
AB′	+ 1,950	+ 500	− 2,450	− 780	− 1,890	{ + 2,450 { − 500
CD′	+ 1,220	− 450	− 5,670	− 1,670	− 4,780	{ + 1,220 { − 4,450
EF	+ 3,390	− 1,340	− 2,890	− 2,670	− 2,780	+ 3,390
GH	+ 3,390	− 2,890	− 1,340	− 4,340	− 1,220	{ + 3,390 { − 950
I′J	+ 1,220	− 5,670	− 450	− 6,450	− 230	{ + 1,220 { - 5,230
K′L	+ 1,950	− 2,450	+ 500	− 4,000	+ 670	{ + 2,620 { − 2,050
BB′	0	0	+ 9,120	0	+ 8,340	+ 9,120
DD′	0	0	+ 11,450	0	+ 11,670	+ 11,670
FF′	0	0	+ 7,000	0	+ 7,110	+ 7,110
HI	+ 890	0	+ 4,000	0	+ 4,110	+ 5,000
JK	+ 1,670	0	+ 2,780	0	+ 2,400	+ 4,450
BC	+ 1,670	+ 2,780	0	+ 2,170	0	+ 4,450
DE	+ 890	+ 4,000	0	+ 3,780	0	+ 4,890
F′G	0	+ 7,000	0	+ 7,000	0	+ 7,000
II′	0	+ 11,450	0	+ 11,780	0	+ 11,780
KK′	0	+ 9,120	0	+ 9,560	0	+ 9,560

CHAPTER VIII.

BRACED GIRDERS WITH PARALLEL FLANGES.

FIG. 147 represents a cantilever loaded at the apices. The load $\frac{W}{2}$ at the apex o and the stresses in the bars $o1$ and $o2$ are in equilibrium; they may therefore be represented by the triangle of forces abc, in which ac represents the load $\frac{W}{2}$, Fig. 148.

The stress in $o1$ ($o1_1$), may be found thus—

$$o1_1 : \frac{W}{2} :: ab : ac \qquad \therefore o1_1 = \frac{W}{2} \sec \theta$$

$$\text{also } o2_1 : \frac{W}{2} :: bc : ac \qquad \therefore o2_1 = \frac{W}{2} \tan \theta$$

We see also, from the direction of the forces in the triangle abc, that $o2$ is in tension, and $o1$ is in compression.

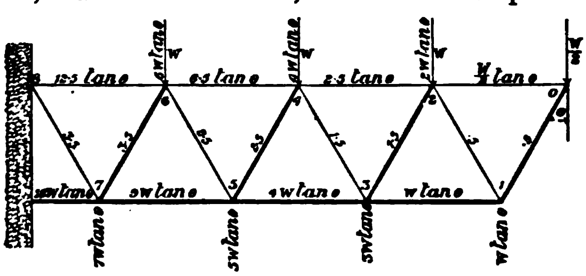


FIG. 147.

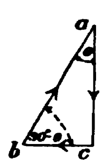


FIG. 148.

The stress in 1 2 is clearly the same as in $o1$. The stress in 2 3 due to the load at the apex 2 is W sec θ, and the total stress on 2 3 is therefore $\frac{3W}{2}$ sec θ, which is also the stress in the bar 3 4; the former is in compression, and the latter in tension.

The stress in 1 3 is the sum of the horizontal components of the stresses in $o1$ and 1 2.

$$\therefore 1\ 3_1 = W \tan \theta \text{ (compression)}$$

The stress $2\ 4_1$ is the sum of the horizontal components of the stresses in 1 2 and 2 3, plus the stress in $o2$.

$$\therefore 2\ 4_1 = 2W \tan \theta + \frac{W}{2} \tan \theta \text{ (tension)}$$

In a similar manner we may find the stresses in the remaining bars.

We see that the increment of stress in the web is W sec θ, so that we may write down the stresses in the remaining bars thus—

In the bars 4 5 and 4 6 the stress is $\frac{5W}{2}$ sec θ

" " 6 7 " 7 8 " " $\frac{7W}{2}$ sec θ

The increment of flange stress at the apex 1 we found to be W tan θ, and at the apex 2, 2W tan θ. In a similar manner we could have shown that the increment of flange stress at the apex 3 is 3W tan θ, and at the apex 4 it is 4W tan θ, and so on for the remaining bars.

Let W = 10 tons, and $\theta = 30°$.

then sec $\theta = 1{\cdot}154$, and tan $\theta = 0{\cdot}577$

The increment of web stress W sec $\theta = 11{\cdot}54$ tons, and W tan $\theta = 5{\cdot}8$ tons.

In Fig. 147 we may write down the stresses in the diagonals by multiplying the number written on the bars by 11·54, thus—

Stress in $o1 = 0{\cdot}5 \times W \sec \theta = -\ 5{\cdot}77$ tons, compression
" " $1\ 2 = 0{\cdot}5 \times W \sec \theta = +\ 5{\cdot}77$ " tension
" " $2\ 3 = 1{\cdot}5 \times W \sec \theta = -\ 17{\cdot}31$ " compression
" " $3\ 4 = 1{\cdot}5 \times W \sec \theta = +\ 17{\cdot}31$ " tension
" " $4\ 5 = 2{\cdot}5 \times W \sec \theta = -\ 28{\cdot}85$ " compression
" " $5\ 6 = 2{\cdot}5 \times W \sec \theta = +\ 28{\cdot}85$ " tension
" " $6\ 7 = 3{\cdot}5 \times W \sec \theta = -\ 40{\cdot}39$ " compression
" " $7\ 8 = 3{\cdot}5 \times W \sec \theta = +\ 40{\cdot}39$ " tension

The stresses in the flanges are written down in a similar manner, thus—

Stress in $o2 = +\ 2{\cdot}89$ tons.	Stress in $23 = -\ 5.77$ tons
„ „ $2\ 4 = +\ 14{\cdot}45$ „	„ „ $35 = -\ 23{\cdot}08$ „
„ „ $4\ 6 = +\ 37{\cdot}58$ „	„ „ $57 = -\ 51{\cdot}93$ „
„ „ $6\ 8 = +\ 72{\cdot}25$ „	„ „ $79 = -\ 92{\cdot}32$ „

Fig. 149 represents a triangular girder supported at both ends and loaded symmetrically; such a girder is termed a

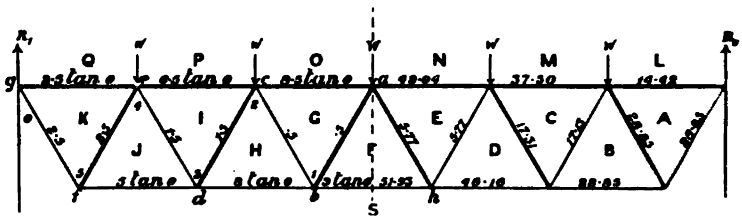


FIG. 149.

"Warren girder." It would be used for a deck bridge, the deck being carried by the top member.

The reactions are each equal to half the load. The stresses in the bars are found thus—

Stress in KS $= R_1 \sec\theta \qquad = +\ 2{\cdot}5W \sec\theta$ (tension)
„ „ KJ $= R_1 \sec\theta \qquad = -\ 2{\cdot}5W \sec\theta$ (compression)
„ „ IJ $= (R_1 - W)\sec\theta \qquad = +\ 1{\cdot}5W \sec\theta$
„ „ JH $= (R_1 - W)\sec\theta \qquad = -\ 1{\cdot}5W \sec\theta$
„ „ HG $= (R_1 - W - W)\sec\theta = -\ 0{\cdot}5W \sec\theta$
„ „ GF $= (R_1 - W - W)\sec\theta = -\ 0{\cdot}5W \sec\theta$

Hence we may write coefficients against the bars, and multiply by W sec θ to find the stress as before. It should be noted that the bars which slope towards the centre are in compression, while those which slope away from the centre are in tension.

The stresses in the flanges are found by adding the horizontal components of the stresses in the web, as before, thus—

Stress in QK $= \qquad = -\ 2{\cdot}5W \tan\theta$
„ „ PI $= -\ (2{\cdot}5 + 4)W \tan\theta = -\ 6{\cdot}5W \tan\theta$
„ „ OG $= -\ (6{\cdot}5 + 2)W \tan\theta = -\ 8{\cdot}5W \tan\theta$
„ „ JS $= \qquad = +\ 5{\cdot}0W \tan\theta$
„ „ HS $= +\ (5 + 3)W \tan\theta \quad = +\ 8{\cdot}0W \tan\theta$
„ „ FS $= +\ (8 + 1)W \tan\theta \quad = +\ 9{\cdot}0W \tan\theta$

Here we note that by adding the coefficients on two bars which meet at a vertex, and writing the result on the vertex, we have the coefficient for the increment of flange stress, which requires to be multiplied by W tan θ and added to the stress in the flange preceding it, counting from the left support, to find the stress.

Let $\theta = 30°$, and W = 10 tons.

$$\text{Then sec } \theta = 1{\cdot}154 \quad \therefore \text{W sec } \theta = 11{\cdot}54 \text{ tons}$$
$$\tan \theta = 0{\cdot}577 \quad \therefore \text{W tan } \theta = 5{\cdot}77 \text{ tons}$$

The actual stresses are written on the right-hand half of the girder, Fig. 149.

Fig. 150 shows a lattice girder loaded symmetrically on the top and bottom flanges.

Here we may consider each triangulation separately, and

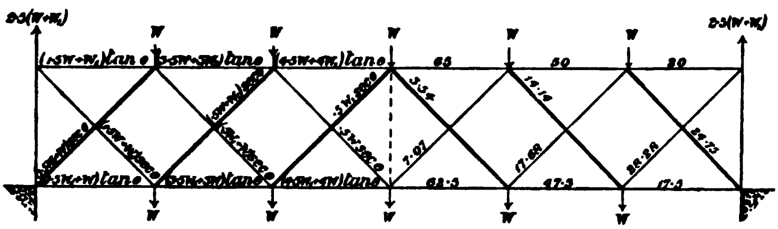


FIG. 150.

apply the foregoing principles. The results are written on the left-hand half of the girder, Fig. 150.

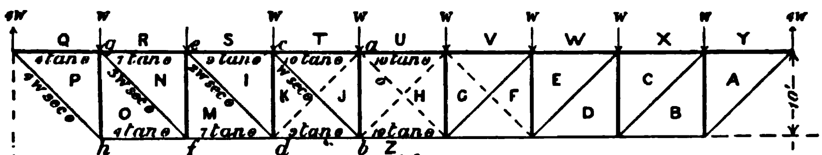
Let $W_1 = 5$ tons, and W = 10 tons, $\theta = 45°$.

$$\text{Then sec } \theta = 1{\cdot}414$$
$$\therefore W_1 \text{ sec } \theta = 7{\cdot}07; \text{ W sec } \theta = 14{\cdot}14; \tan \theta = 1$$

The results are written on the right-hand half of the girder, Fig. 150.

Fig. 151 represents a truss known as the Murphy-Whipple, or Pratt, truss. The vertical members are constructed as compression members, while the diagonal members are in tension. The truss as shown would be used as a deck bridge like the Warren girder, Fig. 149. Fig. 152 represents the same truss inverted, but the vertical members are in tension, and the diagonal members are in compression. In this form it is termed the Howe truss, and is much used for composite structures, the vertical members consisting of one or more bolts, the diagonal and top members of timber, while the bottom member may be constructed of timber or iron. The Howe truss, as shown in Fig. 152, would be used for a through bridge, and the deck would be carried by the bottom member. In both Figs. 151 and 152, the diagonals in the centre bay are unstressed, with the loads symmetrically arranged on either side of the centre,

FIG. 151.



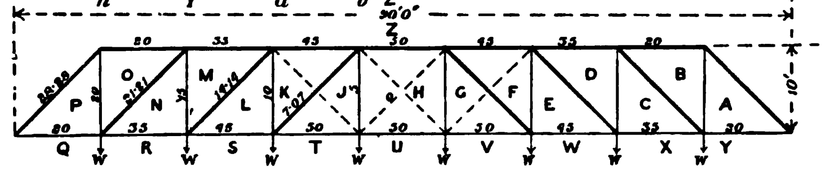


FIG. 152.

as shown; they will be referred to in regard to the live load. We may write coefficients against the bars and determine the stresses as before; thus in Fig. 151 the stresses in the verticals are—

$$ab_1 = \text{W}, \ cd_1 = 2\text{W}, \ ef = 3\text{W}, \ gh = 4\text{W}$$

The stresses in the diagonals are—

$$cb = \text{W} \sec \theta, \ de = 2\text{W} \sec \theta, \ fg = 3\text{W} \sec \theta, \ hk = 4\text{W} \sec \theta$$

The horizontal components of the stresses in the diagonals will produce stresses in the top and bottom members, thus—

$$\begin{aligned} kg_1 &= 4 \tan \theta \\ ge_1 &= 4 \tan \theta + 3 \tan \theta = 7 \tan \theta \\ ec_1 &= 7 \tan \theta + 2 \tan \theta = 9 \tan \theta \\ ca_1 &= 9 \tan \theta + \tan \theta = 10 \tan \theta \end{aligned}$$

In a similar manner we may find the stresses in the bottom member; they are written in Fig.151.

If $\text{W} = 5$ tons, and $\theta = 45°$

then $\text{W} \sec \theta = 5 \times 1{\cdot}414 = 7{\cdot}07$ tons; $\text{W} \tan \theta = 5$

The results are written on Fig. 152.

We may draw reciprocal figures to find the stresses in the various members as illustrated in Figs. 153 and 154.

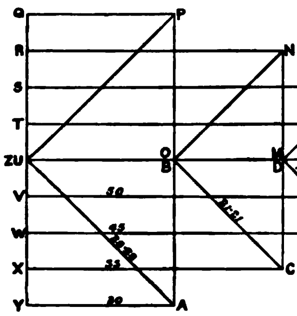


FIG. 153.

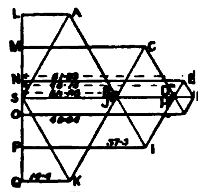


FIG. 154.

Fig. 153 represents the stresses in the truss illustrated in Fig. 152, and Fig. 154 the stresses in that illustrated in Fig. 149.

Braced Girders with Parallel Flanges subjected to a Travelling Load.—We have seen in Chapter IV., Fig. 54, that the maximum shearing stress at any point of a girder subjected to a travelling load occurs when the load extends from either abutment to the point in question, the remainder of the girder being unloaded. Hence in the Warren girder, Fig. 149, the maximum compressive stress in the bar FG occurs when the apices on the right half of the girder are loaded. The maximum tensile stress in FG occurs when the remaining apices are loaded, and the apices on the right half of the girder are unloaded, thus—

$$-\text{FG} = \frac{\text{W} \sec \theta}{6}(1 + 2 + 3) = -\text{W} \sec \theta$$

$$+\text{FG} = \frac{\text{W} \sec \theta}{6}(1 + 2) = +\frac{\text{W} \sec \theta}{2}$$

The same stresses will be produced on the bar GH, only with the signs changed, thus—

$$-\text{GH} = -\frac{\text{W} \sec \theta}{2}, \text{ and } +\text{GH} = +\text{W} \sec \theta$$

In a similar manner the stresses in all the inclined bars may be written down; thus for the left half of the girder we have—

$$\text{Stress in HI} = -\frac{W \sec \theta}{6}(1 + 2 + 3 + 4) \text{ and } + \frac{W \sec \theta}{6}(1)$$

$$= -\frac{10W \sec \theta}{6} \text{ and } + \frac{W \sec \theta}{6}$$

$$\text{,, ,, IJ} = +\frac{10W \sec \theta}{6} \text{ and } - \frac{W \sec \theta}{6}$$

$$\text{,, ,, JK} = -\frac{W \sec \theta}{6}(1 + 2 + 3 + 4 + 5) = -\frac{15W \sec \theta}{6}$$

$$\text{,, ,, KS} = +\frac{W \sec \theta}{6}(1 + 2 + 3 + 4 + 5) = +\frac{15W \sec \theta}{6}$$

The maximum stresses in the horizontal members due to the live load will occur when the girder is fully loaded, and they may be found in a similar manner to that explained in the foregoing examples. We may determine the stresses in the inclined members graphically thus: In the Warren girder, Fig. 149, let the dead load at the apices be 5 tons, and the live load 10 tons. Here we draw the ordinary shearing-stress diagrams for the live and dead loads, and set out the triangles as shown in Fig. 155.

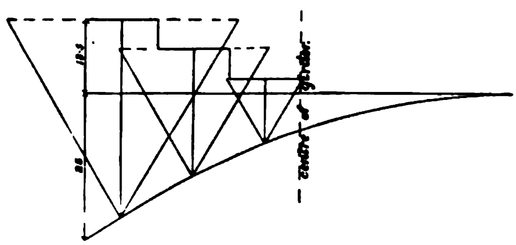


FIG. 155.

In Fig. 151 let the dead load at the apices be 5 tons, and the live load 10 tons, and let $\theta = 45°$. The stresses in the top and bottom members may be found as in the case of the dead load, or we may use the method of sections; thus for the stress in WE, we suppose a vertical section cutting ED, and take moments about the point where ED cuts the bottom member, thus—

$$-10\text{WE} + 15(20 + 10) - 60 \times 30 = 0$$
$$\therefore \text{WE} = -135 \text{ tons}$$

In a similar manner we write down the stresses in all the top and bottom members, thus—

$$YA = \qquad = -60 \text{ tons}$$
$$XC = \frac{-60 \times 20 + 15 \times 10}{10} = -105 \text{ ,,}$$
$$WE = \frac{-60 \times 30 + 15(20 + 10)}{10} = -135 \text{ ,,}$$
$$VG = \frac{-60 \times 30 + 15(30 + 20 + 15)}{10} = -150 \text{ ,,}$$
$$UH = \qquad = -150 \text{ ,,}$$
$$BZ = \qquad = +60 \text{ ,,}$$
$$DZ = \frac{60 \times 20 - 15 \times 10}{10} = +105 \text{ ,,}$$
$$FZ = \frac{60 \times 30 - 15(20 + 10)}{10} = +135 \text{ ,,}$$
$$HZ = \frac{60 \times 40 - 15(30 + 20 + 10)}{10} = +150 \text{ ,,}$$

These stresses are three times those written on Fig. 152.

Stresses. in Diagonal Members.—For the stresses in the diagonal members we proceed as in the case of the Warren girder, Fig. 149—

$$W_1 \sec\theta = 14{\cdot}14 \text{ tons for the live load}$$
$$W \sec\theta = 7{\cdot}07 \text{ ,, ,, dead load}$$
$$\text{and } \frac{14{\cdot}14}{9} = 1{\cdot}572$$

The stresses on NO, for example, may be found most conveniently by writing the equation so that the live loads producing tension and compression may be readily seen—

$$\left.\begin{aligned} NO = 21{\cdot}21 + 1{\cdot}572(1 + \ldots + 7) \\ - 1{\cdot}572(1) \end{aligned}\right\} = 21{\cdot}21 + 44{\cdot}016 - 1{\cdot}57$$

hence $NO = +65{\cdot}22$ tons and $+19{\cdot}64$ tons

The stress in the bar PZ is—

$$PZ = 28{\cdot}28 + 1{\cdot}572(1 + \ldots + 8) = +84{\cdot}84 \text{ tons}$$

Also in the remaining bars we may find the stresses as before—

$$LM = 14{\cdot}14 + 1{\cdot}572(1 + \ldots + 6)$$
$$\qquad - 1{\cdot}572(1 + 2)$$
$$\therefore LM = 14{\cdot}14 + 33{\cdot}012$$
$$\qquad - 4{\cdot}716$$
$$\therefore LM = +47{\cdot}15 \text{ tons and } 9{\cdot}42 \text{ tons}$$

The stresses in the first three bars are not reversed by the live load, and are always tensile. In the bar JK we shall see that the live load puts a compressive stress upon the bar. thus—

$$JK = 7{\cdot}07 + 1{\cdot}572(1 + \ldots + 5) - 1{\cdot}572(1 + 2 + 3)$$

$$\therefore JK = 7{\cdot}07 + 23{\cdot}58 - 9{\cdot}432$$

$$\therefore JK = +\ 30{\cdot}65 \text{ tons and } -\ 2{\cdot}36 \text{ tons}$$

Since JK can only resist tensile stresses, we must counterbrace the panel by inserting a diagonal crossing JK. Since it is hardly possible to ensure that each member shall perform the duty assigned to it, it is better practice to make the counterbrace take all the live-load shear, so that JK would have to sustain + 30·65, and the counterbrace 9·432 tons.

In the central bay there is no stress in the diagonals due to the dead load; the stresses due to the live load are—

$$\pm\ 1{\cdot}572(1 + 2 + 3 + 4) = \pm\ 15{\cdot}72 \text{ tons}$$

Hence the central bay must be counterbraced by inserting two diagonals crossing each other, each designed for a tensile stress of 15·72 tons. These diagonal counterbraces are shown in dotted lines, both in Figs. 151 and 152. The counterbraces only come into action when the live load tends to produce distortion of the panel by putting a compressive stress on the main brace; when this occurs, the main brace bends and throws the tensile stress upon the counterbrace, so that the two braces are never in action simultaneously.

Stresses in Vertical Members.—The stresses in the verticals are the vertical components of the stresses in the diagonals, and one may be found from the other, or we may proceed independently, thus—

$$LK = -\ 10 - \tfrac{10}{9}(1 + \ldots + 6) + \tfrac{10}{9}(1 + 2) \quad \therefore LK = -\ 33{\cdot}33 \text{ tons}$$

$$JH = -\ 5 - \tfrac{10}{9}(1 + \ldots + 5) + \tfrac{10}{9}(1 + 2 + 3) \quad \therefore JH = -\ 15 \text{ and } +\ 1{\cdot}66$$

Hence we see that the live load puts tensile stress upon JH and HG. The maximum stresses are written on Fig. 156.

In designing these girders we must compare the maximum stresses, as written on Fig. 156, with those produced by the dead load only, and thus find the range of stress for those bars where the stress is not reversed by the live load. Where there

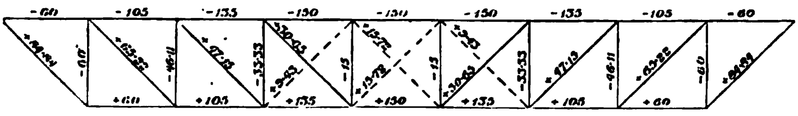


FIG. 156.

is a reversal of stress, as in the two central verticals, the range of stress is shown on Fig. 156. Hence the intensity of working stress is found as explained in Chapter I.

When there are more than one system of triangulation each system may be calculated separately, as in Fig. 157, as the loads upon the apices of a particular system are transmitted by that system to the abutment independently of the others.

Fig. 157 represents a lattice girder or truss of an odd number of panels, which may be subdivided into four different systems, as shown in Figs. 158 to 161. The subdivision illustrated in Figs. 158 and 159 represents the method for partial loading, and Figs. 160 and 161 show a possible division for complete loading; but here there is an ambiguity, as we are not sure whether the system illustrated in Figs. 158 and 159 would not hold. If the stresses are calculated for each case, and the maximum stresses adopted in the design of the girder, this ambiguity will not cause a want of strength. The student may work out the stresses in this case by assuming a live load of 10 tons and a dead load of 5 tons at each bottom apex, $\theta = 30°$, span = 50 feet.

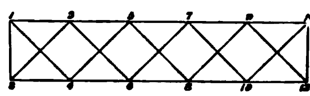


FIG. 157.

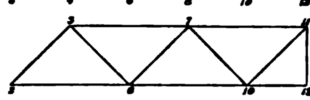


FIG 158.

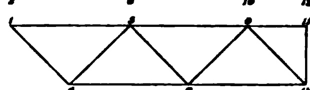


FIG. 159.

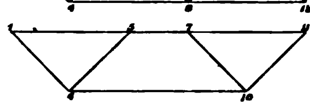


FIG. 160.

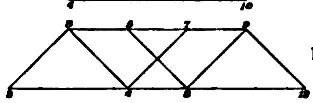


FIG. 161.

This method of subdivision and superposition is often very convenient, and may be used in the two following examples, Figs. 162 and 165. Here, however, there is no ambiguity, as the number of panels is even, and there are only two subdivisions possible.

Figs. 162 and 165 are both modifications of the Pratt or Murphy-Whipple truss, but in these forms they are sometimes

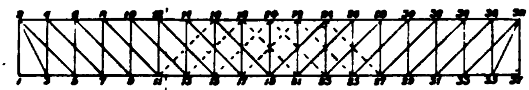


Fig. 162.

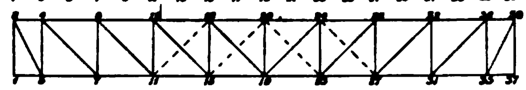


Fig. 163.

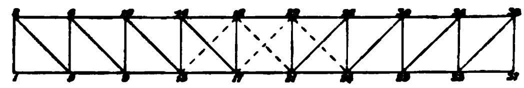


Fig. 164.

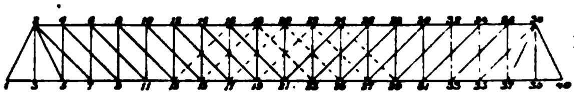


Fig. 165.

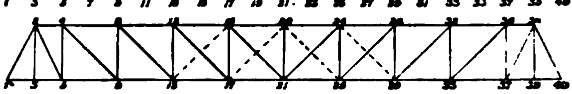


Fig. 166.

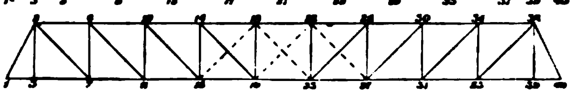


Fig. 167.

called Linville trusses; the former is suitable for a deck bridge, the latter for a through bridge.

Fig. 165 represents the outline of the bridge over the Ohio River, 415 feet span.

Fig. 168 shows a form of timber truss which has been largely used for road bridges in Australia. The vertical tension members

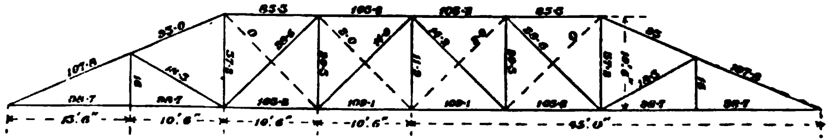


Fig. 168.

are constructed with two or more bolts, which may be screwed up as required, and the diagonal members of the web fit into iron pockets, and are provided with double wedges at the ends for putting the member in initial compression and taking up shrinkage.

The stresses written on the various members of the truss have been determined in the manner sufficiently explained, and may be verified by the student. The dead load is 0·6 tons per foot run, and the live load 0·53 tons per foot run on each truss.

The counterbraces in the four central bays, shown in dotted lines, are chiefly necessary in consequence of the wedges and the stresses, as the various members of the truss may vary from those written on Fig. 168, depending on tightness or slackness of the wedges. The counterbraces should be made about half the sectional area of the main diagonal in the same bay, and should be arranged with their largest lateral dimensions at right angles to the plane of the truss in order to give lateral stiffness.

The cross-beams in this truss are spaced on each side of the panel points; the main tie is subjected to a certain amount of transverse stress.

Fig. 169 shows a Howe truss as used in a highway bridge of 70 feet span, in which the vertical members consist of two or

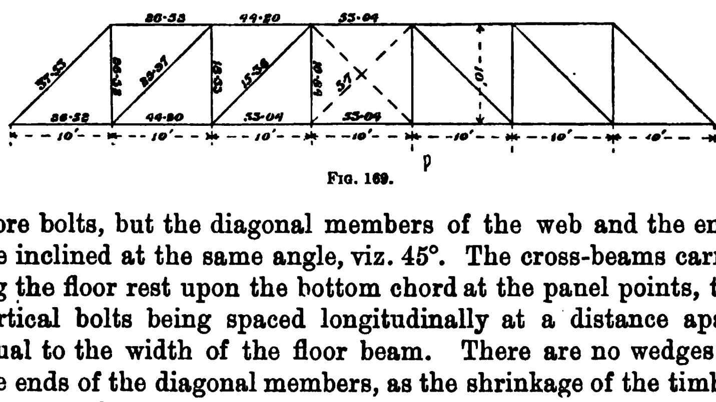


FIG. 169.

more bolts, but the diagonal members of the web and the ends are inclined at the same angle, viz. 45°. The cross-beams carrying the floor rest upon the bottom chord at the panel points, the vertical bolts being spaced longitudinally at a distance apart equal to the width of the floor beam. There are no wedges at the ends of the diagonal members, as the shrinkage of the timber may be taken up completely by screwing up the nuts on the vertical bolts, hence only the central bay is counterbraced.

The stresses written on Fig. 169 have been determined for a dead load of 0·414 tons and a live load of 0·470 tons per foot run for each truss.

This truss is obviously superior to the foregoing for timber bridges.

The method of constructing the joints in the bottom chord of these trusses will be considered in Chapter XIII.

The bottom chord in both these trusses may be constructed of steel, and the compression members only of timber. This composite truss has the advantage of allowing the various timber members to be removed without stopping the traffic over the bridge.

The following example is given to illustrate the method of calculating the stresses in lattice girders subjected to a permanent load due to the weight of the structure, and a live load due to an engine and train. This girder is not given as an example of good proportion, as it is too deep, and is unsuitable for so small a span.

Let the permanent load considered as concentrated at each apex be 2 tons, denoted by W_1, and the live load concentrated in a similar manner be 5 tons, denoted by W. There are 12 bays and 4 systems of triangulation, the bars being inclined at 45°.

$$W \sec \theta = 5 \times 1{\cdot}414 = 7{\cdot}070 \text{ tons, and } W_1 \sec \theta = 2{\cdot}828$$

$$\frac{W \sec \theta}{12} = 0{\cdot}589$$

Span of girder = 60 feet, depth = 10 feet

By referring to Fig. 170 and the subjoined table of stresses, it will be observed that the live load W_1 produces stresses on

No. of Bar	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}	W_{11}	Stress from Live Load Compn. −	Stress from Live Load Tensn. +	Stress from Dead Load Compn. −	Stress from Dead Load Tensn. +	Total Stresses from Live and Dead Loads Compn. −	Total Stresses from Live and Dead Loads Tensn. +
1			−5·30				−2·94				−·59	8·83		2·29		11·12	
2				−4·71				−2·36				7·07		2·29		9·36	
3	+·59				−4·12				−1·77			5·89	·59	2·29		8·18	·59
4		+1·18				−3·53				−1·18		4·71	1·18	1·14		5·85	1·18
5			+1·77				−2·94				−·59	3·53	1·77	0		3·53	1·77
6				+2·36				−2·36				2·36	2·36	0		2·36	2·36
7	+·59				+2·94				−1·77			1·77	3·53	0		1·77	3·53
8		+1·18				+3·53				−1·18		1·18	4·71		1·14	1·18	5·85
9			+1·77				+4·12				−·59	·59	5·89		2·29	·59	8·18
10				+2·36				+4·71					7·07		2·29		9·36
11	+·59				+2·94				−5·30				8·83		2·29		11·12
12		+1·18				+3·53				+5·89			10·60		3·43		14·03
13			+1·77				+4·12				+6·48		12·37		4·58		16·95

Fig. 170.

bars 3, 7, and 11; that W_2 produces stresses on bars 4, 8, and 12; W_3 on bars 1, 5, 9, and 13. W_{13} also produces stresses on bars 1, 5, 9, and 13, thus—

$$-\frac{W \sec \theta}{12} = -0{\cdot}589, \text{ say } -0{\cdot}59$$

W_3 produces—

$$-\frac{9W \sec \theta}{12} = -5{\cdot}30 \text{ on bar 1}$$

$$+\frac{3W \sec \theta}{12} = -1{\cdot}77 \text{ on bars 5, 9, and 13}$$

The stresses produced on each bar by the various loads, considered separately, are tabulated as shown. The stresses on bar 1 for the loads W_3, W_7, and W_{11} which affect it are added together and the result written in the first column, as it is compressive. In a similar manner, the stresses on the other bars are added, and written in the first or second column, according as the stress is compressive or tensile.

The stresses in the third and fourth columns due to dead load are found, as in the foregoing examples, by affixing the proper coefficient to the bar and multiplying it by W_1 sec θ (Fig. 171).

The total maximum stresses are found by adding the stresses of like kind in the first four columns due to live load and dead load. It should be noted that bars from 3 to 9 are subjected to both tensile and compressive stresses.

The stresses in the remaining 13 bars may be written down by numbering them from the right-hand support.

The stresses in the top and bottom horizontal members are found as before, by adding the coefficients written in the lattice bars, Fig. 171, and multiplying by $(W + W_1)$ tan θ for the increment of flange stress $= (2 + 5)$ tan $\theta = 7$ tan θ.

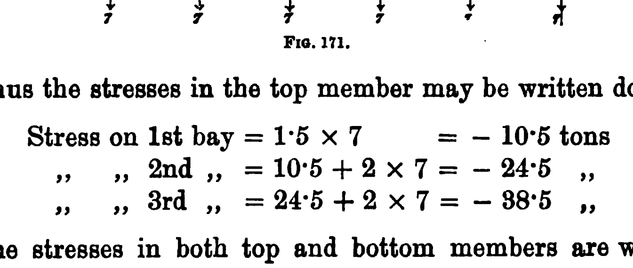


FIG. 171.

Thus the stresses in the top member may be written down—

Stress on 1st bay	$= 1{\cdot}5 \times 7$	$= - 10{\cdot}5$ tons
,, ,, 2nd ,,	$= 10{\cdot}5 + 2 \times 7$	$= - 24{\cdot}5$,,
,, ,, 3rd ,,	$= 24{\cdot}5 + 2 \times 7$	$= - 38{\cdot}5$,,

The stresses in both top and bottom members are written down in Fig. 171.

The stresses in the top and bottom members may be plotted to scale as ordinates on the length of the girder as a base, and the extremities joined, forming two polygons, which are the diagrams of direct stresses due to bending, and may be used for designing the top and bottom flanges as explained in Chapter IV.

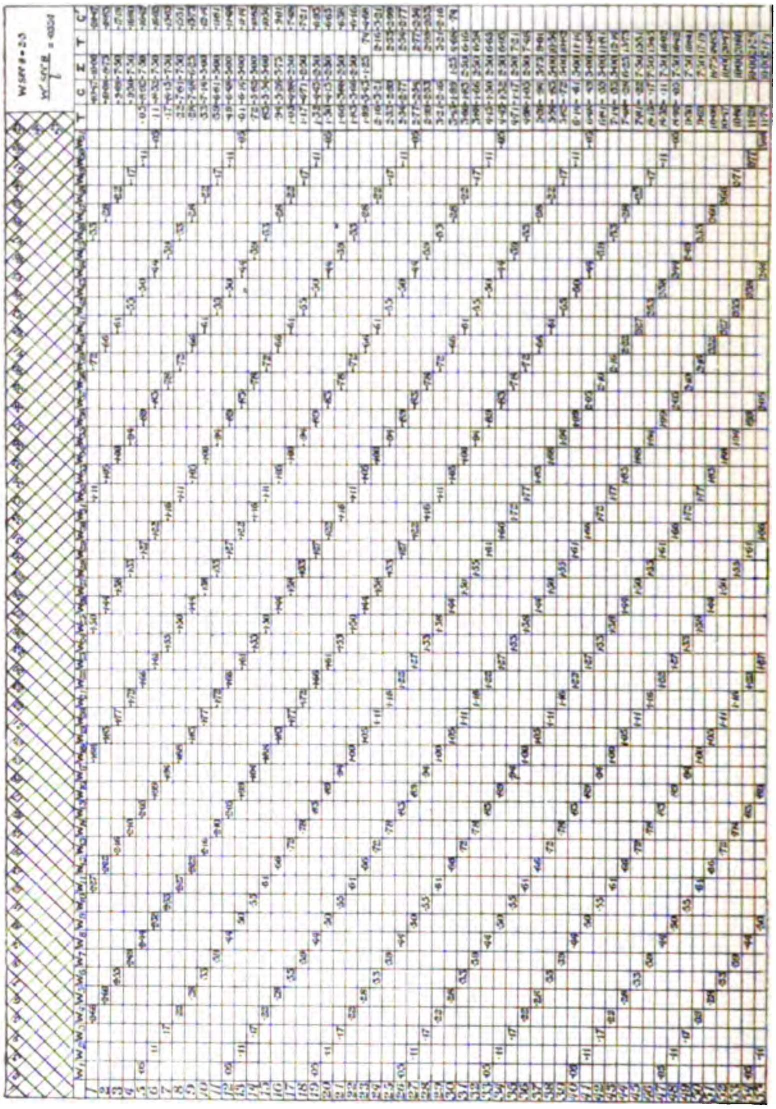


Fig. 172.

The following example is given to illustrate the method of designing a lattice girder or of ascertaining the stresses in a girder already built. Figs. 172 and 173 show the stresses in the web and flanges of a lattice girder in a bridge on the New South Wales Government Railways.[1] The bridge consists of two lattice main girders, forming a clear span of 150 feet. The girders are 161 feet 9 inches long over all, and placed 14 feet apart in the clear. Between them transverse or roadway girders are placed at a distance of 3 feet from centre to centre.

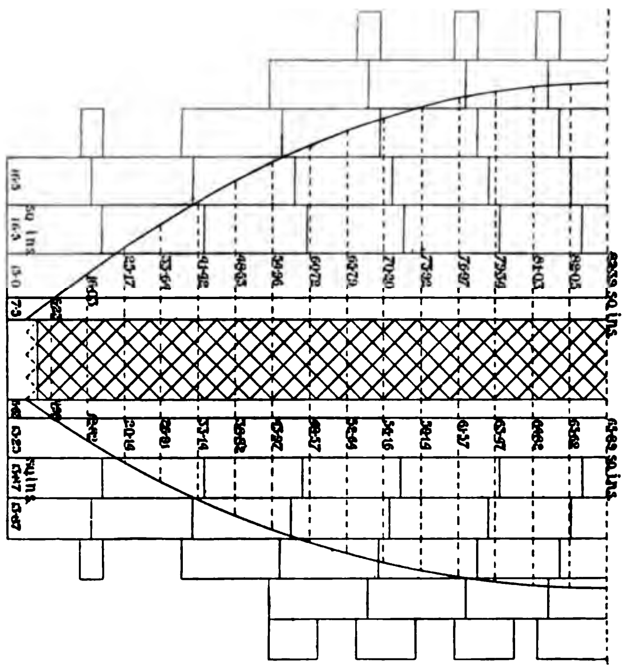


Fig. 173.

The bridge carries a single line of way. The lattice girders rest at each pier on cast-iron bed-plates fixed on the piers, with steel expansion rollers at one pier. Each lattice girder is 12 feet deep between the intersection of the lattice bars. The booms are trough-shaped, and connected with double-lattice webs. The web is formed of a double set of lattice bars riveted to the vertical plates of the booms, and inclined at an angle of 43° 25′. Each set of lattice bars consists of seven systems of triangulation, the flat tension bars varying from 6″ × $\frac{5}{8}$″ to 4″ × $\frac{1}{2}$″, and

[1] Railway Bridges Inquiry Commission, New South Wales, 1887.

the channel irons forming struts from 6″ × $2\frac{1}{2}$″ × $\frac{1}{2}$″ to $3\frac{1}{2}$″ × $1\frac{3}{4}$″ × $\frac{1}{2}$″.

The data for calculations are as follows:—

Effective span = 156′ 0″
,, depth = 72′ 0″
Dead load on each girder = 0·6 tons per foot run
Live ,, ,, ,, = 0·7 ,, ,, ,, ,,

Weight at each apex due to dead load = 0·6 × 3 = 1·8 tons
,, ,, ,, ,, live ,, = 0·7 × 3 = 2·1 ,,

$$\theta = 43° \; 25' \quad \therefore \sec\theta = 1{\cdot}37274$$

$$W_1 \sec\theta = 1{\cdot}8 \times 1{\cdot}37274 = 2{\cdot}5 \text{ tons}$$

$$\frac{W \sec\theta}{l} = \frac{2{\cdot}1 \times 1{\cdot}37274}{52} = {\cdot}0554 \text{ tons}$$

The following table gives the bending moments, stresses, and areas required in the bridge. By comparison of this table with the diagram of areas, Fig. 173, we have—

Maximum stress in compression = 3·72 tons per square inch
,, ,, tension = 4·37 ,, ,, ,,

TABLE XXXII.

$x =$	$M = {\cdot}65(6084 - x^2)$	Stress.	Area required.	
			Top.	Bottom.
0	3954·00	329·55	82·39	65·91
5	3938·35	328·20	82·05	65·64
10	3889·60	324·13	81·03	64·82
15	3808·35	317·36	79·34	63·47
20	3694·60	307·88	76·97	61·57
25	3548·35	295·69	73·93	59·14
30	3369·60	280·80	70·20	56·16
35	3158·35	263·19	65·79	52·64
40	2914·60	242·89	60·72	48·57
45	2638·35	219·86	54·96	43·97
50	2329·60	194·13	48·53	38·82
55	1988·35	165·69	41·42	33·14
60	1614·60	134·53	33·64	26·91
65	1208·35	100·69	25·17	20·14
70	769·60	64·13	16·03	12·82
75	338·35	28·19	7·05	5·62
78	0·00	0·00	0·00	0·00

CHAPTER IX.

BOWSTRING AND POLYGONAL GIRDERS.

THE stresses in girders or trusses with curved or polygonal flanges or chords may be most conveniently ascertained by the method of moments explained in Chapter III.

Let Fig. 174 denote an inverted bowstring girder or truss, in which the bottom chord is a polygon inscribed in a parabolic

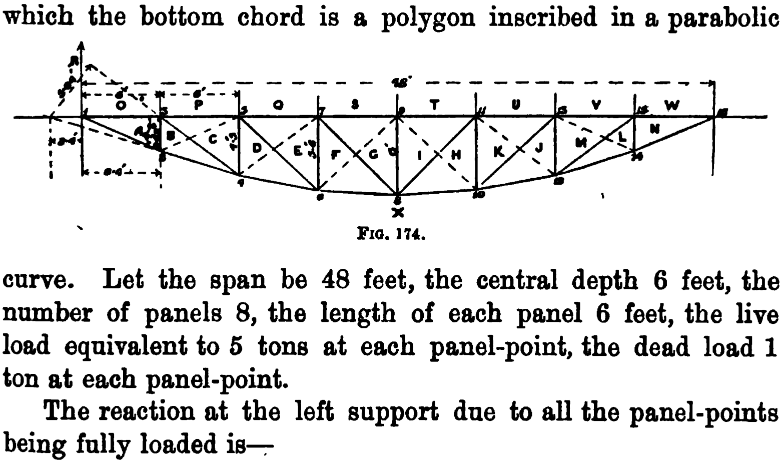


FIG. 174.

curve. Let the span be 48 feet, the central depth 6 feet, the number of panels 8, the length of each panel 6 feet, the live load equivalent to 5 tons at each panel-point, the dead load 1 ton at each panel-point.

The reaction at the left support due to all the panel-points being fully loaded is—

$$R = (\tfrac{1}{8} + \tfrac{2}{8} + \ . \ . \ . + \tfrac{7}{8}) + 5(\tfrac{1}{8} + \tfrac{2}{8} + \ . \ . \ . + \tfrac{7}{8})$$
$$= 3{\cdot}5 + 17{\cdot}5$$

The stress in AO, denoted by AO, is found by taking moments about the point 2, the lever arm or the length 3·2 being 2·62 feet.

$$AO(2{\cdot}62) + R(6) = 0$$

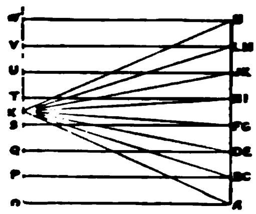
$$\therefore \ AO = -\ (17{\cdot}5 + 3{\cdot}5)\frac{6}{2{\cdot}62} = -\ 48{\cdot}08 \text{ tons}$$

The stress in AX is found by taking moments about the point 3, the lever arm, or the perpendicular distance from 3 on AX, being 2·4 feet—

$$-AX(2{\cdot}4) + R(6) = 0$$

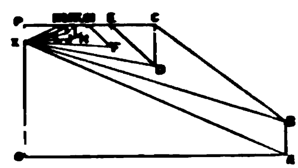
$$\therefore AX = +(17{\cdot}5 + 3{\cdot}5)\frac{6}{2{\cdot}4} = 52{\cdot}5 \text{ tons}$$

The stress in the vertical member AB is found by passing a



DEAD LOAD.

FIG. 175.



LIVE LOAD AT APEX 3.

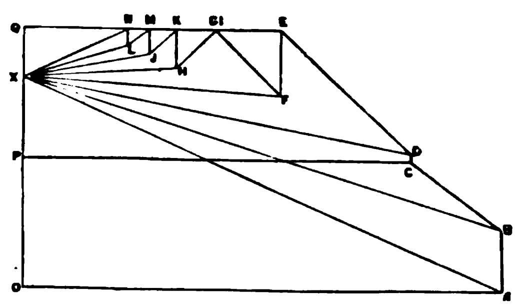
FIG. 176.

plane through AO, AB, and BX, and taking moments about the intersection of AO and BX.

$$-AB(8{\cdot}4) - R(2{\cdot}4) = 0$$

$$\therefore AB = -(17{\cdot}5 + 3{\cdot}5)\frac{2{\cdot}4}{8{\cdot}4} = -6 \text{ tons}$$

The stresses in the three members of the second bay may be



LIVE LOAD AT APEX 3.

FIG. 177.

found by passing a plane through PC, BC, and BX, and taking moments, thus—

For the stress in PC take moments about the point 4, for BX take moments about the point 3, and for BC take moments about the intersection of PC and BX—

$$4{\cdot}5PC + 12R - 6 \times 6 = 0$$

$$\therefore PC = -\frac{21 \times 12 - 36}{4{\cdot}5} = -48 \text{ tons}$$

$$-2{\cdot}5BX \times +6R = 0$$

$$\therefore BX = +\frac{6 \times 21}{2{\cdot}5} = 50{\cdot}4 \text{ tons}$$

$$5{\cdot}04BC - R \times 2{\cdot}4 + 5 \times 8{\cdot}4 + 1 \times 8{\cdot}4 = 0$$

This equation may be written thus—

$$\begin{aligned} O = 5{\cdot}04BC &- \{(\tfrac{1}{8} + \ldots + \tfrac{6}{8})2{\cdot}4 - (8{\cdot}4 - \tfrac{7}{8} \times 2{\cdot}4)\} \\ &- 5\{(\tfrac{1}{8} + \ldots + \tfrac{6}{8})2{\cdot}4 - (8{\cdot}4 - \tfrac{7}{8} \times 2{\cdot}4)\} \\ \therefore O = 5{\cdot}04BC &- (6{\cdot}3 - 6{\cdot}3) \\ &- 5(6{\cdot}3 - 6{\cdot}3) \end{aligned}$$

This equation shows that the diagonals are unstressed when the bridge is completely loaded, either with the dead load only or with the dead and live loads. If the panel-points on the right-hand side of the section plane cutting the three bars in the bay under consideration are fully loaded, and the panel-point on the left is loaded with the dead load only, the equation becomes—

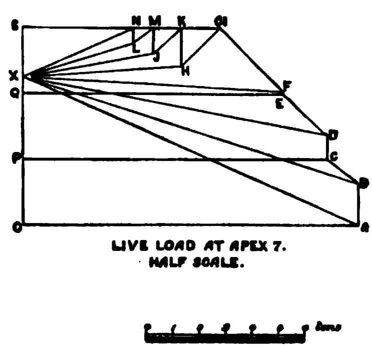


FIG. 178.

$$\begin{aligned} O = 5{\cdot}04BC &- (6{\cdot}3 - 6{\cdot}3) \\ &- 5 \times 6{\cdot}3 \end{aligned}$$

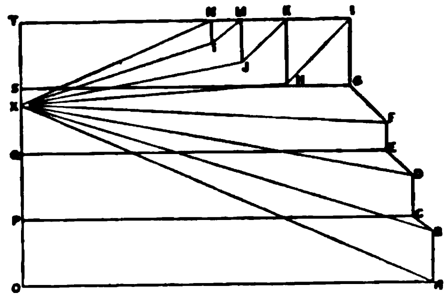
$$\therefore BC = 6{\cdot}25 \text{ tons}$$

Hence the greatest tension on any diagonal occurs when the panel-points on the right are fully loaded. If the panel-point on the left is fully loaded, and those on the right with the dead load only, the equation becomes—

$$\begin{aligned} O = 5{\cdot}04\,BC &- (6{\cdot}3 - 6{\cdot}3) \\ &+ 5 \times 6{\cdot}3 \end{aligned}$$

$$\therefore BC = -6{\cdot}25 \text{ tons}$$

Hence the greatest compression on any diagonal occurs when the panel-points on the left are fully loaded, and those on the right with the dead load only.



LIVE LOAD AT APEX 3.
HALF SCALE.

FIG. 179.

In a similar manner it can be shown that when the diagonals slope in the contrary direction, *i.e.* from 2 to 5, instead of from 3 to 4, the same rule holds good if the word "tension" is altered to "compression," and *vice versâ.*

If both diagonals are present, they should be designed for tension only, in which case only one will be in action at a time, and the vertical members will be in compression.

In a similar manner the stresses in the remaining bars may be found and tabulated, or they may be written on the members of the girder, as in Fig. 180.

If the girder have a single system of triangulation, the stresses may be derived from Fig. 180, thus—

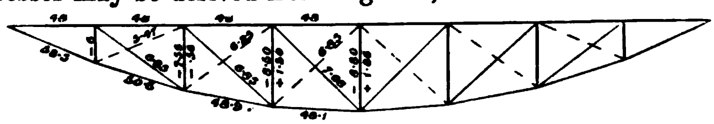


FIG. 180.

The stresses in the bowstring truss shown in Fig. 174 may be determined by drawing reciprocal figures to scale, and measuring the lengths of the lines as explained in Chapter III. One diagram is sufficient to determine the stresses due to the

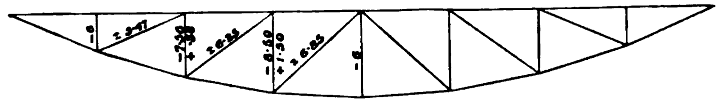


FIG. 181.

dead load, but for the partially distributed live load it is necessary to consider the stresses for the live load at each apex separately, and then to combine the results for the total stress.

Fig. 175 shows the reciprocal figure for the bowstring truss for the dead load; Fig. 176, the live load only at the apex 3; Fig. 177, the live load at apex 5; Fig. 178, the live load at apex

7; and Fig. 179, the live load at apex 9. The sum of the stresses obtained by measuring the lines in the reciprocal figures and tabulating the results may be used as a check on

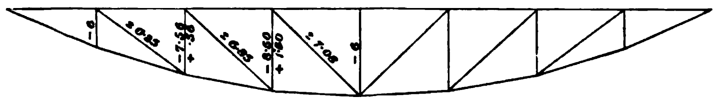


FIG. 182.

the method of moments, in which case the mean value should be taken in designing the structure.

In a polygonal girder in which the panel-points in the polygonal chord do not lie on a parabolic curve, the dead load will produce stresses in the diagonals. Thus Fig. 183 shows a

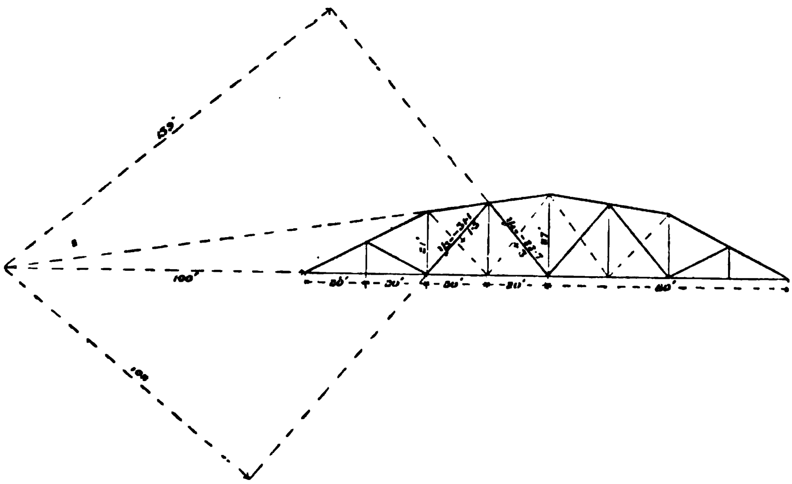


FIG. 183.

form of truss suitable for a timber bridge, and Fig. 184 a truss of the same dimensions suitable for an iron or steel bridge, the compression members in each case being denoted by thicker lines.

For a highway bridge with a roadway 20 feet wide and two overhanging footways each 5 feet wide, the dead load at each panel-point is about 14 tons, and the live load 11·2 tons, or 0·8 of the dead load.

The stress in the diagonal member in the third bay, denoted by Y_3, Fig. 183, is therefore—

$$
\begin{aligned}
O = & - 108y_3 - [14\{(\tfrac{1}{8} + \ldots + \tfrac{5}{8})100 - (120 - \tfrac{7}{8} \times 100) \\
& - (140 - \tfrac{6}{8} \times 100)\}] - 0{\cdot}8[14\{(\tfrac{1}{8} + \ldots + \tfrac{5}{8})100 \\
& - (120 - \tfrac{7}{8} \times 100) - (140 - \tfrac{6}{8} \times 100)\}] \\
\therefore O = & - 108y_3 - 2625 + 1365 \\
& \qquad\qquad - 2100 + 1092 \\
\therefore y_3 = & - 31{\cdot}1 \text{ tons} \\
& + \ 1{\cdot}5 \ \ ,,
\end{aligned}
$$

For the dead load only—

$$
\begin{aligned}
O = & - 108y_3 - 2695 + 1365 \\
\therefore y_3 = & - 11{\cdot}7 \text{ tons}
\end{aligned}
$$

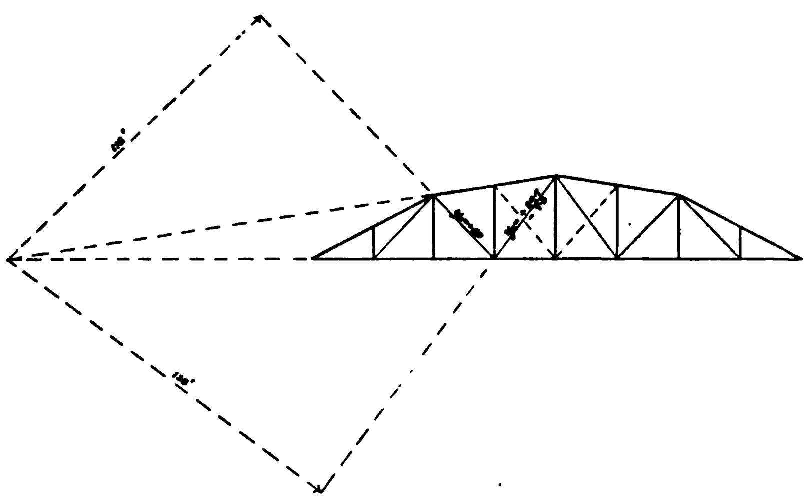


FIG. 184.

In Fig. 184—

$$
\begin{aligned}
O = & \ 116y - [14\{(\tfrac{1}{8} + \ldots + \tfrac{5}{8})100 - (120 - \tfrac{7}{8} \times 100) \\
& - (140 - \tfrac{6}{8} \times 100)\}] - 0{\cdot}8[14\{(\tfrac{1}{8} + \ldots + \tfrac{5}{8})100 \\
& - (120 - \tfrac{7}{8} \times 100) - (140 - \tfrac{6}{8} \times 100)\}] \\
\therefore O = & \ 116y_3 - 2625 - 1365 \\
& \qquad\qquad - 2100 + 1092 \\
\therefore y_3 = & + 29 \text{ tons}
\end{aligned}
$$

The stresses in the diagonals of the fourth bay, Fig. 183, are—

$$\begin{aligned} 0 = 139y_4 &- [14\{(\tfrac{1}{8} + \ldots + \tfrac{4}{8})100 - (120 - \tfrac{7}{8} \times 100) \\ &- (160 - \tfrac{5}{8} \times 100)\}] - 0{\cdot}8[14\{(\tfrac{1}{8} + \ldots + \tfrac{4}{8})100 \\ &- (120 - \tfrac{7}{8} \times 100) - 160 - \tfrac{5}{8} \times 100)\}] \\ 0 = 139y_4 &- 1750 + 455 + 910 + 1365 \\ &- 1400 + 364 + 728 + 1092 \\ \therefore 0 = 139y_4 &- 1750 + 2730 \\ &- 1400 + 2184 \\ \therefore y_4 = &- 22{\cdot}7 \text{ tons} \\ &+ 3{\cdot}0 \;\; ,, \end{aligned}$$

In Fig. 184—

$$\begin{aligned} 0 = 128y_4 &- 1750 + 2730 \\ &- 1400 + 2184 \\ \therefore y_4 = &+ 24{\cdot}7 \text{ tons} \\ &- 7{\cdot}5 \;\; ,, \end{aligned}$$

In a similar manner the remaining stresses may be found.

In Fig. 183 it would be desirable to introduce counterbraces in the four central panels, as indicated in the dotted lines; the largest dimensions should be at right angles to the plane of the truss, for the sake of greater lateral stiffness. The diagonals should be designed for compression only, and provision should be made for taking up the shrinkage of the timber, such as by means of wedges at the joints, which would put the diagonals in initial compression. Otherwise the diagonals should be designed for both tension and compression. The tension members are conveniently constructed with a group of four iron or steel bolts, which may be screwed up as required.

The use of wedges and counterbraces introduces some ambiguity in the determination of the stresses.

In Fig. 184 the two middle panels should be counterbraced by means of rods provided with union screws for putting the counterbrace in initial tension, or these may be omitted and the diagonals designed for compression as well as tension, which may easily be accomplished in an iron or steel bridge, thus avoiding any ambiguity in the determination of the stresses.

The stresses in these trusses might also be found by means of reciprocal figures, as illustrated with respect to the truss shown in Fig. 174.

CHAPTER X.

THE SLOPE AND DEFLECTION OF BEAMS SUPPORTED AT THE ENDS—BENDING MOMENTS, SHEARING STRESSES SLOPE; AND DEFLECTION OF BEAMS FIXED AT ONE OR BOTH ENDS.

THE investigation of the equations of slope and deflection requires a knowledge of the calculus, but the principles on which the investigation is based may be understood without such knowledge; also the results, which will be found tabulated, may be used in connection with the slope and deflection of beams as ordinary formulæ.

By referring to Fig. 75, which represents a beam deflected under the action of transverse loads, we may proceed as follows:—

Let R = the radius of curvature of the neutral axis of the beam.

a = the strain or elongation per unit of length of the fibre.

y = the distance of the extreme fibre from the neutral axis.

l = the length, measured along the neutral axis, between two planes which are parallel in the undeflected beam, but which converge towards the centre of curvature in the deflected beam.

$l + al$ = the increased length due to bending, measured along the extreme fibres.

f = the intensity of stress at the extreme fibres.

E = the modulus of elasticity.

I = the moment of inertia.

i = the slope of the beam at any point x from the origin.

v = the deflection of the beam.

Then it has been proved that—

$$\frac{R+y}{R}=\frac{l+al}{l} \quad \therefore 1+\frac{y}{R}=1+a$$

$$\therefore \frac{y}{R}=a$$

$$\text{but } a=\frac{f}{E} \quad \therefore \frac{f}{E}=\frac{y}{R}$$

but in Chapter V. it has been proved that—

$$M=\frac{fI}{y} \quad \therefore \frac{M}{I}=\frac{f}{y}$$

$$\therefore \frac{1}{R}=\frac{f}{Ey}=\frac{M}{EI}$$

It is proved in books on the Differential Calculus that—

$$\frac{1}{R}=\frac{\frac{d^2v}{dx^2}}{\left\{1+\left(\frac{dv}{dx}\right)^2\right\}^{\frac{3}{2}}}$$

In the case of beams the deflection is small, so that the value of $\left(\frac{dv}{dx}\right)^2$ is so small that it may be neglected, in which case—

$$\frac{1}{R}=\frac{d^2v}{dx^2}=\frac{M}{EI}$$

If i denote the circular measure of the slope of the beam at a distance x from the origin of co-ordinates, since—

$$i=\tan i=\frac{dv}{dx} \text{ nearly}$$

$$\frac{d^2v}{dx^2}=\frac{M}{EI}$$

$$\therefore i=\frac{dv}{dx}=\int\frac{M}{EI}\,.\,dx$$

$$v=\int\int\frac{M}{EI}\,.\,dx^2$$

E is a constant for the same material, and I is a constant if the section of the beam is uniform; M can also be expressed in terms of x; so that the above equations may be written—

$$i = \frac{1}{EI}\int M \, . \, dx, \text{ and } v = \frac{1}{EI}\int\int M \, . \, dx^2$$

The method of using these equations is illustrated in the following examples.

Case I.—Cantilever loaded at the extremity.

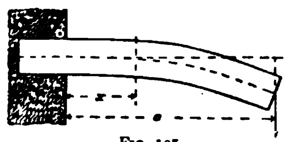


FIG. 185.

In this case—

$$M = -W(c - x)$$

$$\therefore i = -\frac{W}{EI}\int (c - x)dx$$

$$= -\frac{W}{EI}\left(cx - \frac{x^2}{2}\right) + C$$

When $x = 0$, $i = 0$. $\therefore C = 0$

$$\therefore i = -\frac{W}{EI}\left(cx - \frac{x^2}{2}\right) \qquad (1)$$

When i is the slope at a distance x from the origin—

$$v = \int i \, . \, dx = -\frac{W}{EI}\int\left(cx - \frac{x^2}{2}\right)dx = -\frac{W}{EI}\left(\frac{cx^2}{2} - \frac{x^3}{6}\right) + C$$

When $x = 0$, $v = 0$. $\therefore C = 0$

$$\therefore v = -\frac{W}{EI}\left(\frac{cx^2}{2} - \frac{x^3}{6}\right) \qquad (2)$$

To find the greatest slope and deflection, we have only to remember that both equations (1) and (2) reach their maximum value when $x = c$; hence, denoting the greatest slope and deflection by i_0 and v_0 respectively, we have—

$$i_0 = -\frac{Wc^2}{2EI}; \text{ and } v_0 = -\frac{Wc^3}{3EI}$$

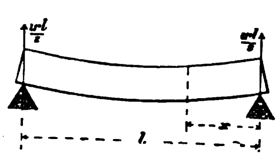


FIG. 186.

Case II.—Beam supported at both ends and loaded uniformly, the load being w per unit of length.

In this case—

$$M = \frac{wlx}{2} - \frac{wx^2}{2} = \frac{w}{2}(lx - x^2)$$

$$i = \frac{w}{2EI}\int (lx - x^2)dx = \frac{w}{2EI}\left(\frac{lx^2}{2} - \frac{x^3}{3}\right) + C$$

When $x = \frac{l}{2}$, $i = 0$.

$$\therefore 0 = \frac{w}{2EI}\left(\frac{l^3}{8} - \frac{l^3}{24}\right) + C$$

$$\therefore C = -\frac{w}{2EI}\left(\frac{l^3}{8} - \frac{l^3}{24}\right) = -\frac{Wl^2}{24EI}$$

$$\therefore i = \frac{w}{2EI}\left(\frac{lx^2}{2} - \frac{x^3}{3}\right) - \frac{wl^3}{24EI} = \frac{w}{24EI}\{6lx^2 - 4x^3 - l^3\}$$

$$\therefore v = \int i \, . \, dx = \frac{w}{24EI}\int(6lx^2 - 4x^3 - l^3)dx$$

$$= \frac{w}{24EI}(2lx^3 - x^4 - l^3x) + C$$

When $x = 0$, $v = 0$. $\therefore C = 0$

$$\text{and } v = \frac{w}{24EI}(2lx^3 - x^4 - l^3x)$$

To find the greatest slope, put $x = 0$, or $x = l$.

$$\therefore i_0 = \frac{wl^3}{24EI} = \frac{Wl^2}{24EI}, \text{ where } W = wl$$

For the greatest deflection, put $x = \frac{l}{2}$.

$$\therefore v_0 = \frac{w}{24EI}\left(\frac{5l^4}{16}\right) = \frac{5wl^4}{384EI} = \frac{5Wl^3}{384EI}$$

In a similar manner, the greatest slope and deflection may be found for any beam loaded in any way whatever.

In the following table $W = wl =$ the total load will be used :—

TABLE XXXIII.

TABLE OF MAXIMA SLOPES AND DEFLECTIONS FOR BEAMS OF UNIFORM SECTION.

Description of beam.	Greatest slope.	Greatest deflection.
Fixed at one end and loaded at the other ...	$\frac{Wl^2}{2EI}$	$\frac{Wl^3}{3EI}$
Fixed at one end and loaded uniformly ...	$\frac{Wl^2}{6EI}$	$\frac{Wl^3}{8EI}$
Supported at both ends and loaded at the centre	$\frac{Wl^2}{16EI}$	$\frac{Wl^3}{48EI}$
Supported at both ends and loaded uniformly ...	$\frac{Wl^2}{24EI}$	$\frac{5Wl^3}{384EI}$

By referring to the results in the foregoing table, it will be seen that both the slope and deflection are proportional to the load, but that the slope is proportional to the square of the length, while the deflection is proportional to the cube of the length. Hence we may express the slope and deflection thus—

$$i_0 = \frac{nWl^2}{EI} \qquad v_0 = \frac{mWl^3}{EI}$$

where n and m have the numerical values given in the foregoing table; thus in the case of a beam supported at both ends and loaded uniformly, $n = \frac{1}{24}$, and $m = \frac{5}{384}$.

It is frequently convenient to express the slope and deflection of a beam in terms of the maximum intensity of working stress; thus in the case of a beam supported at both ends and loaded uniformly—

$$\frac{Wl}{8} = \frac{fI}{y} \qquad \therefore W = \frac{8fI}{ly}$$

$$\text{Hence the slope } i_0 = \frac{fl}{3Ey}$$

$$\text{and the deflection } v_0 = \frac{5fl^2}{48Ey}$$

In this case the slope is proportional to the length, while the deflection is proportional to the square of the length.

In a similar manner, the slopes and deflections of the other three cases may be expressed in terms of the maximum intensity of working stress. Hence we may express i_0 and v_0 thus—

$$i_0 = \frac{n'fl}{Ey}, \text{ and } v_0 = \frac{m'fl^2}{Ey}$$

In rectangular beams—

$$I = \frac{1}{12}bd^3 \qquad y = \frac{d}{2}$$

By substituting these values in the foregoing table, we obtain formulæ applicable to rectangular beams; thus in the case of a beam supported at both ends and loaded uniformly—

$$i_0 = \frac{Wl^2}{2Ebd^3}, \text{ and } v_0 = \frac{5Wl^3}{32Ebd^3}$$

If we denote the intensity of stress at the extreme fibre by f, we have—

$$i_0 = \frac{2fl}{3Ed}, \text{ and } v_0 = \frac{5fl^2}{24Ed}$$

In a similar manner we may find the value of i_0 and v_0 for the other three cases—

Beams of Uniform Strength.—In a beam of uniform strength the moment of resistance of any cross-section is proportional to the bending moment at that cross-section.

In Figs. 187 to 194 we have various forms in which either the breadth or the depth is constant, except in Fig. 189, which is a beam of similar cross-section. Figs. 187, 188, and 189

FIG. 187. FIG. 188. FIG. 189.

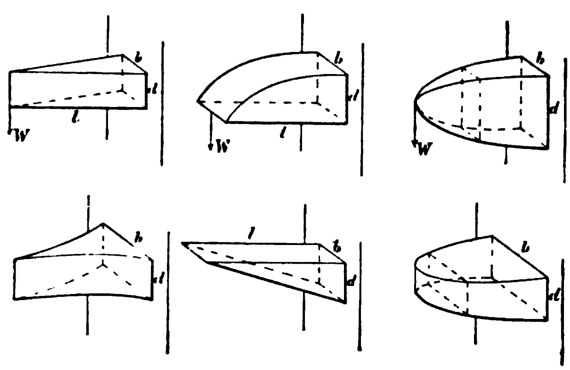


FIG. 190. FIG. 191. FIG. 192.

represent cantilevers of uniform strength when loaded at the extremity with a concentrated load; and Figs. 190, 191, and 192 represent cantilevers loaded with a uniform load.

Fig. 193 shows the shape of a beam of uniform strength,

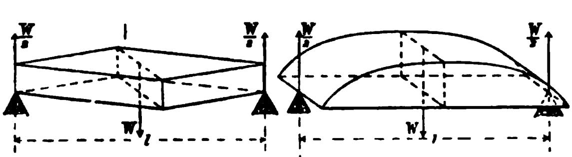


FIG. 193. FIG. 194.

supported at both ends and loaded in the centre when the depth is constant.

Fig. 194 shows a beam when the breadth is constant.

Fig. 195 shows the shape of a beam for similar cross-section.

In practice the above forms could not be carried out, as it is necessary to provide for the resistance to the shearing stresses as well as the bending moments; consequently they may be modified as shown in Figs. 196 and 197, where the sectional areas over the supports are made in accordance with the shearing stresses.

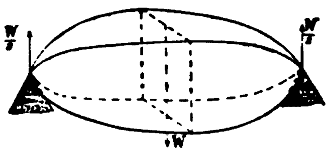


FIG. 195.

In beams of uniform strength the moment of inertia is not constant, and the term $\frac{M}{EI}$ contains two variables.

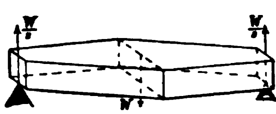


FIG. 196.

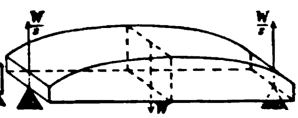


FIG. 197.

In the case of Fig. 187, d is constant, and b varies; here, if b_1 denote the breadth of any section, $b_1 = b\frac{x}{l}$.

If $b = b_1$, and d varies, as in Fig. 188, then $d_1^2 = d^2\frac{x}{l}$, where d_1 denotes the depth at any section. If both b and d vary, as in Fig. 189—

$$\frac{b}{d} = \frac{b_1}{d_1} \qquad \therefore b_1 = \frac{bd_1}{d_1}$$

In Fig. 187—

$$Wx = \frac{2fI}{d_1} \qquad \therefore f = \frac{Wxd_1}{2I}$$

where $d_1 = 2y$; but in rectangular sections—

$$f = \frac{6Wx}{b_1h_1^2} = \frac{6Wl}{bh^2}$$

since f is a constant;

$$\therefore \frac{b_1d_1^2}{bd^2} = \frac{x}{l}$$

$$\text{Also } \frac{d^2y}{dx_2} = -\frac{M}{EI} = -\frac{Wx}{\frac{1}{12}Eb_1d_1^3}$$

where b and h vary.

If the height is constant and equal to d, then $b_1 = \frac{bx}{l}$

$$\therefore \frac{d^2v}{dx^2} = -\frac{12Wl}{Ed^3b}$$

Integrating this expression, and remembering that $\frac{dy}{dx} = 0$ when $x = l$, we have—

$$\frac{dv}{dx} = -\frac{12Wlx}{Ed^3b} + \frac{12Wl^2}{Ed^3b}$$

Integrating again, and remembering that $v = 0$ when $x = l$, we have—

$$v = -\frac{6Wlx^2}{Ed^3b} + \frac{12Wl^2x}{Ed^3b} - \frac{6Wl^3}{Ed^3b}$$

The maximum deflection is $v_0 = \frac{6Wl^3}{Ed^3b}$, whereas, for a uniform cross-section we found $v_0 = \frac{4Wl^3}{Ed^3b}$, or the beam of uniform strength deflects one and a half times as much as the beam of uniform cross-section.

In a similar manner, it may be proved that the beam of constant breadth (Fig. 188) deflects $v_0 = \frac{8Wl^3}{Ed^3b}$, or twice as much as the beam of uniform cross-section.

Also, for similar cross-section (Fig. 189), we have $v_0 = \frac{36Pl^3}{5Ed^3b}$

For any cross-section, using the general formulæ—

$$i_0 = \frac{nWl^2}{EI}, \text{ and } v_0 = \frac{mWl^3}{EI}$$

we have the following values for n and m:—

TABLE XXXIV.

TABLE OF VALUES FOR n AND m FOR BEAMS OF UNIFORM STRENGTH.

Description of beam. Uniform depth.	n.	m.
Fixed at one end and loaded at the other	1	$\frac{1}{2}$
Fixed at one end and loaded uniformly	$\frac{1}{2}$	$\frac{1}{4}$
Supported at both ends and loaded at the centre	$\frac{1}{8}$	$\frac{1}{32}$
Supported at both ends and loaded uniformly	$\frac{1}{16}$	$\frac{1}{64}$

TABLE XXXV.

Description of beam. Uniform breadth.	n.	m.
Fixed at one end and loaded at the other	2	$\frac{2}{3}$
Supported at both ends and loaded at the centre	$\frac{1}{4}$	$\frac{1}{24}$
Supported at both ends and loaded uniformly	[illegible]	0·018

Deflection of Beams under Loads developing a Known Intensity of Stress.—In this case we use the formula—

$$r = \frac{m f l^2}{E y}$$

f denotes the maximum intensity of stress on the weaker side of the beam.

If f_t denote the maximum intensity of tensile stress, and f_c the maximum intensity of compressive stress, then $f = \frac{f_t + f_c}{2}$. If d denote the depth of the beam—

$$r = \frac{m(f_t + f_c)l^2}{E d}$$

This formula may be applied to wrought-iron or steel girders. The values of m are given in the following table :—

TABLE XXXVI.

No.	Uniform cross-section.	m
1	Cantilever fixed at one end and loaded at the other	$\frac{1}{3}$
2	Cantilever fixed at one end and loaded uniformly	$\frac{1}{4}$
3	Beam supported at both ends and loaded in the centre ...	$\frac{1}{12}$
4	Beam supported at both ends and loaded uniformly	$\frac{5}{48}$
	Uniform Strength and Depth.	
5	Under either of the conditions Nos. 1 to 4	$\frac{1}{8}$
	Uniform Strength and Breadth.	
6	Cantilever fixed at one end and loaded at the other	$\frac{2}{3}$
7	Cantilever fixed at one end and loaded uniformly	1
8	Beam supported at both ends and loaded at the centre ...	$\frac{1}{6}$
9	Beam supported at both ends and loaded uniformly	·1426

In beams of uniform cross-section the formulæ given may be used to calculate the deflection either under a known intensity of stress or under a known load; but the formulæ given for beams of uniform strength, if used to calculate the deflection of beams which have been designed in the ordinary way with sufficient material at the ends to resist the maximum shearing stress, and the moment of resistance of the flanges made as far as practicable proportional to the bending moments, will give results which are in excess of those obtained by experiment. In plate, web, and lattice girder bridges the breadth is uniform, and the moments of resistance of the cross-section are made as far as practicable proportional to the bending moments. The deflection in these girders is assumed to be produced by the elongation and compression of the flanges, as the effect of the web in resisting deflection is so small that it may be neglected without causing appreciable error. The deflection may be calculated by assuming that the girder is of uniform strength and breadth, using the formula—

$$v_0 = \frac{0{\cdot}1426(f_t + f_c)l^2}{\mathrm{E}d}$$

where f_t = the maximum intensity of tensile stress.
f_c = " " compressive stress.
l = the span.
d = the depth.
E = the modulus of elasticity.

This formula will also give results in excess of those obtained from experiment for the reasons already explained.

The camber of a girder may be calculated from this formula, which will always ensure that the girder will never be deflected quite to the horizontal line, and an allowance should be made of about 25 per cent. more than the calculated maximum deflection.

The deflection of girders is usually allowed to be from $\frac{1}{1200}$ to $\frac{1}{2400}$ of the span, according to circumstances.

The following examples of the deflection of railway bridges under a known intensity of flange stress are given to show how far the formula agrees with experiment. The loads consisted of locomotive engines, and the deflections were measured with multiplying levers.

The test load is expressed in terms of tons per foot run, and it was arrived at by calculating the actual bending moments produced by the engine or engines standing on the bridge, the loads on the driving-wheels and the distance between centres being known.

The bridges were carefully measured and calculated in order to determine f_t and f_c.[1]

The modulus of elasticity in a riveted girder will depend upon the number of joints, and the degree of tightness of the rivets, as well as the material. Experiments made by Sir B. Baker, M.Inst.C.E., on small riveted girders show that the rivets behaved as clamps for ordinary working stresses, the deflections being practically the same as if the girders were rolled in one piece.

In a plate web girder bridge, however, it will generally happen that the modulus is 20 per cent. less than the material of the girder.

For iron E varies from 10,000 to 12,000 tons per square inch.

For mild steel E varies from 11,000 to 13,500 tons per square inch.

For a riveted iron girder, from 8000 to 9500 tons per square inch.

Example I.—A plate web-girder bridge, single web.

Effective span	60 feet.
„ depth	4·7 „
Dead load...	0·76 tons per foot run.
Test load	1·50 „ „
Ratio of test to total load	0·663 „ „
Maximum intensity of stress in tension flange ...	6·75 tons per square inch.
„ „ „ compression flange	4·30 „ „
Mean intensity of stress in flanges	5·535 „ „
„ „ due to test load ...	3·67 „ „

In these girders E will be taken as 9000 tons per square inch.

$$v = \frac{0{\cdot}1426 \times 3{\cdot}67 \times 60 \times 60}{9000 \times 4{\cdot}7} = 0{\cdot}0445 \text{ feet} = 0{\cdot}534 \text{ inches}$$

The average deflection obtained on six bridges by taking the mean of the right and left hand girder was 0·472 inches. The maximum deflection obtained was 0·513 inches.

On thirty-three spans, not differing much from the above example in design, but slightly better in workmanship, agreeing in loads and dimensions, the average deflection was 0·439 inches, and the maximum was 0·516 inches.

Example II.—A plate web girder, single web.

[1] "Report of Railway Bridges Inquiry Commission," New South Wales. 1886.

Effective span	61 feet.
„ depth	6 „
Dead load	0·64 tons per foot run.
Test load	1·50 „ „
Ratio of test to total load	0·7 „ „
Maximum intensity of stress in tension flange ...	4·42 tons per square inch.
„ „ „ compression flange	3·50 „ „
Mean intensity of stress in flanges	3·46 „ „
„ „ due to test load ...	2·42 „ „

E is taken at 9000 tons, although the girders were much better designed.

$$v = \frac{0{\cdot}1426 \times 2{\cdot}42 \times 61 \times 61}{9000 \times 6} = 0{\cdot}0238 \text{ feet} = 0{\cdot}286 \text{ inches}$$

The average observed deflection was 0·283 inches.

Example III.—A lattice girder bridge; double lattice webs, trough-shaped booms.

Effective span	156 feet.
„ depth	12 „
Dead load	1·2 tons per foot run.
Test load	1·4 „ „
Ratio of test to total load	0·54 „ „
Maximum intensity of stress in tension boom ...	4·37 tons per square inch.
„ „ „ compression boom	3·72 „ „
Mean intensity of stress in flanges	4·045 „ „
„ „ due to test load ...	2·18 „ „

E will be taken at 8000 tons in this case.

$$v = \frac{0{\cdot}1426 \times 2{\cdot}18 \times 156 \times 156}{8000 \times 12} = 0{\cdot}0788 \text{ feet} = 0{\cdot}945 \text{ inches}$$

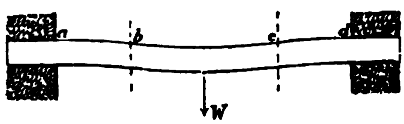
The observed deflection with three engines on the bridge, equivalent to 1·4 tons per foot run, was 0·88 inches, and with the same engines moving at about 40 miles an hour the deflection was one inch.

In a girder bridge similar in every respect to the above, but with fewer loose rivets and slightly better workmanship, the deflection obtained with the stationary load was 0·82 inches, and with the moving load at 40 miles an hour, 0·90 inches.

Beams fixed at the Ends.—A beam is said to be fixed when its ends are loaded in such a manner as to make the tangents at points of support horizontal. The fixing may be effected by building the ends of the beam into a wall (Fig. 198), or by loading the ends as shown in Fig. 199.

When a beam is fixed by either of the above methods there is a downward deflection due to the load, and an upward deflection due to the fixing; hence it follows that there will be two points, *b* and *c*, where the downward deflection changes to an upward deflection; these points are termed "points of inflexion" or of "contra-flexure." The portions from *a* to *b* and from *c* to *d* act as cantilevers, while the portion from *b* to *c* acts as a beam supported at *b* and *c* and loaded as shown. The upward deflection caused by the upward bending moment is uniform from *a* to *d*. Since the tangents at *a* and *d* are horizontal, it follows that the upward slopes at these points, due to the upward bending moment, must be equal to the downward slope due to the load upon the beam.

FIG. 198.



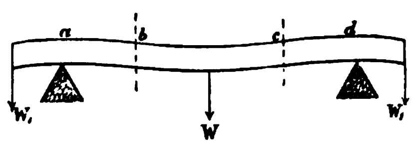


FIG. 199.

Case I.—A beam of uniform section, fixed at both ends and loaded in the centre (Fig. 198).

In the case of a beam supported at both ends, it has been shown that the slope at *a* and *d* is—

$$i_1 = -\frac{Wl^2}{16EI}$$

If M denote the upward bending moment, which is uniform from *a* to *d*, the slope at *a* and *d* produced by it may be proved to be—

$$i_2 = -\frac{Ml}{2EI}\ [1]$$

---

[1] $i = \frac{M}{EI}\int dx = \frac{Mx}{EI} + C$

$\therefore i = 0 \text{ when } x = \frac{l}{2}$

$\therefore C = -\frac{Ml}{2EI}$

$\therefore i = \frac{Mx}{EI} - \frac{Ml}{2EI} = \frac{M}{EI}\left(x - \frac{l}{2}\right)$

The slope at *a* and *d* is found by putting $x = 0$, or $x = l$;

$\therefore i_2 = -\frac{Ml}{2EI}$

and since $i_1 + i_2 = 0$—

$$-\frac{Wl^2}{16EI} - \frac{Ml}{2EI} = 0$$

$$\therefore M = -\frac{Wl}{8}$$

which is the bending moment at either a or d.

If M_1 denote the downward bending moment at any point distant x from a or d, the total bending moment M_2 at x will be—

$$M + M_1 = M_2$$

$$\therefore M_2 = \frac{Wx}{2} - \frac{Wl}{8}$$

When $x = \frac{l}{2}$, we obtain the central bending moment, which we will denote by M_3.

$$M_3 = \frac{Wl}{4} - \frac{Wl}{8} = \frac{Wl}{8}$$

Hence $M = -M_3$, and the bending moments may be represented graphically as in Fig. 200.

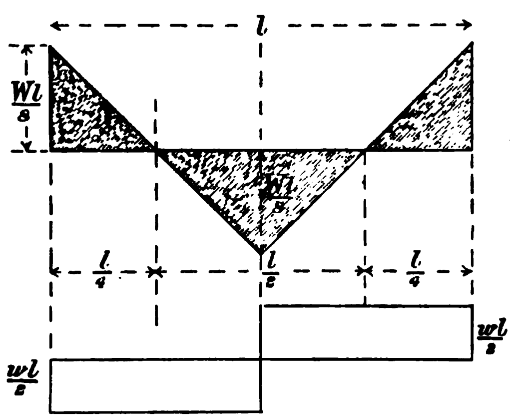


FIG. 200.

FIG. 201.

To find the points of contra-flexure, we observe that the bending moments must be zero at these points; hence $M_2 = 0$,

$$\therefore \frac{Wx}{2} - \frac{Wl}{8} = 0, \text{ and } x = \frac{l}{4}$$

This is also clear from the above diagrams, Figs. 200 and 201.

Slope and Deflection under Given Load.

$$i = \int \frac{M}{EI} dx = \frac{1}{EI} \int \left(\frac{Wx}{2} - \frac{Wl}{8} \right) dx = \frac{Wx^2}{4EI} - \frac{Wlx}{8EI} + C$$

When $x = 0$, $i = 0$. $\therefore c = 0$

$$\therefore i = \frac{dv}{dx} = \frac{W}{8EI}(2x^2 - lx)$$

$$v = \frac{W}{8EI}\left(\frac{2x^3}{3} - \frac{lx^2}{2} \right) + C$$

and $v = 0$ when $x = 0$. $\therefore C = 0$

$$\therefore v = \frac{W}{8EI}\left(\frac{2x^3}{3} - \frac{lx^2}{2} \right)$$

The maximum slope occurs at the points of contra-flexure, where $x = \frac{l}{4}$; the maximum deflection when $x = \frac{l}{2}$;

$$\therefore i_0 = -\frac{Wl^2}{64EI}$$

$$v_0 = -\frac{Wl^3}{192EI}$$

Let f denote the maximum intensity of working stress, and W the central breaking load, then—

$$\frac{Wl}{8} = \frac{fI}{y} \qquad \therefore W = \frac{8fI}{ly}$$

$$\therefore i_0 = -\frac{fl}{8Ey};\ v_0 = \frac{fl^2}{24Ey}$$

Case II.—Beam of uniform section, fixed at the ends and loaded with a uniformly distributed load.

Proceeding in the same manner as before, we obtain the downward slope at the ends, considering the beam as supported—

$$i_1 = -\frac{Wl^2}{24EI}$$

The upward slope at the ends due to the uniform bending moment M is—

$$i_2 = -\frac{Ml}{2EI}$$

Therefore, since the slope at the ends must be zero—

$$i_1 + i_2 = 0$$

$$\therefore -\frac{Wl^2}{24EI} - \frac{Ml}{2EI} = 0; \text{ and } M = -\frac{Wl}{12}$$

The bending moment at the supports is therefore $-\frac{Wl}{12}$; the downward bending moment at any point distant x from the origin is—

$$M_1 = \frac{W}{2l}(lx - x^2)$$

the total bending moment will therefore be—

$$M_2 = M_1 + M = \frac{W}{2l}(lx - x^2) - \frac{Wl}{12}$$

This is a maximum when $x = 0$, hence the greatest value of the bending moment is $\frac{Wl}{12}$.

The bending moment at the centre is found by putting $x = \frac{l}{2}$.

$$M_2 = \frac{W}{2l}\left(\frac{l^2}{2} - \frac{l^2}{4}\right) - \frac{Wl}{12} = \frac{Wl}{24}$$

The bending moments may be represented graphically as in Fig. 202.

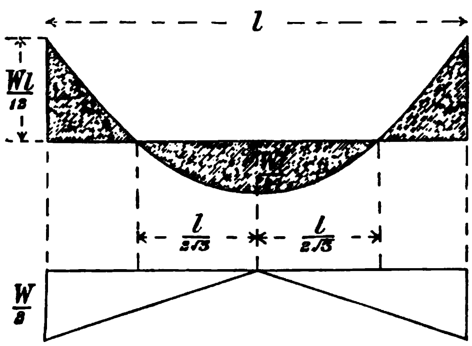


Fig. 202.

Fig. 203.

To find the points of contra-flexure, we have—

$$M_2 = 0 = \frac{W}{2l}(lx - x^2) - \frac{Wl}{12}$$

$$\therefore x = \frac{l}{2} \pm \frac{l}{\sqrt{12}}$$

which may be written—

$$x = \frac{l}{2} \pm \frac{l}{2\sqrt{3}}$$

which is represented in Fig. 202.

The diagram of shearing stress is shown in Fig. 203.

Slope and Deflection.

$$i = \int \frac{M}{EI} dx = \frac{W}{12lEI}(3x^2 l - 2x^3 - l^2 x)$$

the constant vanishing when $i = 0$ and $x = 0$.

$$v = \frac{W}{12lEI}\left\{lx^3 - \frac{x^4}{2} - \frac{l^2x^2}{2}\right\}$$

the constant vanishing when $v = 0$ and $x = 0$.

Hence for the maximum slope and deflection we have, for slope putting $x = \frac{l}{2}\left(1 \pm \frac{1}{\sqrt{3}}\right)$, and for deflection putting $x = \frac{l}{2}$—

$$i_0 = -\frac{Wl^2}{72\sqrt{3}EI}$$

$$\text{and } v_0 = -\frac{Wl^3}{384EI}$$

The slope and deflection under the maximum intensity of working stress, denoted by f, may be found thus—

$$\frac{Wl}{12} = \frac{fI}{y} \quad \therefore W = \frac{12fI}{yl}$$

$$\therefore l_0 = -\frac{fl}{6\sqrt{3}Ey}$$

$$v_0 = -\frac{fl^2}{32EI}$$

Case III.—Beam of uniform section supported at one end and fixed at the other, and loaded in the centre.

This case may be investigated in a similar manner to Cases I. and II. The results are recorded in Figs. 204, 205, and 206.

The deflection at the centre is—

$$v = \frac{0 \cdot 0091Wl^3}{EI}$$

The maximum deflection is—

$$v_0 = \frac{0\cdot0093Wl^3}{EI}$$

Case IV.—Beam of uniform section supported at one end and fixed at the other, and loaded uniformly with a distributed load. The results of this case are recorded in Figs. 207 and 208.

$$v_0 = \frac{0\cdot0054Wl^3}{EI}$$

Summary of Results.—For beams fixed at both ends and loaded at the centre the strength is twice as great, and the stiffness four times as great, as the same beam merely supported at the ends.

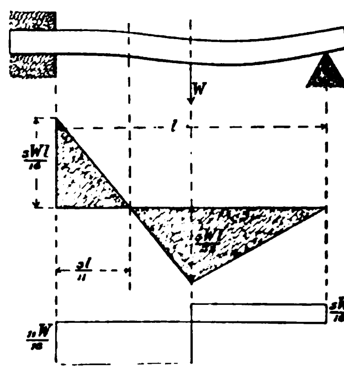


FIG. 204.

FIG. 205.

FIG. 206.

For beams fixed at both ends and loaded with a uniformly distributed load the strength is one and a half times as great, and the stiffness five times as great, as when merely supported at the ends.

The shearing stresses are not affected by fixing in either of the above cases.

For beams fixed at one end and supported at the other: For a central load the strength is increased one and a third times, and the stiffness 2·23 times, over that of a beam supported at both ends.

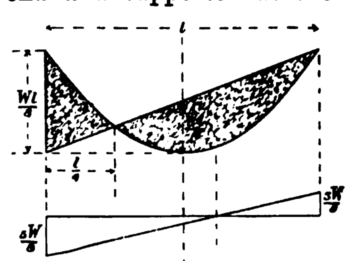


FIG. 207.

FIG. 208.

The shearing stress is increased at the fixed end and diminished at the supported end. For a distributed load the strength is not increased by fixing one end, but the stiffness is increased 2·39 times. The shearing stress is increased at the fixed and diminished at the supported end.

It is not correct to build in the ends of cast-iron beams in order to fix them, as the smaller flange is put in tension at the fixed points. Wrought-iron beams, whether built or rolled, may be fixed by supporting them on a bed-stone, and putting a similar stone on the top flange to discharge the weight of the wall on the ends of the beam.

Timber beams should not be built into walls in order to fix them, as dry rot is almost certain to occur from the want of proper ventilation.

The cross-girders of box web-girder bridges may be riveted to the main girders at the ends in such a manner as to fix them, in which case they should be designed as fixed beams.

The fixing tends to cant the main girders inwards, and the advantage of fixing is doubtful.

CHAPTER XI.

CONTINUOUS GIRDERS AND THEIR APPLICATION IN ROAD AND RAILWAY BRIDGES—BRESSUMMER BEAMS AND WAREHOUSE FLOORS.

A GIRDER supported at each end and at one or more intermediate piers is said to be continuous over these piers. In the case of a girder of 150 feet span, supported at each end and at two intermediate piers spaced 50 feet centres, the girder is said to be continuous over two piers in spans of 50 feet.

The complete investigation of the stresses in continuous girders, and the slopes and deflections, will not be attempted in this work, but the results of these investigations will be made use of sufficiently to illustrate the methods of designing continuous girders for various purposes.

Two cases will be considered—

1. When the load on any span is uniformly distributed over that span.

2. When the loads are concentrated at one or more points in a span.

A third case might occur with both distributed and concentrated loads, but this could obviously be treated by combining the results of each loading considered separately.

The assumptions made in the various examples considered in this chapter are—

1. That all the supports are on the same level or on the same uniform gradient.

2. That the girder is uniform in section throughout its length.

With regard to the second assumption, which is never realized in bridges, in which the material is disposed in accordance with the variations in the stresses, the errors in the stresses need not lead to errors in the design of the various members of any practical importance.

Let M_0, M_1, M_2, and M_3, Fig. 210, denote the bending moments at the points of support A, B, C, and D respectively.

Let l_1, l_2, and l_3 be the lengths of the spans, and w_1, w_2, and w_3 denote the loads per lineal foot.

Then, on the assumptions stated, it can be proved that—

$$M_0 l_1 + 2M_1(l_1 + l_2) + M_2 l_2 + \tfrac{1}{4}(w_2 l_2^3 + w_1 l_1^3) = 0$$

This equation expresses the relation between the bending moments at any three consecutive piers; it is known as the equation of three moments, and was first demonstrated by Clapeyron.

The moments M_1 and M_2 in Fig. 210 tend to bend the girders upwards; thus in Fig. 209, which shows a continuous girder of three spans, the deflection over the piers B and C is upwards, and the diagram of upward bending moments is represented by the figure A*bc*D, Fig. 210. The bending moments over the points A and D are zero.

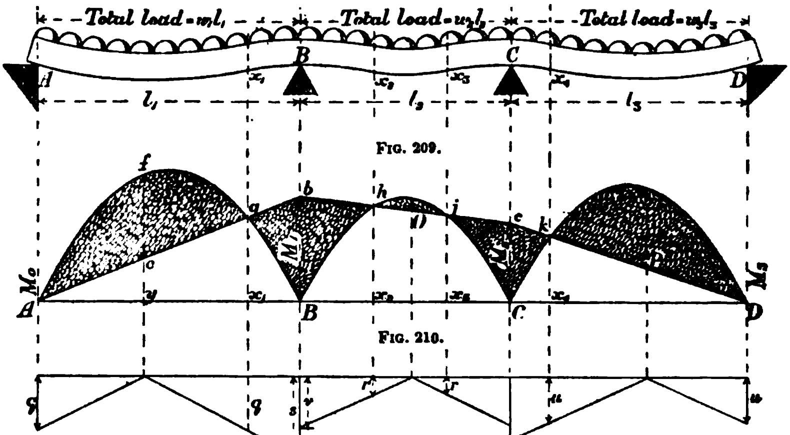


FIG. 209.

FIG. 210.

FIG. 211.

Beside the upward bending moments, which attain their maximum value for adjacent spans at the points B and C, there are downward bending moments between the supports, which act in precisely the same manner as in ordinary beams supported at AB, BC, and CD. The resultant bending moment at any point is the difference between the upward and downward bending moments at that point.

Let the curves described upon AB, BC, and CD denote the bending moments over these spans respectively considered as detached beams. In the case of uniform loading these curves are parabolas, and since the upward bending moments are denoted by the figure A*b6*D, the shaded areas represent the resultant bending moment; thus the bending moment at the point y is denoted by the ordinate ef.

At the points g, h, j, and k, corresponding with the points x_1, x_2, x_3 and x_4 on the girder, the bending moment is zero, or the upward bending moment is exactly balanced by the downward bending moment. The points x_1, x_2, x_3, and x_4 are termed points of inflection or contra-flexure. The portion of the span AB between A and x_1 is subjected to the same bending moments as an ordinary girder supported at A and x_1 and loaded with the same load per foot run as that on the span AB. The portions between x_2 and x_3 in the span BC, also between x_4 and D in the span CD, are similarly subjected to the same bending moments as in ordinary beams supported at points corresponding with the points of contra-flexure, x_2, x_3, and x_4, and loaded with the same load per foot run as that on the spans BC and CD respectively.

The portions x_1B, Bx_2, x_3C, and Cx_4 act as cantilevers, each loaded with a uniform load corresponding with that on the particular span in which the portion referred to occurs, in addition to a load at their extremities corresponding with the reactions at x_1, x_3, x_3, and x_4 considered as ordinary detached girders supported at Ax_1, x_2x_3, and x_4D respectively.

The shearing stresses may therefore be derived by considering the whole girder between A and D as made up of detached girders and cantilevers in the manner described, thus: The shearing stresses q, q, u, and u are each half the load on the spans Ax_1 and x_4C; the portions rr are the shearing stresses at x_2 and x_3, and are equal to half the load on the portion x_2x_3.

The shearing stresses on the girder are represented by the diagram, Fig. 211.

The pressures on the supports A and D are equal to the shearing stresses at these points, and the pressures at B and C to the sum of the shearing stresses from the adjacent spans; thus at B the pressure is $s + v$.

The method of calculating the stresses in continuous girders may be summarized as follows :—

1. Find the bending moments over the piers, and draw the diagram of upward bending moments A*bc*D, Fig. 210.

2. Draw the diagram of downward bending moments on the same datum line, AD, Fig. 210, and on the same side of it as the diagram of upward bending moments; then find the points of contra-flexure and the resultant bending moments in each span.

3. Draw the shearing-stress diagrams for each span, Fig. 211, and obtain the pressures on the piers.

When the bending moments over the piers are obtained, the equations of bending moments and shearing stresses for each span can be easily written down and the results tabulated.

Example.—A railway bridge for a single line is constructed with two main girders, each 480 feet long, continuous over two piers, forming three spans each 159 feet centres.

The effective depth measured between the centres of gravity of the top and bottom chords, or booms, is 12 feet.

The dead load on each main girder is 0·6 ton per foot run.

The equivalent uniformly distributed load is 0·7 ton per foot run.

The total load on a span is therefore 1·3 ton per foot run.

In the equation of three moments $l_1 = l_2 = l_3$ and $M_0 = M_4 = 0$.

Case I.—The first span loaded with the live and dead load ($w_1 = 1·3$), the remaining spans loaded with the dead load only ($w_2 = w_3 = 0·6$).

The substituting in the three-moment equation, we obtain—

$$\left.\begin{aligned} 2M_1(2l) + M_2l &= -\tfrac{1}{4}(w_2l_2^3 + w_1l_1^3) \\ M_1(l) + 2M_2(2l) &= -\tfrac{1}{4}(w_3l_3^3 + w_2l_2^3) \end{aligned}\right\}$$

$$\therefore 16M_1 + 4M_2 = -0·6 \times \overline{159}^2 - 1·3 \times \overline{159}^2 = -\overline{159}^2(1·9)$$

$$4M_1 + 16M_2 = -2 \times 0·6 \times \overline{159}^2 = -\overline{159}^2(1·2)$$

$$\therefore M_1 = -2696·64 \text{ foot-tons}$$

$$\text{and } M_2 = 1222 \text{ foot-tons}$$

The equation of bending moments for the first span is—

$$y = 0·65(159x - x^2) - 16·96x$$

To find the point of contra-flexure, let $y = 0$, then $x = 132·9$ feet; hence the point of contra-flexure is 26·1 feet from the first pier.

The equation of bending moments in the middle span is—

$$y = 0·3(159x - x^2) - 1222 - 9·27x$$

When $y = 0$, $x = 89{\cdot}55$ and $58{\cdot}65$ feet; hence the points of contra-flexure occur 89·55 feet from the first pier, and 58·65 feet from the second pier.

It will not be necessary to consider the third span, as, when the live load approaches from the right instead of the left, the stresses will be the same as in the first span already considered, which will exceed considerably the stresses due to the dead load only on this span.

Shearing Stresses.

First span at abutment $= \dfrac{132{\cdot}9 \times 1{\cdot}3}{2} = 86{\cdot}38$ tons.

,, ,, first pier $= 86{\cdot}38 + 26{\cdot}1 \times 1{\cdot}3 = 120{\cdot}31$ tons.

Second span at first pier $= 95{\cdot}85 \times 0{\cdot}6 = 57{\cdot}5$ tons.

,, ,, second pier $= 64 \times 0{\cdot}6 = 55{\cdot}38$ tons.

The diagram of bending moments and shearing stresses is shown in Figs. 212 and 213.

If the live load had extended from the middle of the first

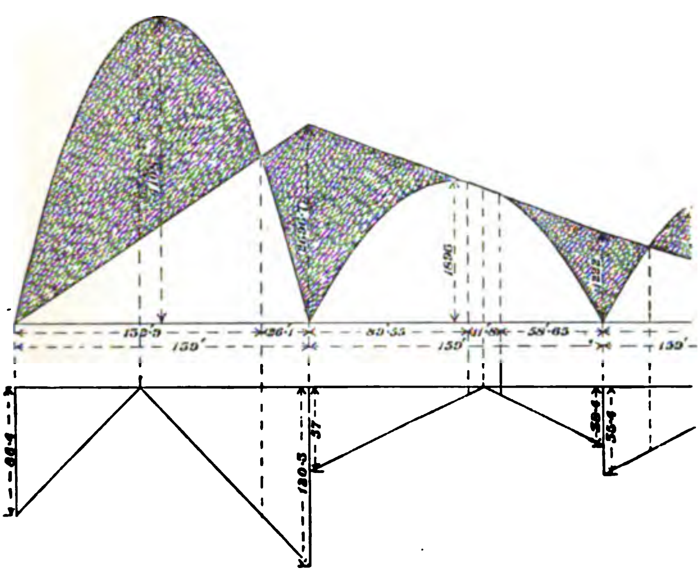


Fig. 212.

Fig. 213.

span to the middle of the second span, the moment over the first pier would have been about 1½ per cent. greater than that found in Case I., but partially distributed live loads will not be considered. If the train approaches from the right headed by two engines, the whole bridge may be covered with the live load; this may be taken as uniform throughout.

Case II.—Three spans loaded with 1·3 ton per foot run.

The three-moment equation becomes—

$$2M_1(2l) + M_2 l = -\tfrac{1}{4}(2w_1 l^3)$$
$$\therefore 8M_1 + 2M_2 = -w_1 l^2$$
$$\text{also } 8M_2 + 2M_1 = -w_1 l^2$$
$$\text{hence } M_1 = M_2 = -\frac{w_1 l^2}{10} = -\frac{1{\cdot}3 \times \overline{159}^2}{10} = -3286{\cdot}5 \text{ foot-tons}$$

The equation of bending moments for the first and third spans becomes—

$$y = 0{\cdot}65(159x - x^2) - 20{\cdot}67x$$

where $x =$ the distance from the left and right abutment.

$$\text{If } y = 0,\ x = 127{\cdot}2 \text{ feet}$$

The equation of bending moments in the middle span is—

$$y = -3286{\cdot}5 + 0{\cdot}65(159x - x^2)$$

from which we find, by making $y = 0$, $x = 115$ and 44 feet; hence the points of contra-flexure occur 44 feet from each pier.

Shearing Stresses.

$$\text{First and third spans over abutments} = \frac{127{\cdot}2 \times 1{\cdot}3}{2} = 82{\cdot}68 \text{ tons.}$$

$$\text{,, ,, over piers} = 95{\cdot}4 \times 1{\cdot}3 = 124{\cdot}02 \text{ ,,}$$

$$\text{Middle span over piers} = \frac{159 \times 1{\cdot}3}{2} = 103{\cdot}35 \text{ tons.}$$

Comparing Cases I. and II., we observe that the shearing stresses and bending moments over the piers is greater in Case II., while the bending moments between the left abutment and point of contra-flexure is greater in Case I.

Case III.—First and second spans loaded with 1·3 ton per foot run; third span loaded with 0·6 ton per foot run.

$$\text{Here } 8M_1 + 2M_2 = -w_1 l^2$$
$$16M_2 + 4M_1 = -(w_2 l^2 + w_3 l^2) \quad \text{where } w_1 = w_2$$
$$\therefore M_1 = -(7w_1 - w_3)\frac{l^2}{60} = -(9{\cdot}1 + 0{\cdot}6)\frac{25281}{60} = -3581{\cdot}3 \text{ foot-tons}$$
$$M_2 = -2106{\cdot}7 \text{ foot-tons}$$

Equation to first span is—

$$y = 0\cdot65(159x - x^2) - 22\cdot52x$$

therefore the point of contra-flexure is $x = 124\cdot35$.

Equation to second span is—

$$y = 0\cdot65(159x - x^2) - 2106\cdot7 - 9\cdot27x$$

therefore the points of contra-flexure occur at 42·65 feet from the first pier, and 28·35 feet from the second pier.

Case IV.—First and third spans loaded with 0·6 ton per foot run; middle span loaded with 1·3 ton per foot run.

$$\text{Here } \left.\begin{aligned} 16M_1 + 4M_2 &= -\ (w_1l^2 + w_2l^2) \\ 16M_2 + 4M_1 &= -\ (w_2l^2 + w_1l^2) \end{aligned}\right\}$$
$$\therefore M_1 = M_2 = -\ 2401\cdot7 \text{ foot-tons}$$

Only the middle span need be considered for this case, the equation of which is—

$$y = 0\cdot65(159x - x^2) - 2401\cdot7$$

therefore the points of contra-flexure occur at 28·27 feet from the piers.

The shearing stress over the piers for the middle span is the same as in Case II.

Case V.—First and third spans loaded with 1·3 ton per foot run; middle span loaded with 0·6 ton per foot run.

Here $M_1 = M_2$, as in Case IV., viz. 2401·7 foot-tons.

Equation of first and third spans is—

$$y = 0\cdot65(159x - x^2) - 15\cdot105x$$

therefore the points of contra-flexure occur at $x = 135\cdot76$ feet from either abutment.

The shearing stresses are less than in the cases already considered.

Comparing the foregoing cases, we see that the maximum bending moment for the first and third spans between the point of contra-flexure and the abutments occurs for the loading considered under Case V., which gives the longest effective span.

The maximum moment over the piers and in the cantilever portions occurs for the loading considered in Case III. The middle span is subjected to greatest bending moment about the centre in Case IV., and in the cantilever portions in Cases I., III., and V.

The maximum shearing stresses in the side spans occur in Cases III. and V., and in the middle span in Cases III. and IV.

The diagram of shearing stresses for a moving load will be semi-parabolas between the abutments and points of contra-flexure in the first and third spans, and straight lines tangential to the parabolas for the cantilever portions. In the central span the shearing stresses in the portion between the two points of contra-flexure will be represented by parabolas and the two cantilevers on each side by straight lines.

The diagram of maxima bending moments is shown in Fig.

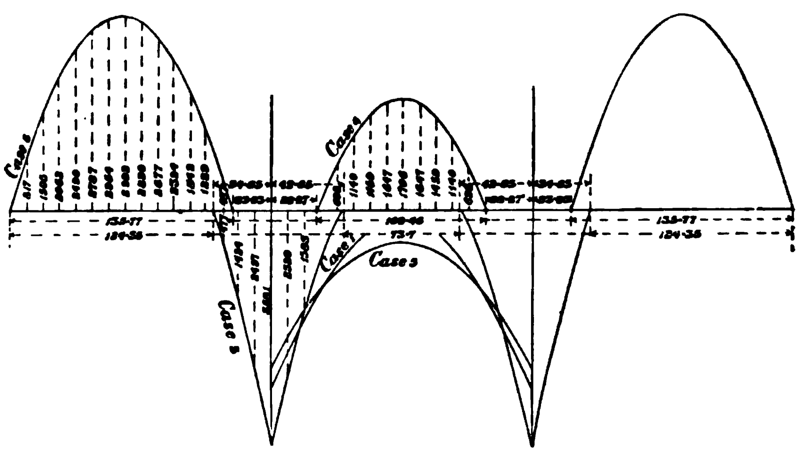


FIG. 214.

214. The moments of resistance of the flange plates provided to resist them may be plotted as in Fig. 173, Chapter VIII.

The diagram of maxima shearing stresses is shown in Fig. 215.

It will be observed that the points of contra-flexure are continually changing during the passage of the live load, hence the working stress must be taken much lower about the region of the points of contra-flexure, and the diagonal members of the web must be counterbraced for reversals of stress, or designed for compression as well as tension.

Concentrated Loads.—The equation of three moments for concentrated loads, for the case when all the supports are on the same level or on the same uniform gradient, and the section of the girder is uniform throughout its length, may be expressed as follows :—

$$\frac{M_2}{3}(l_1 + l_2) + \frac{M_1 l_1}{6} + \frac{M_3 l_2}{6} + \Sigma\frac{W_1'x'}{6l_2}(l_2^2 - x'^2) + \Sigma\frac{Wx}{6l_1}(l_1^2 - x^2) = 0$$

where—

$$\Sigma\frac{Wx}{6l_1}(l_1^2 - x^2) = \frac{W_1 x_1}{6l_1}(l_1^2 - x_1^2) + \frac{W_2 x_2}{6l_1}(l_1^2 - x_2^2) + \ . \ . \ .$$

$$\Sigma\frac{W_1'x'}{6l_2}(l_2^2 - x_1^2) = \frac{W_1'x_1'}{6l_2}(l_2^2 - x'^2_1) + \frac{W_2'x_2'}{6l_2}(l_2^2 - x_2'^2) + \ . \ . \ .$$

The use of this equation will be illustrated in the following example.

A bressummer beam of two spans, continuous over one sup-

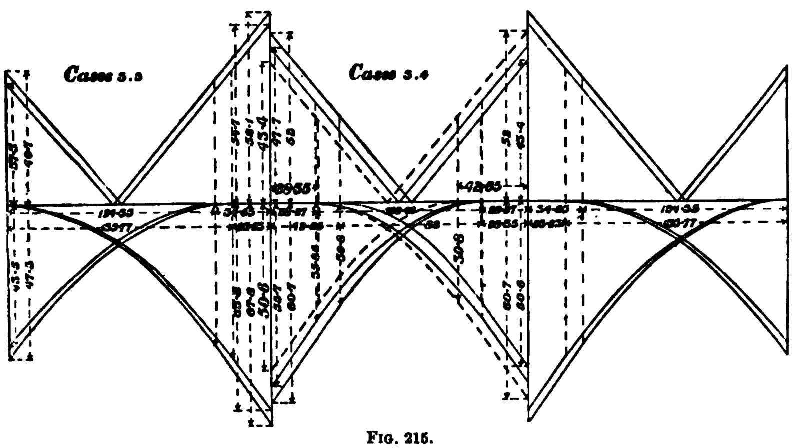


FIG. 215.

port, carries a shop-front and portions of the floors above. The whole of the load is applied to the beam through masonry piers built upon the upper flange. The magnitudes and points of

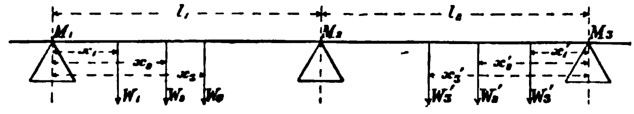


FIG. 216.

application of these loads are illustrated in Figs. 216 and 217. In this case $M_1 = M_3 = 0$, and the equation becomes—

$$\frac{23\cdot1}{3}M_2 + \frac{67\times10}{6\times12\cdot5}(12\cdot5^2 - 10^2) + \frac{30}{6\times12\cdot5}(12\cdot5^2 - 1^2)$$
$$+ \frac{11\cdot5\times9}{6\times10\cdot6}(10\cdot6^2 - 9^2) + \frac{11\cdot5\times45}{6\times10\cdot6}(10\cdot6^2 - 4\cdot5^2)$$
$$+ \frac{60\times2\cdot2}{6\times10\cdot6}(10\cdot6^2 - 2\cdot2^2) = 0$$

$$\therefore 7\cdot7M_2 + 502\cdot5 + 62\cdot1 + 51\cdot03 + 74\cdot94 + 223\cdot1 = 0$$
$$\therefore 7\cdot7M_2 = -913\cdot67$$
$$\therefore M_2 = -118\cdot6 \text{ foot-tons}$$

The diagram of bending moments A*bcd*B and B*fg*C, for the spans AB and BC considered as detached girders, should be plotted to a sufficiently large scale, as well as the diagram of upward bending moments APC, and their intersections noted, from which the points of contra-flexure x_1 and x_2 are obtained, 3·7 feet from B in the first span, and 1·8 foot from B in the second span.

The shearing-stress diagram is drawn by considering Ax, and Cx_2 as detached spans. The cantilever x_1B is loaded at its

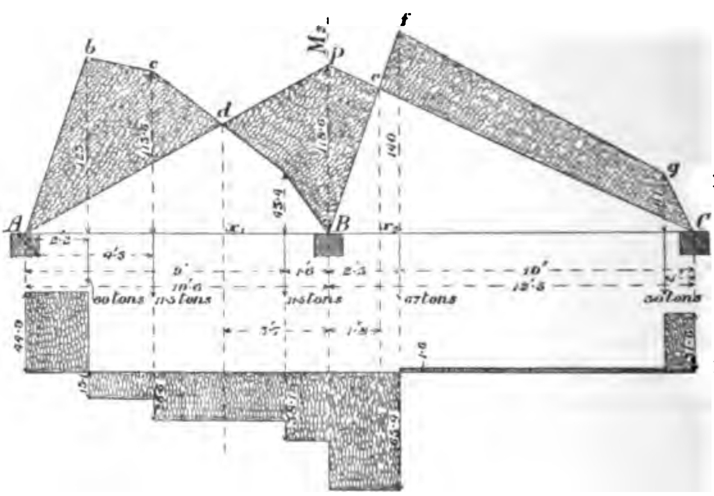


FIG. 217.

FIG. 218.

extremity with the shearing stress at x; it is also loaded with 11·5 tons concentrated at a point 1·6 foot from B. The cantilever Bx_2 is loaded at its extremity with 65·4 tons.

The diagrams of bending moments and shearing stresses are illustrated in Figs. 217 and 218.

The load upon the central support is—

$$65{\cdot}4 + 38{\cdot}1 = 103{\cdot}5 \text{ tons}$$

The maximum bending moment occurs at B, viz. 118·6 foot-tons.

The beam is constructed of two Dorman rolled steel girders 12 inches deep, with flanges 6 inches × 1 inch, and webs $\frac{3}{4}$ inch thick. The flanges are connected together on the top by means of a plate running the whole length. The web is stiffened by means of channel steel stiffeners. The working stress is 9 tons per square inch, the tensile strength of the steel being 32 tons per square inch.

In consequence of the change in the position of the points of contra-flexure during the passage of a rolling load, the shearing stresses cannot be accurately determined without considerable labour.

In the first example, when the uniform live load of 0·7 ton per foot run advances from one abutment and gradually covers the three spans, the point of contra-flexure in the first span moves through a distance of 135·8 – 124·3 = 11·5 feet.

If we draw the diagrams of shearing stresses for the extreme positions of the point of contra-flexure, we see that the nearer the point is to the pier the smaller the stress on the pier and the greater the stress on the abutment, so that if we use the point nearer the pier for determining the shearing stresses on the half of the effective span nearest the abutment, and the point farther from the pier for determining the shearing stresses for the remainder of the effective span and cantilever portions, we shall obtain stresses which do not differ much from the true maxima. We can deal with the central span in a similar manner. If the girder is of the lattice type, we first find the extreme positions of the points of contra-flexure, then apply the methods explained in Chapter VIII.

Concentrated rolling loads may be dealt with by calculating the bending moments and shearing stresses for various positions of the wheel loads in the manner illustrated in the foregoing example of the bressummer beam. The positions of the wheel loads which produce maxima stresses could, of course, be found if the points of contra-flexure are known; but this is not the most convenient method, excepting in the case of a traction engine with only two axles, as shown in Figs. 323 and 324.

In this case the points of contra-flexure for the dead load will not be moved very much with the traction engine in any position, so that, if we make two calculations for the extreme positions of the point of contra-flexure, we generally cover the maxima stresses in this case also.

Economy of the Continuous Girder.—In a three-span bridge of the best proportions, *i.e.* in which the effective spans are about equal in length under the loads which produce maxima stresses, or in which the side spans are about four-fifths of the central span, the economy of material due to continuity is about 50 per cent. for the dead load, and 16 per cent. for the live load, over three independent spans; hence the advantage of continuous girder road bridges where the dead load is considerable. This advantage will vanish if the piers settle unequally.

Mechanical fixing of the Points of Contra-flexure and Cantilever Bridges.—We have seen that the points of contra-flexure are continually changing in position during the passage of the live load, and that in a bridge of three spans there will generally be four points of contra-flexure, viz. two in the central span and one in each of the side spans. The points of contra-flexure may be fixed mechanically by uniting the girder at the proposed points by some form of hinged connection. The two points may be fixed either in the central or side spans, but not in both, or the stability of the bridge will be destroyed. The fixing of the points of contra-flexure for any span considerably simplifies the calculations for that span.

In the Kentucky bridge, United States, America, which consists of three equal spans, each 375 feet long, there is a hinge in each of the side spans, situated at a distance of 75 feet from the pier, thus reducing the effective length of the side spans to 300 feet. If the two hinges are located in the centre span, we have a form of the cantilever bridge in which the central span consists of two cantilevers projecting from the piers, with an independent girder resting upon their extremities and completing the span. The side spans in this case should be sufficiently long to balance the central span under all conditions of loading, otherwise the extremities must be anchored down.

The Ploughkeepsie bridge over the Hudson River, United States, America, consists of cantilever and rigid spans of almost equal length arranged alternately, so that there is no necessity for anchorages, as the rigid span balances the cantilevers on each side of it.

In the more usual form of cantilever bridge the cantilevers projecting from each side are generally of equal length, so that they balance each other for the dead load; the independent girder rests on the extremities of the cantilevers as before, while the extremities of the side cantilevers must be anchored down to balance the central portion under all conditions of loading.

The ratio of the length of the independent girder to that of the cantilevers varies in different bridges. Examples of this type occur in the bridges over the St. John's, Niagara, and the Frazer rivers, and also the Red Rock Cantilever Bridge, United States, America.

The celebrated bridge over the Forth consists of two cantilever spans, each 1700 feet long; the cantilevers project 680 feet on each side of the piers, with a maximum depth of 343 feet; the independent central girders are each 340 feet span. The Forth Bridge has been fully illustrated in the various engineering journals, and its detailed description will not be attempted here. It is in every respect the greatest constructional achievement in the world.

The cantilever bridge is one of the most economical types for long spans. The calculations may be made by first considering the independent girder, the reactions of which upon the extremities of the cantilevers must be combined with the panel loads of the cantilever.

It will be generally most convenient to consider each panel, proceeding from the extremity of the cantilevers step by step to the pier. Owing to the varying depth, the panel loads due to the weight of the panels are not equal.

Cantilever bridges possess the advantage of being easily erected by building outwards from the piers, and are especially applicable for long spans over deep gorges or rivers, where ordinary scaffolding would be too expensive or subject to great risks. Cantilever bridges should not generally be less than 500 feet span.

CHAPTER XII.

STRENGTH OF COLUMNS.

Short Columns.—Fig. 219 represents a rectangular prism subjected to compressive stress between the flat plates of a testing-machine. If the compressive force is applied along the axis of the prism, and if P denote the total load, A the area of the prism, and p the intensity of stress, then—

$$\frac{P}{A} = p_1$$

The stress p_1 is uniformly distributed over the area of the section.

If the load is applied at some point other than the centre of gravity of the section, such as C, Fig. 219, we may conceive two opposite forces applied along the axis AB equal to p; then we have a uniform compressive stress over the area of the column—

$$p_1 = \frac{P}{A}$$

also a couple, the moment of which is—

$$M = Py$$

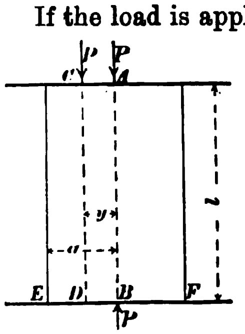


FIG. 219.

The maximum compressive stress p_2 due to the couple Py will occur at E, and will be—

$$p_2 = \frac{Pya}{I}$$

where $a =$ BE, and I is the moment of inertia of the cross section of the prism.

If r denote the radius of gyration, then—

$$p_2 = \frac{Pya}{Ar^2}$$

Hence the maximum intensity of compressive stress at E is—

$$p = \frac{P}{A}\left(1 + \frac{ya}{r^2}\right)$$

Long Columns.—In a long column the ratio of length to the least radius of gyration may be sufficient to cause the column to fail by lateral flexure rather than by direct crushing, in which case we have a compound stress somewhat similar to that existing in the short columns with eccentric loading. The tendency to lateral flexure in a long column may be increased by the eccentricity of the loading caused by the direction of the load not coinciding with the true axis of the column. The strength of long columns has been investigated mathematically by Euler and Rankine, Professor R. Smith, Mr. Claxton Fidler, and others, and experimentally by Hodgkinson, Christie, and at the Watertown Arsenal, United States, America.

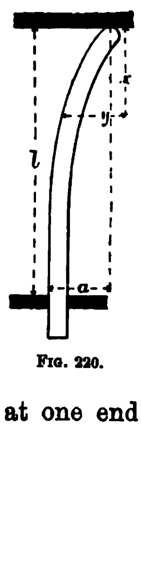


FIG. 220.

Euler's Formulæ for Columns.—Euler's theory of the strength of columns may be stated as follows:—

Fig. 220 represents a column fixed in direction at one end only, which bends as shown in the figure.

If the column fails by direct crushing, then—

$$P_1 = fA$$

If by flexure, then it can be proved that—

$$P_2 = \frac{\pi^2}{4l^2}EI \quad (1)$$

[1]

[1] Let ρ = radius of curvature, M = bending moment at xy, then—

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{Py}{EI}; \quad \frac{1}{\rho} = -\frac{d^2y}{dx^2}$$

$$\therefore -\frac{d^2y}{dx^2} = \frac{P}{EI}y$$

add $\int \frac{dy}{dx}dx$ to each side—

$$-\int \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} \cdot dx = \frac{P}{EI}\int y \cdot \frac{dy}{dx} \cdot dx$$

Then, according to Euler, the actual breaking strength will be the smaller of the two values given for P_1 and P_2 in the foregoing equations.

If the column is rounded or hinged at each end (Fig. 221), then—

$$P_2 = \frac{\pi^2}{l^2}EI \qquad (2)$$

(here length $= 2l$).

If the column is fixed at each end (Figs. 222 and 223), either by building in or by means of flat ends—

$$P_2 = \frac{4\pi^2}{l^2}EI \qquad (3)$$

(here length $= \frac{l}{4}$ in (1)).

These formulæ are strictly true only when applied to an ideal column consisting of uniformly elastic material in which the axis of the straight column coincides with the line of pressure of the load. Professor Smith and Mr. Claxton Fidler[1] have, however, pointed out that these conditions cannot be realized in

$$\therefore \left(\frac{dy}{dx}\right)^2 = -\frac{P}{EI}y^2 + C$$

and since for $y = a$, $\frac{dy}{dx} = 0$; $\therefore C = \frac{P}{EI}a^2$

$$\therefore \left(\frac{dy}{dx}\right)^2 = \frac{P}{EI}(a^2 - y^2)$$

$$\frac{dy}{dx} = \sqrt{\frac{P}{EI}}\sqrt{a^2 - y^2}$$

$$\therefore \int \frac{dy}{\sqrt{a^2 - y^2}} = \int \sqrt{\frac{P}{EI}} \cdot dx$$

$$\therefore \sin^{-1}\frac{y}{a} = \sqrt{\frac{P}{EI}}x + C$$

and since when $x = 0$, $y = 0$; $\therefore C = 0$

$$\text{and } \sin^{-1}\left(\frac{y}{a}\right) = \sqrt{\frac{P}{EI}} \cdot x$$

When $y = a$, we have $x = l$.

$$\text{Hence } \frac{\pi}{2} = \sqrt{\frac{P}{EI}} \cdot l$$

$$\text{and } P = \frac{\pi^2}{4l^2} \cdot EI$$

[1] Professor R. H. Smith in *Engineering*, October 14, 1887; Mr. Claxton Fidler, *Min. Proc. Inst. C.E.*, vol. lxxxvi.

practice, as there is nearly always some eccentricity due to imperfections of workmanship, want of uniformity in the modulus of elasticity throughout the column, journal friction, occurring

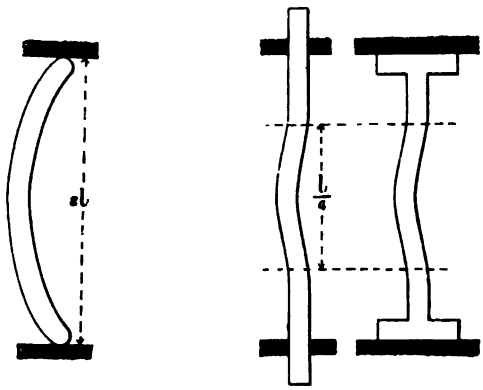


FIG. 221. FIG. 222. FIG. 223.

for instance in connecting and eccentric rods. The strength of columns is therefore more a matter of probability than of exact theory.

In the case of a column hinged at each end, $P = \frac{\pi^2}{l^2}EI$, and, dividing by the area, we have—

$$\rho = \frac{P}{\text{area}} = \pi^2 E \frac{r^2}{l^2}$$

where r denotes the radius of gyration. If we plot the curve represented by this equation, using $\frac{l}{r}$ for abscissæ, and ρ for ordinates, we obtain BHC (Fig. 224), which represents the higher limit of strength.

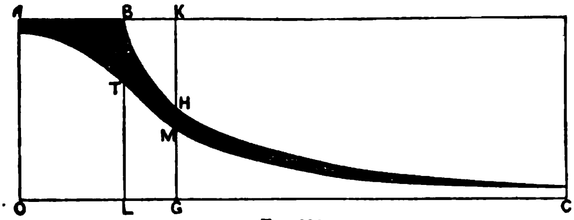


FIG. 224.

The straight line ABK represents the constant value of f, the ultimate crushing strength. GH denotes the stress due to the direct load; HK, the stress due to the ultimate deflection or bending moment.

In the case of a girder-shaped strut, such as would be used as a compression member in a bridge, Mr. Claxton Fidler points out that if the ends are rounded, the eccentricity due to the variation in the modulus may be treated in the following manner:—

Let ε_1 = the specific compression in the inner flange $= \frac{1}{E_1}$

ε_2 = " " " " outer " $= \frac{1}{E_2}$

ε = average compression $= \frac{\varepsilon_1 + \varepsilon_2}{2}$

δ = the central deflection due to load P.

y = the distance from the neutral axis to the extreme fibre.

θ = the eccentricity.

r = the radius of gyration = half the depth in a girder-shaped strut.

$p = \frac{P}{A}$ = direct load in pounds per square inch.

$$\text{Then } \theta = r\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

and it can be proved that—

$$\delta = \frac{\pi r}{2} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot \frac{p}{\rho - p} = \frac{\theta\pi}{2} \cdot \frac{p}{\rho - p}$$

$$\text{Since } M = P\delta = \frac{f_1 I}{y} = \frac{f_1 A r^2}{y}$$

$$\pm f_1 = \frac{P\delta y}{Ar^2} = \frac{p\delta y}{r^2}$$

$$\therefore \pm f_1 = \frac{\pi}{2} \cdot \frac{y}{r} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \cdot \frac{p^2}{\rho - p}$$

$$\text{If } \phi = \frac{y}{r} \cdot \frac{\pi}{2} \cdot \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$\pm f_1 = \frac{\phi p^2}{\rho - p}$$

$$\text{and since } f = p + f_1 = p\left(1 + \frac{\phi p}{\rho - p}\right)$$

$$\text{hence } p = \frac{f + \rho - \sqrt{(f + \rho)^2 - 4f\rho(1 - \phi)}}{2(1 - \phi)}$$

This equation, when plotted as before, using $\frac{l}{r}$ for abscissæ, gives a wavy line, ATMC, which Mr. Fidler calls the curve of the lower limit of the strength of the column, and he states that "between the shaded area included between the two curves, the results of individual experiments," plotted in a similar manner, "may be expected to range themselves at haphazard. The upper limit will be the line of the ideal column ABC, and it remains to determine for each material the position of the lower limit ATC, which must evidently be regarded as the greatest reliable strength of the column."

The diagram, Fig. 224, shows that the strength of a long column depends more upon the modulus of elasticity than upon the crushing resistance of the material, thus: GM denotes the stress due to the direct load, and MK the stress due to bending moment. The diagram also shows that a very long or very short column will not differ much from the ideal column, but that for intermediate values of $\frac{l}{r}$ the effect of eccentricity, or inequality in the modulus of elasticity, becomes most marked; the maximum effect occurs at B, where $\rho = f$, and $p = f\frac{1 - \sqrt{\phi}}{1 - \phi}$.

In Fig. 225, GM denotes the stress at the centre of the column due to direct load, and MK that due to bending moment; hence the graphical representation of the stress in the column is as shown in Fig. 225, and if the column is braced, the stresses in the lattice bars may be deduced from the flange stresses, as shown by the curve AMBK.

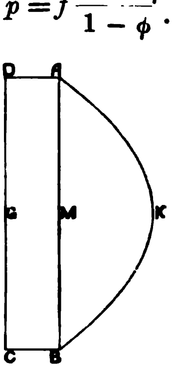


FIG. 225.

Mr. Fidler has calculated the strength of columns for various values of the ratio $\frac{l}{r}$, by means of the formulæ referred to, and he states that the results agree closely with those obtained from experiments made by Mr. Christie and others.

These results are tabulated.

Table XXXVII.

Strength of Columns with Round Ends in Pounds per Square Inch of Sectional Area, according to Mr. Claxton Fidler.

Ratio $\frac{l}{r}$	Cast iron.	Wrought iron.	Mild steel.	Hard steel.
20	72,300	35,200	46,700	67,200
40	50,800	32,600	42,700	58,600
60	30,000	28,400	36,000	45,500
80	17,600	23,200	28,300	33,000
100	11,700	18,200	21,500	23,700
120	8,300	14,100	16,400	17,500
140	6,300	11,100	12,700	13,300
160	4,900	8,800	10,100	10,400
180	3,900	7,200	8,160	8,360
200	3,200	5,900	6,710	6,850
220	2,680	4,970	5,620	5,710
240	2,270	4,210	4,750	4,820
260	1,950	3,640	4,080	4,130
280	1,690	3,140	3,550	3,570
300	1,480	2,750	3,100	3,130
320	1,300	2,430	2,730	2,740
340	1,160	2,160	2,430	2,440
360	1,040	1,940	2,190	2,190
380	940	1,730	1,960	1,960
400	850	1,570	1,760	1,760

Table XXXVIII.

Strength of Columns with Fixed Ends in Pounds per Square Inch of Sectional Area, according to Mr. Claxton Fidler.

Ratio $\frac{l}{r}$	Cast iron.	Wrought iron.	Mild steel.	Hard steel.
20	77,600	35,800	47,400	68,700
40	67,800	34,900	45,700	65,800
60	54,700	33,400	43,300	60,500
80	42,000	31,100	39,900	53,600
100	30,000	28,400	36,000	45,500
120	21,200	25,300	31,000	37,400
140	16,000	22,200	26,500	30,500
160	12,600	19,200	22,500	25,000
180	10,200	16,500	19,100	20,900
200	8,300	14,100	16,400	17,500
220	6,900	12,100	13,900	14,900
240	5,700	10,500	12,000	12,600
260	5,000	9,300	10,400	11,000
280	4,400	8,200	9,100	9,500
300	3,900	7,200	8,200	8,400
320	3,400	6,300	7,200	7,300
340	3,000	5,600	6,300	6,500
360	2,700	5,100	5,500	5,700
380	2,470	4,600	5,100	5,200
400	2,270	4,210	4,750	4,800

Hodgkinson's Formulæ for Columns.—The principal practical use of Euler's formulæ was to furnish a general form of expression for the breaking load to the experiments of Eaton Hodgkinson.

Applying Euler's formula to solid cylindrical columns with flat end, and substituting for $I = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$, we have—

$$P_1 = \left(\frac{2\pi}{l}\right)^2 EI = \tfrac{1}{16} E\pi^3 \frac{d^4}{l^2}$$

But Hodgkinson's experiments gave—

$$P_1 = (\text{constant}) \times \frac{d^{3 \cdot 55}}{l^2} \text{ for wrought iron}$$

$$P_1 = (\text{constant}) \times \frac{d^{3 \cdot 55}}{l^{1 \cdot 7}} \text{ for cast iron}$$

For a square column, in which the length of side is denoted by b, Euler's formula would give—

$$P_1 = \frac{\pi^2}{3} E \frac{b^4}{l^2}$$

and Hodgkinson's experiments on timber gave—

$$P_1 = (\text{constant}) \times \frac{b^4}{l^2}$$

Inserting the constants determined by Hodgkinson, his formulæ become, when the length exceeds 30 times the diameter—

$P_1 = \frac{44 d^{3 \cdot 55}}{l^{1 \cdot 7}}$ for cast-iron columns with flat ends, where l is in feet and d in inches, and P_1 the breaking load in tons

$P_1 = \frac{134 d^{3 \cdot 55}}{l^2}$ for wrought-iron columns, ditto, ditto

$P_1 = \frac{10 \cdot 95 b^4}{l^2}$ for dry oak, ditto, ditto

$P_1 = \frac{7 \cdot 81 b^4}{l^2}$ for dry fir, ditto, ditto

For hollow columns of cast iron we have, where D and d denote the external and internal diameters respectively in inches—

$$P_1 = 44\frac{D^{3.55} - d^{3.55}}{l^{1.7}}$$

Hodgkinson found that the strength of columns with flat ends, one end flat and the other rounded, both ends rounded, are as 3 : 2 : 1, approximately.

Rankine's Formula.—This formula is based on the assumption that the load acts along the axis of the column, and that there is no eccentricity. It may be investigated in the following manner:—[1]

Let θ = the eccentricity.
- P = the load.
- δ = the central deflection.
- R = the radius of curvature.
- A = the area of the cross-section.
- r = the radius of gyration.
- f^1 = the intensity of stress due to bending within the elastic limit of the material.
- f = the intensity of stress at rupture.
- y = the distance from the neutral axis to the extreme fibre.
- l = the length of the column.

It can be proved that, in a column fixed at one end and free at the other—

$$R = \frac{l^2}{2\delta} \text{ approximately}$$

but—

$$\frac{1}{R} = \frac{M}{EI} = \frac{P(\delta + \theta)}{EI}$$

$$\therefore \frac{2\delta}{l^2} = \frac{P(\delta + \theta)}{EI}$$

$$2\delta EI = P(\delta + \theta)l^2$$

$$\therefore \delta = \frac{P\theta l^2}{2EI - Pl^2}, \text{ or } \delta + \theta = \frac{2EI\theta}{2EI - Pl^2}$$

but since $M = P(\delta + \theta) = \frac{f_1 I}{y}$

$$f_1 = \frac{P(\delta + \theta)y}{I} = \frac{2PE\theta y}{2EI - Pl^2}$$

[1] Dubois.

But $f = f_1 + \frac{P}{A}$ $\qquad \therefore f_1 = f - \frac{P}{A} = \frac{2PE\theta y}{2EI - Pl^2}$

$$\text{or } fA = P\left(1 + \frac{2E\theta yA}{2EI - Pl^2}\right)$$

or $P(2EI - Pl^2 + 2E\theta yA) = fA(2EI - Pl^2)$

Since Pl^2 is small compared with $2EI + 2AE\theta y$, we may neglect it.

$$\therefore \frac{P}{A} = \frac{2fEI}{2E(I + A\theta y) + fAl^2} = \frac{f}{1 + \frac{\theta y}{r^2} + \frac{fl^2}{2Er^2}}$$

If $\theta = 0$—

$$\frac{P}{A} = \frac{f}{1 + \frac{f}{2E} \cdot \frac{l^2}{r^2}} = \frac{f}{1 + c\frac{l^2}{r^2}}$$

C is a constant depending on the material and the conditions of fixing, or otherwise, at the ends. This is Rankine's formula, and it is clear that it is only true when there is no eccentricity, and when the stress produced is within the elastic limit of the material. In order not to exceed the elastic limit, the intensity of buckling stress, $\frac{P}{A}$, should be divided by a suitable factor of safety. If the factor of safety is expressed thus, $\frac{1}{4 + \frac{l}{20d}}$ which is convenient for variable loads, where d is the least dimension of the cross-section, then, if we denote the working stress by W, we have—

$$W = \frac{1}{4 + \frac{l}{20d}}\left(\frac{f}{1 + \frac{cl^2}{r^2}}\right)$$

The usual factor of safety for dead loads is four.

Gordon's Formula.—This formula is of the same form as Rankine's, but, instead of using the least radius of gyration, r, the least dimension of the cross-section is substituted, and the value of c modified accordingly. This will be true for circular and rectangular sections, since r is a function of d, but for all other sections r is not a simple function of d, hence Rankine's formula is more general than Gordon's.

In Rankine's formula f is taken as 36,000 for wrought iron, and $C = \frac{1}{36000}$ for flat ends, $= \frac{1}{18000}$ for round ends, $= \frac{1}{24000}$ for one end flat and the other end round.

In hollow cylindrical struts, if d is substituted for r—

$$\text{for cast iron } f = 72{,}000,\ c = \tfrac{1}{800};\ \tfrac{1}{400};\ \tfrac{3}{1600}$$
$$\text{for wrought iron } f = 36{,}000,\ c = \tfrac{1}{4500};\ \tfrac{1}{2250};\ \tfrac{1}{3000}$$

Professor Bauschinger recommends the following formulæ for columns in fire-proof buildings, as the result of a long series of experiments in which the columns were heated and sprinkled with water—

$$\frac{P}{A} = \frac{14223}{1 + 0{\cdot}0001\frac{l^2}{r^2}} \text{ for wrought iron}$$

$$\frac{P}{A} = \frac{19912}{1 + 0{\cdot}00025\frac{l^2}{r^2}} \text{ for cast iron}$$

where l = the length, and r = the least radius of gyration.

The following tests of full-sized columns were made at the Watertown Arsenal in 1881.

The columns were placed in the testing-machine on accurate bearings formed by inserting thin pieces of rolled brass between the ends of the column and the face of the holder when there was not a perfect fit.

Counterpoise weights were attached to the long columns at two points of their length, to counteract the tendency to bend from the weight of the column.

TABLE XXXIX.

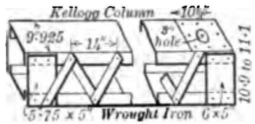
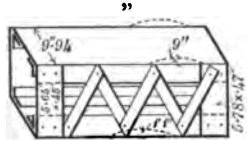
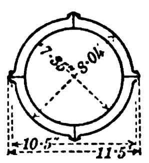
Number of test.	Style of column.	Length in feet and inches.	Sectional area in square inches.	Ultimate strength.		Elastic limit in pounds per square inch.	Remarks.
				Total pounds.	Pounds per square inch.		
1		29′ 9·3″	19·244	480,000	25,980	—	Tested with cap at end A, and pin of hard cast steel 3 inches in diameter at end B. Plate on pin end cracked at middle rivet, also channel beam under this crack.
2	FIG. 226.	27′ 9″	19·4473	550,000	28,280	22,620	Cap and pin not used; both ends take square bearings on compression plate.
3	"	19′ 9″	19·486	590,000	30,740	24,120	Tested in a similar manner to No. 2. Failed suddenly, springing upwards, breaking every brace on each side, excepting in the middle of the column.
4	"	15′ 9″	19·3012	460,000	25,130	—	Cap not used. Pin end tested with pin used in No. 1 column. Cracked upper channel beam through middle rivet, then lower channel on side of hole near pin end.
5	"	11′ 9″	19·5614	498,000	25,460	25,0[illegible]0	Tested in a similar manner to No. 4. Failed at lower channel beam, a crack extending from pin 5 inches long through centre rivet-hole.
6	"	8′ 0″	19·1026	574,500	30,080	24,600	Tested as in Nos. 4 and 5. Maximum deflection 1·15 inch. End braces gave out at rivet-holes.
7	 FIG. 227.	4′ 0″	19·3616	720,000	37,180	24,270	The column deflected upwards as shown in dotted lines.

TABLE XL.

No. of test.	Style of column.	Length in feet and inches.	Sectional area in square inches.	Ultimate strength. Total pounds.	Ultimate strength. Pounds per square inch.	Elastic limit in pounds per square inch.	Deflection in inches. Horizontal.	Deflection in inches. Vertical.
1		28′ 0″	12·062	424,200	55,150	—	4·552	3·296
2		28′ 0″	12·180	416,000	34,150	—	0·05	2·47
3		25′ 0″	12·233	431,500	35,270	27,960	—	—
4		25′ 0″	12·100	424,000	35,040	—	1·715	0·067
5		22′ 0″	12·371	440,000	35,570	—	0·077	0·006
6		22′ 0″	12·311	423,000	34,360	—	0·010	0·391
7		19′ 0″	12·023	425,200	35,365	—	0·092	0·024
8		19′ 0″	12·087	446,000	36,900	29,290	0·016	0·032
9		16′ 0″	12·000	439,000	36,580	—	2·09	0·002
10	FIG. 228.	16′ 0″	12·000	439,000	36,580	—	3·22	0·012
11		13′ 0″	12·185	449,000	36,857	28,890	1·435	0·234
12		13′ 0″	12·069	449,000	37,200	—	2·196	0·336
13		10′ 0″	12·248	446,823	36,480	26,940	0·049	0·030
14	”	10′ 0″	12·339	449,000	36,397	28,360	0·036	0·016
15	”	7′ 0″	12·265	468,000	38,157	29,350	0·535	—
16	”	7′ 0″	11·962	517,000	43,300	29,590	2·110	0·846
17	”	4′ 0″	12·080	598,000	49,500	—	—	—
18	”	4′ 0″	12·119	621,000	51,240	—	—	—
19	”	0′ 8″	11·903	680,000	57,130	—	—	—
20	”	0′ 8″	11·903	682,000	57,300	—	—	—

Compression of wrought-iron columns, lattice, box, and solid web, made at Watertown Arsenal in 1883 and 1884.

This series of tests comprises seventy-four columns made by the Detroit Bridge and Iron Company.

Tabulation of experiments with 3½-inch pins in the centre of gravity of cross-section, except those marked N, in which the loading was eccentric. Sufficient bearing area for the pins and flat ends was provided by means of reinforcing plates.

TABLE XLI.

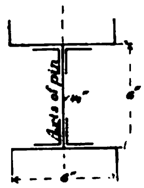
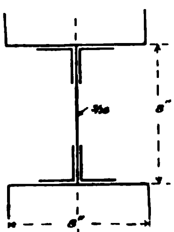
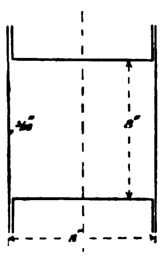
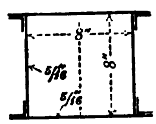
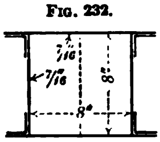
No. of test.	Style of column.	Length centre to centre of pins in inches.	Sectional area in square inches.	Ultimate strength. Total pounds.	Ultimate strength. Pounds per square inch.	Manner of failure.
752	 FIG. 229.	120·20	9·831	297,100	30,220	Deflected perpendicular to axis of pins.
757		120·07	10·199	320,000	31,380	Sheared rivets in eye-plates.
755		180·00	9·977	251,000	25,160	Deflected perpendicular to axis of pins.
756		180·00	9·977	210,000	21,050	Ditto.
753		240·00	9·732	188,600	19,380	Ditto.
754		240·00	9·762	158,300	16,220	Ditto.
751	 FIG. 230.	240·00	16·077	425,000	26,430	Deflected perpendicular to axis of pins.
1642		240·00	16·281	367,000	22,540	Ditto.
1646		320·00	16·179	318,800	19,700	Ditto.
1647		320·10	16·141	283,600	17,570	Ditto.
1653	 FIG. 231.	320·00	17·898	474,000	26,480	Deflected perpendicular to axis of pins.
1654		320·00	19·417	491,000	25,290	Ditto.
1645	 FIG. 232.	319·95	16·168	453,000	28,020	Deflected parallel to axis of pins.
1650		320·00	16·267	454,000	27,910	Deflected perpendicular to axis of pins.
1651	 FIG. 233.	320·00	20·954	540,000	25,770	Deflected in diagonal directions.
1652		320·10	20·613	535,000	25,950	Sheared rivets in eye-plates.

TABLE XLI.—*continued.*

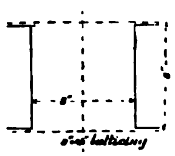
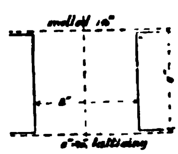
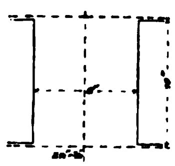
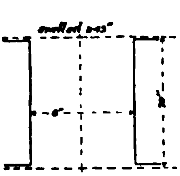
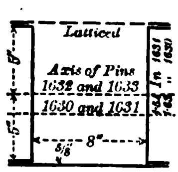
No. of test.	Style of column.	Length centre to centre of pins in inches.	Sectional area in square inches.	Ultimate strength. Total pounds.	Ultimate strength. Pounds per square inch.	Manner of failure.
746	 Fig. 234.	159·20	7·628	258,700	33,910	Deflected perpendicular to axis of pins.
747		159·27	8·056	294,700	36,580	Ditto.
748		239·60	7·621	260,000	34,120	Ditto.
749		239·60	7·621	254,600	33,410	Deflected diagonally.
1648		319 90	7·705	243,600	31,610	Deflected parallel to axis of pins.
1649		319·85	7·673	229,200	29,870	Deflected diagonally.
740	 Fig. 235.	159·90	7·645	262,500	34,340	Deflected perpendicular to axis of pins.
741		159·90	7·624	255,650	33,530	Ditto.
789		239·70	7·517	251,000	33,390	Deflected parallel to axis of pins.
750		239·70	7·531	259,000	34,390	Deflected perpendicular to axis of pins.
1643		319 80	7·691	237,200	30,840	Deflected parallel to axis of pins.
1644		319·92	7·702	237,000	30,770	Deflected horizontally.
1640	 Fig. 236.	199·84	11·944	403,000	33,740	Deflected perpendicular to axis of pins.
1641		200·00	12·302	426,500	34,670	Deflected diagonally.
1634		300·00	12·148	408,000	33,630	Deflected perpendicular to axis of pins.
1635		300·00	12·175	395,000	32,440	Ditto.
1638	 Fig. 237.	199·25	12·366	385,000	31,130	Deflected perpendicular to axis of pins.
1639		199·50	12·659	405,000	31,990	Ditto.
1636		300·20	11·920	391,400	32,830	Deflected in diagonal direction.
1637		300·15	11·932	390,700	32,740	Ditto.
1630	 Fig. 238.	300·00	17·622	461,500	26,190	Deflected perpendicular to axis of pins.
1631		300·00	17·231	485,000	28,150	Ditto.
1632		300·00	17·570	306,000	17,420	Ditto.
1633		300·00	17·721	307,000	17,270	Ditto.

Table XLII.

Wrought-iron Columns with Flat Ends.

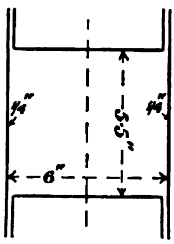
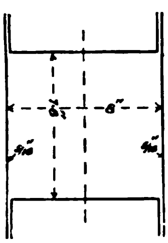
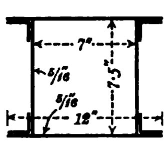
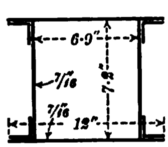
No. of test.	Style of column.	Total length in inches.	Sectional area in square inches.	Ultimate strength. Total pounds.	Ultimate strength. Pounds per square inch.	Manner of failure.
377	 Fig. 239.	127·900	12·08	383,200	31,722	Buckling of ¼-inch plate between riveting.
378		127·900	11·11	372,900	33,564	Buckling plates.
379	 Fig. 240.	167·80	17·01	594,500	34,950	Buckling plates between the riveting.
380		167·80	17·80	633,600	35,595	Triple flexure.
346	 Fig. 241.	167·90	15·74	517,000	32,846	Buckling plates.
347		167·90	15·84	555,200	35,050	Ditto.
342		247·62	15·68	517,500	33,003	Deflected upwards.
344		247·80	15·56	536,900	34,505	Buckling plates.
348	 Fig. 242.[1]	167·75	21·02	708,000	33,682	Buckling plates.
349		167·75	21·46	709,500	33,061	Triple flexure.
341		247·60	21·20	700,000	33,019	Deflected upwards.
343		247·62	21·49	729,450	33,943	Deflected downwards.

TABLE XLII.—*continued.*

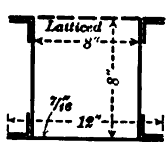
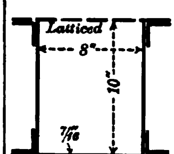
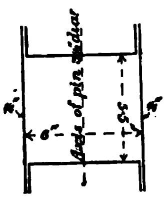
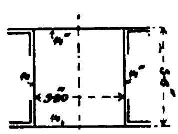
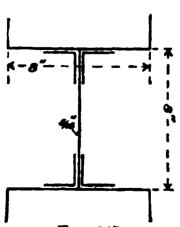
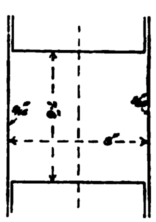
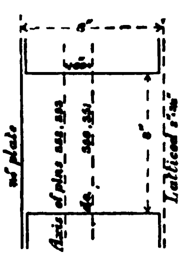
No. of test.	Style of column.	Total length in inches.	Sectional area in square inches.	Ultimate strength. Total pounds.	Ultimate strength. Pounds per square inch.	Manner of failure.
339	 FIG. 243.	247·94	12·64	412,900	32,666	Deflected upwards, latticing on the concave side.
340		247·94	12·74	431,400	33,862	Ditto.
337	 FIG. 244.	307·75	16 90	582,400	34,279	Deflected downwards and sideways, continuous place on the concave side.
338		307·87	17·40	580,000	33,333	Deflected diagonally, latticing on the concave side.

Table XLIII.

Wrought-Iron Columns with 3½-inch Pins.

No. of test.	Style of column.	Length centre to centre of pins in inches.	Sectional area in square inches.	Ultimate strength. Total pounds.	Ultimate strength. Pounds per square inch.	Manner of failure.
368	 Fig. 245.	180·10	11·42	379,200	33,205	Horizontal deflection perpendicular to plane of pins.
367		180·00	11·42	369,200	32,329	Ditto.
356		240·00	11·42	342,000	29,947	Ditto.
357		240·00	11·31	330,100	29,186	Ditto.
371	 Fig. 246.	119·90	9·14	286,100	31,302	Buckling the plates between the rivets.
372		120·00	10·07	319,200	31,698	Ditto.
370		180·00	9·21	291,500	31,650	Horizontal deflection and buckling between the rivets.
369		180·00	9·44	290,000	30,720	Ditto.
354		240·00	9·24	267,500	28,950	Triple flexure.
355		240·00	9·36	279,700	29,879	Horizontal deflection.
360	 Fig. 247.	160·12	15·34	475,000	30,965	Deflected upwards in the plane of the 3½-inch pins.
361		160·00	15·40	485,000	31,494	Horizontal deflection perpendicular to the plane of the pins.
358	 Fig. 248.	240·00	17·77	570,000	32,077	Horizontal deflection perpendicular to the plane of the pins.
359		240·00	17·22	555,400	32,253	Ditto.
350	 Fig. 249.	240·25	12·48	202,700	16,242	Horizontal deflection, concave on lattice side.
351		240·00	10·84	208,200	19,207	Ditto.
352		240·25	12·65	350,000	27,668	Ditto.
353		240·25	12·76	390,400	30,596	Horizontal deflection perpendicular to the plane of the pins, convex on lattice side.

The tests on full-sized columns recorded in the foregoing tables furnish the most reliable data for the determination of the strength of columns.

If we apply Rankine's or Gordon's formulæ to calculate the strength of these columns, we shall find that the results differ from those obtained by experiment.

The following empyrical formulæ represent more nearly the results of tests recorded in the foregoing tables.

Mr. Theodore Cooper has proposed the following formulæ for Phœnix columns :—

$$\frac{P}{A} = \frac{36000}{1 + \frac{\left(\frac{l}{r} - 80\right)}{18000}} \text{ for flat or square-ended columns}$$

$$\frac{P}{A} = \frac{36000}{1 + \frac{\left(\frac{l}{r} - 32\right)^2}{18000}} \text{ for pin-ended columns}$$

For lattice columns with pin ends—

$$\frac{P}{A} = \frac{34000}{1 + \frac{\left(\frac{l}{r} - 60\right)^2}{12000}}$$

For box and solid web columns—

$$\frac{P}{A} = \frac{33000}{1 + \frac{\left(\frac{l}{r} - 80\right)^2}{10000}} \text{ for flat ends}$$

$$\frac{P}{A} = \frac{31000}{1 + \frac{\left(\frac{l}{r} - 60\right)^2}{6000}} \text{ for pin ends}$$

Although not mathematically correct, the following formulæ are used to proportion the compressive members of bridges :—[1]

[1] Mr. Theodore Cooper's "General Specifications for Railway and Highway Bridges."

Railway Bridges.—Chord segments—

$$I = 8000 - 30\frac{l}{r} \text{ for live load stresses}$$

$$I = 16{,}000 - 60\frac{l}{r} \text{ ,, dead ,, ,,}$$

Columns—

$$I = 7000 - 40\frac{l}{r} \text{ of live load stresses}$$

$$I = 14{,}000 - 80\frac{l}{r} \text{ ,, dead ,, ,,}$$

$$I = 10{,}500 - 60\frac{l}{r} \text{ ,, wind stresses}$$

Lateral Struts—

$$I = 9000 - 50\frac{l}{r} \text{ for assumed initial stress}$$

Highway Bridges.—Chord segments—

$$I = 10{,}000 - 40\frac{l}{r} \text{ for live load stresses}$$

$$I = 20{,}000 - 80\frac{l}{r} \text{ ,, dead ,, ,,}$$

Columns—

$$I = 8750 - 50\frac{l}{r} \text{ for live load stresses}$$

$$I = 17{,}500 - 100\frac{l}{r} \text{ ,, dead ,, ,,}$$

$$I = 13{,}000 - 75\frac{l}{r} \text{ ,, wind stresses}$$

Lateral Struts—

$$I = 11{,}000 - 60\frac{l}{r} \text{ for assumed initial stress}$$

where I = the maximum intensity of working stress in pounds per square inch.

l = the length of the compression member in inches.

r = the least radius of gyration in inches.

l must not exceed forty-five times the least width of the compression member.

The areas obtained by dividing the live-load stresses by the live load must be added to the areas obtained by dividing the dead-load stresses by the dead load.

The following formulæ are used for steel struts :—

$$I = 8500 - 55\frac{l}{r} \text{ for live load stresses}$$

$$I = 17{,}000 - 110\frac{l}{r} \text{ ,, dead ,, ,,}$$

$$I = 13{,}000 - 85\frac{l}{r} \text{ ,, wind stresses}$$

Members subjected to alternate stress of tension and compression shall be proportioned to resist each kind of stress. Both of the stresses shall, however, be considered as increased by an amount equal to $\frac{8}{10}$ of the least of the two stresses.

The following formulæ were suggested in *Engineering* [1] for designing struts in which the length does not exceed forty times the least transverse dimensions.

$$I = \left(5 - \frac{0{\cdot}2l}{r}\right)\left(1 + \frac{\text{min.}}{\text{max.}}\right) \text{ for columns with fixed ends}$$

$$I = \left(5 - \frac{0{\cdot}3l}{r}\right)\left(1 + \frac{\text{min.}}{\text{max.}}\right) \text{ for columns with rounded ends}$$

For alternating stresses substitute $\left(1 + \frac{\text{min.}}{2\ \text{max.}}\right)$ for $\left(1 + \frac{\text{min.}}{\text{max.}}\right)$ in the foregoing formulæ.

It should be noted that l = the length in feet, and r = the least radius of gyration in inches. I = the safe working stress.

All the foregoing straight-line formulæ should only be used within the limits specified in regard to the ratio of length to least width of section or least radius of gyration.

[1] *Engineering*, January 5, 1892.

CHAPTER XIII.

SIZES OF PLATES AND BARS—JOINTS AND CONNECTIONS.

In designing built-up structures of wrought iron and steel, we make use of certain rolled sections which consist chiefly of plates, angles, T, channel, and flat bars; also of rolled girders, bulb T-irons, and occasionally other specially rolled sections connected together by means of rivets, bolts and nuts, pins, eyes, gibs and cotters, union screws, etc. It is necessary to know how to arrange the lengths and sections of the various riveted joints and other connections in the most convenient manner, not merely with a view to resist the various stresses developed in them, but also to reduce the cost of manufacture, carriage by land and sea to the site of the structure, and its subsequent erection.

The designer will find it convenient to have by him a list of sizes of plates and bars, made by different manufacturers, such as the following: The Butterley Iron Company, Measures Bros., Carnegie, Dalzell Steel and Iron Works, and many others.

Iron and steel plates vary in thickness from $\frac{1}{4}$ to $1\frac{1}{2}$ inch, increasing by $\frac{1}{16}$ of an inch. Plates less than $\frac{1}{4}$ inch are termed sheets, and are generally more expensive. The following maximum sizes from the Butterley Iron Company's list have been selected, and indicate the limits of area, length, and width:—

TABLE XLIV.

Thickness.	Area in square feet.	Length.	Width.	Remarks.
$\frac{1}{4}''$	72	33′	6′ 0″	The sizes are the maximum in each case; thus, if we require to know the greatest length of plate obtainable $\frac{5}{8}$ inch thick by 4 feet wide, we have $\frac{140}{4} = 35$ feet.
$\frac{3}{8}''$	94	40′	6′ 9″	
$\frac{1}{2}''$	130	45′	8′ 0″	
$\frac{5}{8}''$	140	45′	8′ 6″	
$\frac{3}{4}''$	180	40′	9′ 0″	
1″	125	33′	9′ 0″	

The best Yorkshire ironmakers roll plates for boilers 20 per cent. larger in area than given in the above table.

The following sizes have been selected from the Dalzell Steel and Iron Works list :—

TABLE XLV.

Thickness in inches.	Maximum lengths in feet.				
	4 ft. wide.	5 ft. wide.	6 ft. wide.	7 ft. wide.	7 ft. 6 in. wide.
$\frac{13}{16}$ to $1\frac{1}{2}$	30 to 40	24	20	16	15
$\frac{3}{8}$ to $\frac{3}{4}$	33	27	21	16	15
$\frac{1}{4}$ to $\frac{11}{32}$	20	16	12	—	—

Extras are charged for thicknesses of iron and steel plates under $\frac{1}{4}$ inch, and for lengths over 25 feet; for widths over 4 feet 6 inches in iron and over 6 feet in steel; for weights over 10 cwt. in iron and 20 cwt. in steel; and for areas over 60 square feet.

L and T bars are rolled in a great variety of sections. For ordinary sections the lengths may be 30 feet without extra charge; generally the length and depth united should not exceed 9 inches, or extras will be charged. Dalzell Steel Company, however, allow 11 inches for angles and 10 for T's without extras.

The following are probably the largest sections obtainable in iron and steel angles: 10″ × 3½″ from ½″ to ¾″ in thickness. In steel, 10″ × 4″ and 11″ × 4″ from ½″ to ¾″ may be obtained. T's may be obtained in steel 10″ in the stalk by ⅞″ thick, and 7″ in the table by ¾″ thick. Channel bars are rolled to a maximum size of 15″ × 4″ × ½″, and rolled girders to a maximum size of 20″ × 8″, at a mean weight of 100 lb. per foot, but extras are charged when the sum of the depth and width exceeds 16 inches.

Flat bars are rolled up to a maximum width of 12 inches, and may be obtained from ⅛ of an inch in breadth and ⅛ of an inch in thickness, from 1″ × ¼″ to 6″ × 1″ without extra charge.

Round and square iron and steel bars may be obtained from ⅜″ to about 3″ without extra charge; the maximum sizes are 10″ diameter or 10″ square. The relative cost of the various rolled sections may be approximately expressed, for the purpose of estimating the relative cost of various designs, as follows:—

Flat, round, and square bars	1·00
L, T, and [bars	1·12
Plates	1·18
Rivets	1·45

Economy may be effected by making use of angle and flat bars in preference to plates wherever they can be used without a corresponding disadvantage, and by reducing the riveting as

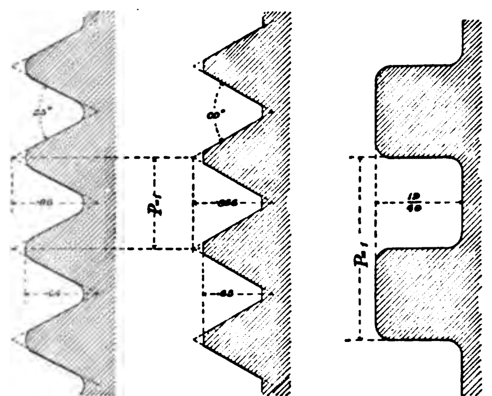


FIG. 250. FIG. 251. FIG. 252.

much as possible consistent with strength. It is sometimes desirable to incur the extra charges for longer bars and plates, rather than to increase the number of joints, as the total cost is reduced.

Unequal-sided angles should be avoided unless used in large quantities, and generally the fewer the sections used in a particular work the better. It is most desirable to reduce the amount of smith's work, such as cranking, bending, joggling, as much as possible, as the iron is injured in the process and the cost of the work increased. Welding should never be resorted to in girder-work if it can possibly be avoided.

Bolts, Nuts, Union Screws.—Bolts and nuts, suspension and tie-rods, are specified by English engineers to have Whitworth threads, but American engineers generally use the Seller thread (see Figs. 250, 251, and 252).

Whitworth threads are rounded at the tops and bottoms, while Seller's threads are flat at top and bottom, and consequently they can be cut with one tool.

Strength of Bolts and Screwed Rods.—Let P denote the tensile stress on the rod or bolt; *f*, the safe intensity of working stress; *a*, the area at the bottom of the thread given in the following table for Whitworth threads; then—

$$P = af$$

Table XLVI.

Table of Whitworth V Screw Threads.[1]

Diameter in inches.		Number of threads per inch.	Pitch in inches.	Diameter at bottom of thread in inches.	Area of section at bottom of thread in inches.
½	0·500	12	0·0833	0·393	0·121
⅝	0·625	11	0·0909	0·509	0·204
¾	0·750	10	0·1000	0·622	0·304
⅞	0·875	9	0·1111	0·733	0·422
1	1·000	8	0·1250	0·840	0·554
1⅛	1·125	7	0·1430	0·942	0·697
1¼	1·250	7	0·1430	1·067	0·894
1⅜	1·375	6	0·1670	1·161	1·058
1½	1·500	6	0·1670	1·286	1·299
1⅝	1·625	5	0·2000	1·369	1·472
1¾	1·750	5	0·2000	1·494	1·753
1⅞	1·875	4½	0·2222	1·590	1·986
2	2·000	4½	0·2222	1·715	2·311
2¼	2·250	4	0·2500	1·930	2·926
2½	2·500	4	0·2500	2·180	3·733
2¾	2·750	3½	0·2860	2·384	4·464
3	3·000	3½	0·2860	2·634	5·450
3¼	3·250	3¼	0·3080	2·855	6·402
3½	3·500	3¼	0·3080	3·105	7·563
3¾	3·750	3	0·3333	3·323	8·673
4	4·000	3	0·3333	3·573	10·027
4¼	4·250	2⅞	0·3480	3·804	11·365
4½	4·500	2⅞	0·3480	4·054	12·908
4¾	4·750	2¾	0·3640	4·284	14·404
5	5·000	2¾	0·3640	4·534	16·146
5¼	5·125	2⅝	0·3810	4·762	17·810
5½	5·500	2⅝	0·3810	5·012	19·720
5¾	5·750	2½	0·4000	5·239	21·570
6	6·000	2½	0·4000	5·489	23·640

f should not be taken more than 4 tons per square inch for a steady load, since the stress is not uniformly distributed in consequence of the abrupt change of section from the full area of the bolt to the area at the bottom of the thread.

[1] Unwin's "Machine Design," part i. p. 145.

In bridge-work the safe working stress must be determined from the range of stress, as explained in Chapter I. If the ratio of the minimum to the maximum stress is $\frac{1}{4}$, then f may be taken at 3 tons per square inch. When the bolt is screwed up torsional stress will be developed, which, if the maximum load producing tension is also on the bolt, will necessitate a further reduction in the working stress. Professor Unwin has shown[1] that this stress requires the diameter of the bolt to be increased about 15 per cent., and he gives the following formulæ for machine bolts where the range of stress varies from zero to a maximum :—

$$P = 2400d^2 \text{ to } 3000d^2$$

where $P =$ the total load in pounds, and $d =$ the full diameter of the bolt in inches.

In suspension bolts for timber truss bridges, and in cable suspension bridges, also in the tension rods in iron roofs and wind-bracing, the rods should be upset before the thread is cut, so that the diameter at the bottom of the thread is never less than the full diameter of the bolt. Under these circumstances the effect of unequal distribution of stress may be neglected. The torsion may also be neglected, since it is not likely to be applied when the full load is on the rod. The screwing up, however, will in most cases put initial tension upon the rod, the exact amount of which is difficult to determine. In roofs and wind-bracing this initial stress may be neglected in fixing the intensity of working stress, since the maximum load is rarely applied. It will generally have to be considered in addition to the range of stress due to dead and live loads.

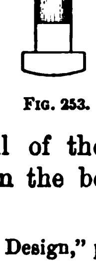


Fig. 253.

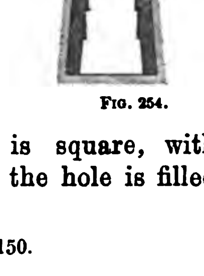


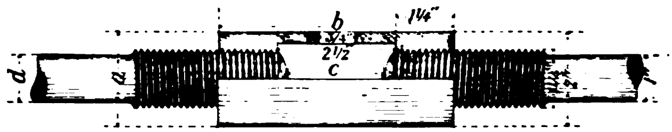
Fig. 254.

Fig. 253 shows a common bolt and nut; Fig. 254, a rag bolt which is sometimes used to secure foundation-plates to the masonry or concrete bed upon which it rests. A taper hole is cut in the masonry, and the tail of the bolt is square, with jagged ends; the space between the bolt and the hole is filled with molten lead.

[1] Unwin's "Machine Design," part i. p. 150.

Union Screw Coupling.—Figs. 255 and 256 represent two union-screw couplings; the threads cut upon the upset ends of the rods are right and left handed, so that when the coupling is turned the rods are caused to approach each other until they are tight. In Fig. 255 the coupling is turned by inserting a bar in the centre hole, and in Fig. 256 the middle of the coupling is forged hexagonal, so that it may be turned by means of a

FIG. 255.



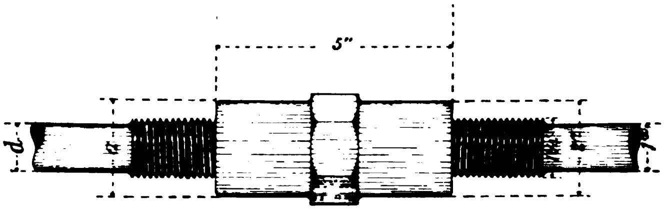


FIG. 256.

screw-key. The outside diameter of the screwed portion is equal to the diameter of the rod + 2 × depth of thread, which in Whitworth threads is $\frac{5}{8}$ of the pitch; hence, if d_1 = the outside diameter, and d = the diameter of the rod—

$$d_1 = d + \frac{5}{8} \text{ pitch}$$

Table 44 will give us at once the outside diameter if we look out the diameter which has the diameter at the bottom of the thread equal to, or not less than, the diameter of the rod; thus if the rod is 1 inch in diameter, we find that $1\frac{1}{4}$ will give 1·067 at the bottom of the thread.

The depth of the thread in the coupling should be made equal to the outside diameter of the thread, to ensure that the threads will not strip, and the total length of the coupling will be twice the outside diameter of the thread plus the portion, C, left for adjustment. There only remains, therefore, the outside diameter of the coupling, a, to be determined, so that the sectional area through the hole in Fig. 255 may be equal to that of the rod. Let d_2 = the inside diameter of the coupling, d_3 = the outside diameter; let the diameter of the hole be $\frac{3}{4}$ of an inch; then d_2 will equal the outside diameter of the screwed portion of the thread plus the clearance, so that d_2 may equal 1½ inch if $d = 1$ inch ; hence—

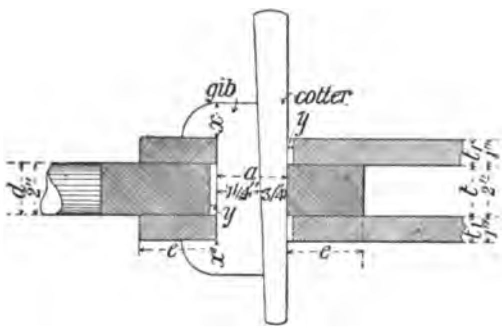
$$\frac{\pi}{4} = \frac{\pi}{4}\{d_3^2 - (\tfrac{3}{2})^2\} - 2 \times 0{\cdot}75(d_3 - \tfrac{3}{2})$$

$$\therefore d_3 = 2 \text{ inches}$$

Examples of union screws, such as are generally used in bridge-work, are given on Plate II.

Jib-and-Cotter Joints.—The tie-rod of an iron roof is frequently attached to the principal, and to the shoe at the abutments, by means of a gib-and-cotter joint. Figs. 257 and 258 show this kind of joint. The outside of the gib is made parallel to the outside of the cotter, and the taper of the cotter is made from

Fig. 257.



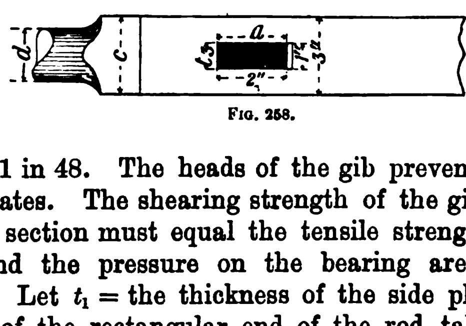


Fig. 258.

1 in 24 to 1 in 48. The heads of the gib prevent any spread of the side plates. The shearing strength of the gib and cotter at its central section must equal the tensile strength of the main tie-rod, and the pressure on the bearing area must not be excessive. Let t_1 = the thickness of the side plates, and t the thickness of the rectangular end of the rod, taken as equal to the diameter of the rod; let c = the breadth of the rectangular end of the rod, a = the length of the gib and cotter at the central section, t_3 = the thickness.

Hence, if $t = 2t_1$, the tensile and bearing strengths of the side plates will equal those of the rectangular end of the rod, and if we take 5 tons as the intensity of working stress in the tie-rod, 4 tons as the safe shearing stress of the gib and cotter, and 8 tons as the safe intensity of pressure on the bearing area, then, since the gib and cotter are in double shear and must therefore shear at two sections at the same time—

$$2at_3 \times 4 = \frac{\pi}{4}d^2 \times 5$$

$$\therefore at_3 = \frac{5\pi}{32}d^2$$

If t_3 is known, a can be found from the above equation. Let $d = 2$ inches, then the safe stress on the tie-rod is—

$$4\frac{\pi}{4} \times 5 = 15{\cdot}7 \text{ tons} = 8at_3$$

$$\therefore at_3 = 2 \text{ square inches (about)}$$

If $t_3 = \frac{3}{4}$ of an inch, $a = 2{\cdot}7$ inches.

The pressure on the bearing area will be—

$$P \times \tfrac{3}{4} \times 2 = 15{\cdot}7$$

$$\therefore P = 10{\cdot}4 \text{ tons per square inch}$$

which is too great. Let $t_3 = 1$ inch, then a would require to be 2 inches, and—

$$P \times 1 \times 2 = 15{\cdot}7$$

$$\therefore P = 7{\cdot}8 \text{ tons.}$$

Hence if the gib and cotter are made 1 inch thick, the pressure will not be excessive.

To find the breadth c, so that the sectional area shall be the same as the tie-rod after deducting for the hole—

$$2t_1(c - t_3) = \frac{\pi}{4}d^2$$

or, if $d = t = 2t_1$, we may write—

$$d(c - t_3) = \frac{\pi}{4}d^2$$

$$\therefore c - t_3 = \frac{\pi}{4}d$$

$$\therefore c = t_3 + \frac{\pi}{4}d$$

If $d = 2$ inches, and $t_3 = 1$ inch, $c = 1 + 1{\cdot}57 = 2{\cdot}57$ inches.

It is desirable to have a margin here for possible inaccuracy in forging, hence c should be made 3 inches.

The depth *e* in Fig. 257 must be sufficient to avoid shearing a piece out the width of the cotter.

Drilled and Punched Holes (Riveting).—Although punching is largely used for rivet-holes, it is objectionable for the following reasons:—

The spacing of the holes is not done so accurately as with drilling, and when two or more plates are used, as in the flanges of a girder, the holes frequently overlap, and require to be rhymered before the rivets can be inserted. This inequality in the spacing is partly due to the stretching of the plate in the process of punching, and it increases with the length of the bar or plate, also with the ductility of the material; it is most decided in curved girders. As a consequence of the unequal spacing, it is practically impossible to ensure closely butted joints with punched work. Another objection to punching is that the material round the hole is injured by the plastic flow of the metal under the pressure of the punch; in general the harder the material the greater the injury from this cause, but steel is injured by punching more than iron, and should only be used in thin plates. This injury may be removed by drilling or rhymering out the hole generally to about $\frac{1}{16}$ of an inch in diameter, and to some extent by annealing. The method of driving in a conical pin, known as drifting, should never be allowed, as it causes considerable damage to the material. For ordinary wrought-iron girder-work with not more than two flange plates, punching is good enough if the work is done properly, but when several plates are used in bridge-work the holes should be drilled, as it is almost impossible to ensure accurate work of this class with punching. Drilling the rivet-holes by means of multiple drilling-machines ensures accurate spacing and avoids the injury to the material round the hole as in punching; the sharp edge, however, left by the drill should be rounded off, as it facilitates the shearing of the rivet.

Machine-riveting, such as by means of hydraulic pressure, is much superior to hand-riveting, as the plates are brought in closer contact and the rivets are made to fill the holes more completely; moreover, the pressure exerted upon the rivets is under control, and may be regulated to any extent. Steel rivets require greater care in closing by hand than iron rivets, as they are more liable to be injured if hammered at a dull red heat. With machine-riveting the plates should be well bolted together, or the metal will be squeezed out between the plates, forming collars, and it is generally very difficult to cut out a rivet closed by hydraulic machinery.

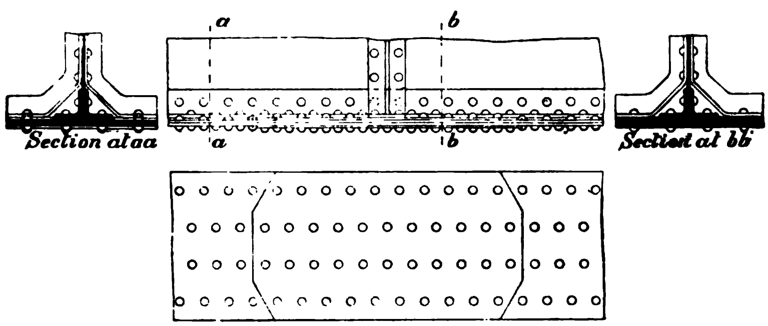
In both hand and machine riveting the contraction of the rivet longitudinally in cooling causes it to exert a pressure on the plates like a clamp, and with accurately drilled holes and hydraulic riveting the shearing resistance of the rivets may never be developed under the ordinary working stress in the structure; but the amount of frictional resistance due to this cause is uncertain, and may be reduced by vibrations. Again, if the elastic limit of the material of the rivet is exceeded, the grip upon the plates must gradually diminish; it is, therefore, in general neglected in designing riveted joints. Machine-riveting is used more in the bridge yard than in the jointing of the various pieces during erection, where it would be of the greatest service, but it has not generally been found convenient to use portable riveting-machines for this purpose, although they will probably be used more extensively in the future. When a bridge or girder is finished, there will generally be a certain proportion of loose rivets, which, if decidedly loose, should be cut out and replaced with well-fitting rivets. Loose rivets may be easily detected by tapping the rivet on one side with a small hammer and holding the finger on the other, but apparently tight rivets may be rendered loose by too frequent tapping, and absolute tightness cannot be expected from the nature of the process.

In designing riveted joints it is generally assumed that the size of the finished rivet is the same as that figured on the drawings. Sometimes, however, the nominal size of the rivet is $\frac{1}{32}$ of an inch smaller than the actual size before it is closed, and if the holes are drilled, they are usually made the correct size; so that there is no practical difference in the figured from the actual sizes, excepting that no rivet can completely fill the hole if put in hot, in consequence of the lateral contraction in cooling. In punched work the holes are generally from 10 to 20 per cent. larger than the nominal sizes of the rivets, and the shearing resistance of the rivets in the joint is consequently always in excess of that calculated, but, since the stresses are not likely to be so well distributed in punched as in drilled work, this excess may be neglected.

Some engineers specify that the rivets shall be the actual sizes figured on the drawings before being inserted in the work, in which case the finished size is $\frac{1}{32}$ of an inch larger, giving an excess in both drilled and punched work. It will be assumed in the following calculations that the rivets are of the actual sizes figured, which appears to be the general practice.

Joints in Flange Plates and Webs of Girders.—Figs. 259 and 260 represent the methods commonly adopted for covering the joints in the flanges and webs of girders. In each case the total

Fig. 259.



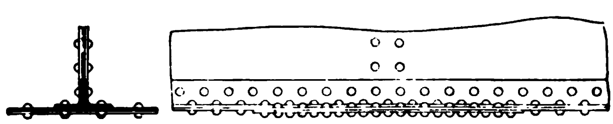


Fig. 260.

shearing resistance of the rivets on one side of the joint should be at least equal to the tensile resistance of the net section of the plate through the rivet-holes; or, in other words, the total shearing resistance of the rivets on one side of the joint should not be less than the stress developed at the joint. It is a good rule to make the strength of the rivets about 10 per cent. greater than that of the net section of the plate, as, from the nature of the process of riveting, the stress is not often uniformly distributed over the rivets, some being more severely stressed than others. In Fig. 259 there is a cover-plate on each side of the joint, and the rivets are in double shear, as they must shear at two sections at the same time. In Fig. 260 the cover-plate is on one side only, and the rivets can shear at only one section, consequently the rivets in double shear ought to be twice as strong as those in single shear. Again, in single shear the material is not symmetrical with reference to a line passing through the centre of the thickness of the plate, and there is a tendency towards an unequal distribution of stress over the joint. In double shear it is usual to allow 1·75 times the strength in single shear.

In order to obtain cover-plates of the full width of the girder on each side of the joint, it is necessary to insert a packing-piece between the angle irons and the flange plates equal in thickness to the inside cover-plates, and butting against it as shown in Fig. 259. An intermediate case is one in which narrow cover-plates are riveted on each side of the angle irons; but if the rivets are considered to be in double shear, the flanges of the angle irons ought not to be considered in the net sectional area of the bottom flange. Sometimes a strip is placed at the bottom of the girder of the same width as the flanges of the two angle irons in addition to the two inside strips and outside cover. The cover-plates need not be so long when the rivets through them are in double shear. In Fig. 259 there are two plates, and the joint occurs on the inside; hence the moments of the area of each cover about the centre of the joint ought to be equal, from which it follows that the inside cover should be thicker than the outside, because its lever arm is shorter, while the total thickness of the two covers need not be greater than that of the plate covered, thus: If the plate covered is $\frac{1}{2}$ inch thick, and the plate between it and the outside cover the same thickness, and t_1 and t_2 the thickness of the inside and outside covers respectively—

$$t_1 + t_2 = \frac{1}{2}$$

$$\frac{t_1}{2} = t_2$$

hence the inside cover should be twice as thick as the outside cover, $t_1 = \frac{1}{3}$, $t_2 = \frac{1}{6}$, and the covers may be made $\frac{3}{8}$ inch and $\frac{1}{4}$ inch respectively.

The greater the number of plates between the joint and the outside cover, the smaller the thickness of the outside relatively to that of the inside cover. In a pile of plates with a joint in the centre, or in a joint consisting of a single plate with a cover on each side, the thicknesses of the cover-plates should be equal. The covers to joints in web plates are generally double, so that the rivets are in double shear, and their joint thickness is slightly greater than that of the web plate. The resistance of iron and steel bars to shearing is about ⅘ of the tensile strength of the material; but rivet iron is of better quality than plate-iron, the tensile strength being about 24 tons per square inch, while rivet-steel is about 26 to 30 tons per square inch, so that the shearing strength of rivet-iron is about equal to the tensile strength of plate-iron. But the shearing strength of steel is only about 23 tons, hence the ratio of rivet area to plate area in single shear should be one of equality for iron plates and rivets, but as 5 : 4 in steel plates and rivets; but, in order to allow for imperfections in workmanship, and the variations from other causes, the ratio for average cases may be taken as about 1·1 for iron, and about 1·4 for steel. Although the ratio of rivet area to plate area is the most important consideration in the strength of riveted joints, still, under some circumstances, the joint may fail in other ways even when this ratio is as above stated. The plates or rivets may be crushed due to excessive pressure on the bearing area, or the plate may break away in front of the rivet.

The value of the crushing stress which produces injury to the tenacity or shearing resistance of the joint is stated by Professor Unwin to be very uncertain, but that there is no indication of injury in the case of steel joints with crushing pressure of 50 tons per square inch. If, therefore, the working crushing pressure is not greater than 8 tons per square inch in iron joints, and 10 tons in steel, there will be no danger of the rivets crushing, or the holes elongating.

If b = the breadth of a plate after deducting the diameters of the rivet-holes;
t = the thickness of the plate;
a = the area of the rivet;
n = the number of rivets required on each side of the joint;

then, neglecting the pressure on the bearing area, we have, for iron rivets—

$$1{\cdot}1bt = na, \text{ single shear}; \quad \therefore n = \frac{1{\cdot}1bt}{a}$$

$$1{\cdot}1bt = 1{\cdot}75na, \text{ double shear}; \quad \therefore n = \frac{0{\cdot}628bt}{a}$$

for steel rivets—

$$1{\cdot}4bt = na, \text{ single shear}; \quad \therefore n = \frac{1{\cdot}4bt}{a}$$

$$1{\cdot}4bt = 1{\cdot}75na, \text{ double shear}; \quad \therefore n = \frac{0{\cdot}8bt}{a}$$

The areas of $\frac{3}{4}$ inch, $\frac{7}{8}$ inch, and 1 inch rivets may be taken as 0·44, 0·6, and 0·78 square inches respectively. The pressure on the bearing area should then be calculated to see if it is within the safe working stress above referred to; if it is not, the number or diameter of the rivets should be increased. It will not generally be found necessary to increase the diameter or the number of rivets in order to increase the bearing area in single-shear joints, but with thin plates and double-shear joints the bearing area must not be neglected.

Fig. 261 represents a group of joints in the tension boom of a girder with cover-plates on each side, so that the rivets are in double shear. The thicknesses of the two covers should be equal, as the lever arm with reference to the joint in the upper plate on the right and the top cover is the same as the lever

FIG. 261.

arm with reference to the joint in the lower plate on the left and the bottom cover. If the group of plates consists of equal thicknesses, there will be a slight excess in strength if each cover is made equal in thickness to one of the plates; if the plates vary in thickness it will depend upon their position, in so far as they affect the length of the lever arms of the covers about the joint. If each of the five plates shown in the figure is $\frac{1}{2}$ inch in thickness, the thickness of each cover should be made $\frac{1}{2}$ inch; the excess in area may be neglected. With reference to the group of joints shown in the sketch—

1. Failure may occur by the shearing of all the rivets in the joints between 1 and 7.

2. Failure may occur by the shearing of the rivets between 2 and 6, and the tearing of both cover-plates at the same time.

3. Failure may occur by the rivets between 1 and 7 cutting into the plates from insufficiency of, or excessive pressure on, the bearing area.

4. Failure may occur by the rivets cutting into the covers between 1 and 2 and 6 and 7 at the same time that the rivets

cut into the plates between 2 and 6 and 2 and 9, or between 2 and 6 and 2 and 10.

With regard to the first mode of failure.

Let N = the total number of rivets in the joint.
p = the number of plates.
b = the breadth of each plate less rivet-holes.
t = the thickness of each plate.
m = the number of rivets in each end group.
r = the number of rivets in each central group.
d = the diameter of the rivets.
a = the area of one rivet.

$$\text{Then } N = \frac{0{\cdot}628}{a} pbt \text{ for iron} = 2m + (p-1)r \qquad (1)$$

$$N = \frac{0{\cdot}8}{a} pbt \text{ for steel} = 2m + (p-1)r \qquad (2)$$

With regard to the second mode of failure. If the thickness of each cover be made equal to t, then—

$$\text{Cover-plate area} = 2bt$$

$$\text{Number of rivets in the central groups} = (p-1)r$$

$$\text{Rivet area in the central groups} = (p-1)ra$$

$$\text{Equivalent plate area} = \frac{(p-1)ra}{0{\cdot}628}$$

$$\therefore\ 2bt + \frac{(p-1)ra}{0{\cdot}628} = pbt \text{ (iron)} \qquad (3)$$

$$2bt + \frac{(p-1)ra}{0{\cdot}8} = pbt \text{ (steel)} \qquad (4)$$

Equations 3 or 4 may be solved for r, which may be substituted in equations 1 or 2 to find m. It will generally be more convenient to calculate the ratio of the intensity of pressure on the bearing area to the tensile strength of the plates, in order to see whether the third and fourth methods of failure are possible with the joint under consideration.

If we take 8 and 10 tons as the greatest permissible pressure in the case of iron and steel respectively, this ratio should not exceed 1·6; let q denote the ratio referred to. The total area exposed to pressure is for the third mode of failure—

$$pmtd + (p-1)rtd + \ldots + rtd = pmtd + \frac{p}{2}(p-1)rtd$$

$$\therefore q = \frac{pb}{\frac{pd}{2}\{2m + r(p-1)\}}$$

For the fourth mode of failure the area exposed to pressure is—

$$2mtd + (p-1)rtd + \ldots + rtd = 2mtd + \frac{p}{2}(p-1)rtd$$

$$\therefore q = \frac{pb}{d\left\{2m + \frac{p}{2}(p-1)r\right\}}$$

Example I.—A group joint for three plates, each 24 inches wide by $\frac{1}{2}$ inch thick, rivets $\frac{7}{8}$ inch in diameter and 4 inches pitch. We may assume that there will be four rows of rivets longitudinally, and that the plates need not be weakened to a greater extent than by two rivets.

The effective width of the plate is therefore $(24 - 2 \times \frac{7}{8})$, or 22·25 inches, and the effective area 11·125 square inches.

The lever arms for the cover-plates will be about as 1 : 3, so that $\frac{3}{8}$-inch plates will be sufficient. The total number of rivets required is—

$$\left.\begin{aligned} N &= \frac{0{\cdot}628 \times 3 \times 11{\cdot}125}{0{\cdot}6} = 34{\cdot}93 \text{ (iron)} \\ &= \frac{0{\cdot}8 \times 3 \times 11{\cdot}125}{0{\cdot}6} = 44{\cdot}50 \text{ (steel)} \end{aligned}\right\} = 2m + 2r$$

$$\text{Cover-plate area} = \frac{2 \times 3 \times 22{\cdot}25}{8} = 16{\cdot}68 \text{ square inches}$$

$$\left.\begin{array}{l}\text{Rivet area in central} \\ \text{groups}\end{array}\right\} = 2 \times 0{\cdot}6 \times r = 1{\cdot}2r \text{ square inches}$$

$$\text{Equivalent plate area} = \frac{1{\cdot}2r}{0{\cdot}628} = 1{\cdot}91r \text{ (iron)}$$

$$\text{,, \quad ,, \quad ,,} = \frac{1{\cdot}2r}{0{\cdot}8} = 1{\cdot}50r \text{ (steel)}$$

$$\therefore 16{\cdot}68 + 1{\cdot}91r = 11{\cdot}125 \times 3 \qquad \therefore r = 8{\cdot}7 \text{ (iron)}$$

$$16{\cdot}68 + 1{\cdot}50r = 11{\cdot}125 \times 3 \qquad \therefore r = 11{\cdot}1 \text{ (steel)}$$

But r must be some multiple of 2, so also must N, therefore we have—

$$2m + 2 \times 10 = 36 \qquad \therefore m = 8 \text{ (iron)}$$

$$2m + 2 \times 12 = 46 \qquad \therefore m = 11 \text{ (steel)}$$

In this case m may be made equal to $r = 10$ and 12 respectively.

$$q = \frac{3 \times 22{\cdot}25}{\frac{3 \times 7}{8 \times 2}(20 + 20)} = 1{\cdot}2 \text{ (iron)}$$

$$= \frac{3 \times 22{\cdot}25}{\frac{3 \times 7}{8 \times 2}(24 + 24)} = 1{\cdot}06 \text{ (steel)}$$

$$\text{or } q = \frac{3 \times 22{\cdot}25}{\frac{7}{8}(20 + 30)} = 1{\cdot}50 \text{ (iron)}$$

$$= \frac{3 \times 22{\cdot}25}{\frac{7}{8}(24 + 36)} = 1{\cdot}26 \text{ (steel)}$$

Hence the pressure on the bearing area is not excessive, and the joint may be designed as in Figs. 262 and 263.

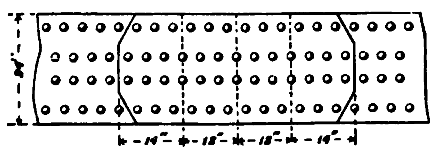


FIG. 262.

FIG. 263.

Group joint for 3 iron plates.

Example II.—A group joint similar to the last, only with five plates instead of three.

$$N = \frac{0{\cdot}628 \times 5 \times 11{\cdot}125}{0{\cdot}6} = 58{\cdot}2, \text{ say 60, for iron}$$

$$= \frac{0{\cdot}8 \times 5 \times 11{\cdot}125}{0{\cdot}6} = 74{\cdot}1, \text{ say 76, for steel}$$

The cover-plates may be taken as the same thickness as the plates in this case.

$$\text{Cover-plate area} = 2 \times 11{\cdot}125 = 22{\cdot}25 \text{ square inches}$$

$$\text{Rivet area in central groups} = 4 \times 0{\cdot}6 \times r = 2{\cdot}4r \text{ square inches}$$

$$\text{Equivalent plate area} = \frac{2{\cdot}4r}{0{\cdot}628} = 3{\cdot}8r \text{ (iron)}$$

$$= \frac{2{\cdot}4r}{0{\cdot}8} = 3r \text{ (steel)}$$

$$\therefore 22{\cdot}25 + 3{\cdot}8r = 11{\cdot}125 \times 5 \qquad \therefore r = 8{\cdot}78 \text{ (iron)}$$

$$22{\cdot}25 + 3r = 11{\cdot}125 \times 5 \qquad \therefore r = 11{\cdot}12 \text{ (steel)}$$

Taking the next greatest multiple of 2, we have, as before—

$$2m + 2 \times 10 = 60 \quad \therefore m = 20 \text{ (iron)}$$
$$2m + 2 \times 12 = 76 \quad \therefore m = 26 \text{ (steel)}$$
$$q = \frac{5 \times 22 \cdot 25}{\frac{5 \times 7}{2 \times 8}(40 + 10 \times 4)} = 0 \cdot 6 \text{ (iron)}$$
$$= \frac{5 \times 22 \cdot 25}{\frac{5 \times 7}{2 \times 8}(52 + 12 \times 4)} = 0 \cdot 5 \text{ (steel)}$$
$$\text{or } q = \frac{5 \times 22 \cdot 25}{\frac{7}{8}(40 + 100)} = 0 \cdot 9 \text{ (iron)}$$
$$= \frac{5 \times 22 \cdot 25}{\frac{7}{8}(52 + 120)} = 0 \cdot 7 \text{ (steel)}$$

Hence the pressure on the bearing area is very small, and the joint may be designed as shown in Figs. 264 and 265.

Generally, if the tension flange is designed for double shear, the compression flange in drilled and planed work may be

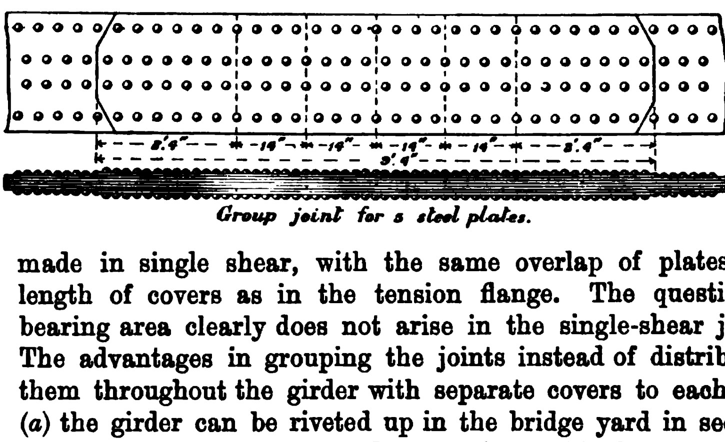


FIG. 264.

FIG. 265.

Group joint for 5 steel plates.

made in single shear, with the same overlap of plates and length of covers as in the tension flange. The question of bearing area clearly does not arise in the single-shear joints. The advantages in grouping the joints instead of distributing them throughout the girder with separate covers to each are: (*a*) the girder can be riveted up in the bridge yard in sections of suitable length with regard to carriage; (*b*) the quantity of material used in the covers is reduced, as well as the number of rivets required to be put in during the erection of the bridge. If separate covers are used for each joint, we should require, in the case of the iron joint, with three plates and rivets in single shear—

$$N = \frac{1 \cdot 1bt}{a} = \frac{1 \cdot 1 \times 22 \cdot 25}{2 \times 0 \cdot 6} = 20 \cdot 4$$

or 22 rivets on each side of the joint, necessitating cover-plates 4 feet long, so that for three joints the total length of covers would be 12 feet. In double shear—

$$N = \frac{0.628bt}{a} = \frac{0.628 \times 22.25}{2 \times 0.6} = 11.8$$

or 12 rivets on each side of the joint, necessitating cover-plates 2 feet 4 inches long, so that three covers would measure 7 feet, whereas the group joint only measures 3 feet 4 inches. In the case of the steel joint, with five plates in double shear—

$$N = \frac{0.8bt}{a} = \frac{0.8 \times 22.25}{2 \times 0.6} = 14.8$$

or say 16 rivets on each side of the joint, necessitating cover-plates 3 feet long each, so that five such joints would measure 15 feet, whereas the group joint only measures 9 feet 4 inches.

Joints in angle irons may be arranged as shown in Figs. 266 and 267.

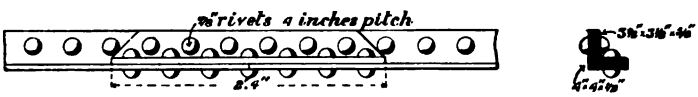


Fig. 266.

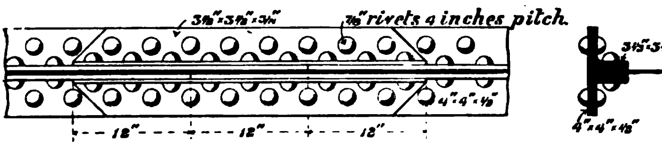


Fig. 267.

In Fig. 266, it is necessary to provide an angle-iron wrapper which shall be equal in area to the main angle iron, thus—

For $4 \times 4 \times \frac{1}{2} = (7\frac{1}{2} - \frac{7}{8})\frac{1}{2} = 3.3$ square inches

„ $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{8} = (6\frac{3}{8} - \frac{7}{8})\frac{5}{8} = 3.4$ „ „

and the number of rivets required on each side of the joint is, since all the rivets are in single shear—

$$N = \frac{1.1 \times 3.3}{0.6} = 6.05, \text{ say 6 rivets}$$

so that the wrapper must be 2 feet 4 inches long.

In Fig. 267 the joints in both angle irons are brought under the same wrappers, so that the area of the two wrappers should be made about 20 per cent. greater than the area of one of the main angle irons. It should be noticed that the rivets through the angle irons and web are in double shear, while those through the flanges are in single shear.

$$\text{Area of 2 angle wrappers, each } 3\tfrac{1}{2} \times 3\tfrac{1}{2} \times \tfrac{3}{8} = 2(6\tfrac{5}{8} - \tfrac{7}{8})\tfrac{3}{8}$$
$$= 4{\cdot}3 \text{ square inches}$$

so that there is an excess of one square inch in area over that of the main angle irons, which is about correct. The rivet area is obviously sufficient for tensile and bearing resistance, and the total weight of covers necessary is less.

The advantages of this joint over the method of providing a separate wrapper for each are: that the centre of gravity does not deviate from the axis of the girder, so that the stresses are uniformly distributed; the smaller wrappers allow the rivet to be closed with a full head, as there is $\frac{1}{4}$ of an inch more space than with the thick single covers, at least half the rivets are in double shear.

Joints between Bracing Bars and Booms.—In riveted girders with parallel, curved, or polygonal booms, it is necessary to so arrange the bars and plates that the vertical or diagonal bars of the web may be conveniently connected to them.

For lattice girders about 80 feet span, the methods illustrated in Figs. 269, 270, and 271 may be conveniently used. T and L irons may be obtained 9 inches deep, and the extra cost more

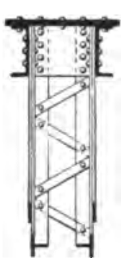


FIG. 268.


FIG. 269.

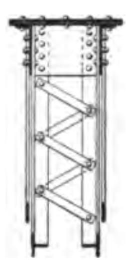


FIG. 270.

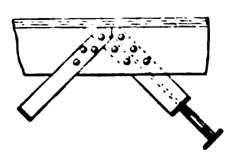


FIG. 271.

than compensates for the convenience in the attachments of the bracing. For girders about 40 feet span, the method illustrated in Figs. 273 and 274 is convenient; it was introduced by Mr. A. M. Rendal for the Warren Girder Bridges, Indian State narrow-gauge railways. The channel iron section shown on Fig. 268 may be used up to about 125 feet. Fig. 272 illustrates a method of forming the booms of a braced girder of from 120 to 200 feet span; the vertical stringer plates may be doubled or made as thick as may be necessary, but they must not be made less than ½ inch thick in any case, as they are liable to twist. Angle irons may be riveted to the bottom of the stringer plate, forming two channel iron sections.

The sections illustrated merely suggest the various ways in which booms of braced girders may be built up of bars and plates in a convenient manner for the attachment of the bracing

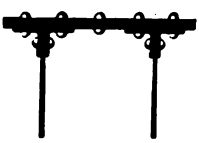


FIG. 272.

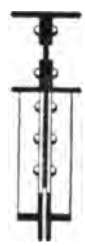


FIG. 273.

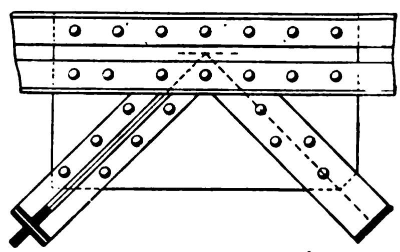


FIG. 274.

bars. Other combinations will suggest themselves. In the bottom boom of a detached girder, for example, a series of vertical plates may be used, retaining one of the designs shown for the top boom. In a continuous girder, in consequence of the reversal of stress which occurs, the bottom boom should be constructed in a similar manner to the top boom.

In designing the attachments of the bracing bars to the booms, the rivets should be symmetrically arranged with reference to the axis of the bracing bars and boom, or the stresses will not be uniformly distributed. Fig. 275 shows a tension bar 6″ × ½″, with 6 rivets 1 inch in diameter, but they are unsymmetrically arranged with reference to the axis of the bar. The centre of gravity of the rivets is ⅜ of an inch from the axis of the bar. If two forces, each equal to the pull upon the bar P, equal say 7½ tons, be supposed to act along the line passing through the centre of gravity of the rivets, they will not disturb the equilibrium, and we have a force P and a couple Px, where $x = \frac{3}{8}$. The force P produces a uniform stress f_1, the intensity of which is—

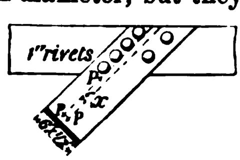


FIG. 275.

$$f_1 = \frac{P}{A} = \frac{7 \cdot 5}{2 \cdot 5} = 3 \text{ tons per square inch}$$

The couple $Px = \frac{2}{3} \times 7\cdot5 = 5$ inch-tons, producing transverse stress upon the bar. If we denote the intensity of stress at the extreme fibres by f_2, we have, since the bar is rectangular, the moment of resistance—

$$\tfrac{1}{6}bd^2f_2 = \frac{6 \times 6}{6 \times 2}f_2 = 3f_2$$

$$\therefore 3f_2 = 3, \text{ and } f_2 = 1 \text{ ton per square inch}$$

Hence the maximum tension produced will be—

$$f_1 + f_2 = 3 + 1 = 4 \text{ tons per square inch}$$

and the maximum compression—

$$f_1 - f_2 = 3 - 1 = 2 \text{ tons per square inch}$$

This unequal distribution of stress is avoided by arranging the rivets symmetrically or by the use of pins, but it is not always possible to obtain perfect symmetry with reference to the line passing through the centre of gravity of the booms. The axes of the bars should intersect on a line parallel to and in the same horizontal plane as the line passing through the centre of gravity of the boom.

Professor Reilly states that "the centre of gravity of the group of rivets must fall on the intersection of the mean fibres of the bars," and he further implies that the first rivet should be on the mean fibre. In Fig. 276, if the ten rivets shown are equal in resistance to the section of the plates less one rivet-hole, the arrangement is as good as it is possible to obtain. Consider the sections a, b, and c. At a the section of the plate is weakened to the extent of one rivet-hole; section b is weakened by two rivet-holes, but the plate cannot tear across through the holes without the rivet on section a shearing at the same time, consequently the shearing resistance of this rivet must be added to that of the plate resistance through b. At section c the plate area is reduced by four rivet-holes, but the two rivets on section b and the one on section a must shear at the same time. If the resistance of the rivets to shearing is equal to the resistance of a strip of plate of a width equal to the sum of the diameters of the rivets, the strength of the joint will be reduced

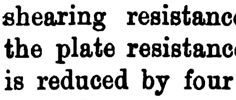


FIG. 276.

by the diameter of one rivet-hole only. The foregoing examples illustrate the principles to be observed in designing the riveted joints and connections in braced girders; other examples are given in the plates.

Eye-bars.—Riveted joints and connections are characteristic of British and European practice in bridge-building, but in America the main connections of tension members, both in the lower chord and diagonal braces, are made by means of pins. The tension members consist of eye-bars, which are designed so that the weakest section is in the bar itself.

Fig. 277 represents a form of hammer-forged eye-bar, which, according to Professor Burr, has stood the test of long American practice. It may be drawn as follows:—

Let w denote the width of the bar, and r the radius of the pin; then make $hb = 0{\cdot}87w$, and abc a semicircle with o_1 as centre and radius $= r + 0{\cdot}66w$; $oo_1 = oo_2$, and the curves ef and

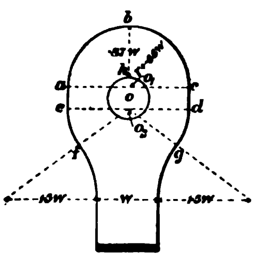
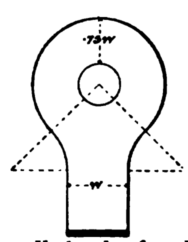
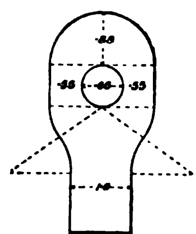


FIG. 277.



Hydraulic forged.

FIG. 278.



Kirkaldy's.

FIG. 279.

dg are portions of a semicircle of the same radius as abc. The curve joining ef and dg to the bar is made so as to give a large amount of metal in the vicinity of o_2f and o_2g.

Fig. 279 shows a form of hammered head based upon experiments made by Mr. Kirkaldy. The head is frequently made circular, as shown in Fig. 278, in which case the eye-bar is forged by hydraulic machinery. If it is desired to thicken the head so as to limit the mean intensity of pressure on the bearing area of the pin, let P denote the safe intensity of pressure; T, the maximum intensity of tensile stress in the bar; t, the thickness of the bar; and t_1, the thickness of the head; then—

$$wtT = 2rpt_1, \text{ and } t_1 = \frac{wtT}{2rp}$$

Size of Pins.—The diameter of the pin should be at least sufficient to resist crushing and to limit the intensity of the pressure on the bearing area of the eye-bar to a safe amount. Let d denote the diameter of the pin, then $tdp = \text{T}tw$.

$$\therefore d = \frac{\text{T}}{p} w$$

The ratio of the safe working intensity of tensile stress, T, to the compressive stress, P, is about $\frac{3}{4}$, so that the smallest permissible diameter of the pin is—

$$d = \tfrac{3}{4} w$$

The smallest diameter usually taken is—

$$d = \tfrac{4}{5} w$$

In Mr. Kirkaldy's eye-bar the diameter of the pin, $d = 0{\cdot}66\,w$, is the smallest which will break the bar. The diameter of the pin is usually greater than this value, as the pull of the eye-bars in the bottom chord and diagonals tend to bend it.

The maximum bending moment on the pin must be found by resolving all the forces which act upon it both horizontally and vertically; thus if M_h denote the sum of the moments of the horizontal forces, and M_v the sum of the moments of the vertical forces, then—

$$\text{M} = \sqrt{\text{M}_h^2 + \text{M}_v^2}$$

$$\text{M} = \frac{f\text{I}}{y}; \text{ which becomes}$$

$$\text{M} = \frac{\pi f d^3}{32} \text{ since the section is circular.}$$

$$\therefore d = \sqrt[3]{\frac{32\text{M}}{\pi f}}$$

The value of f is usually taken as 15,000 lbs. per square inch for iron, and 20,000 lbs. per square inch for steel; so that we have—

$$d = 0{\cdot}089\sqrt[3]{\text{M}} \text{ (iron)}$$
$$d = 0{\cdot}081\sqrt[3]{\text{M}} \text{ (steel)}$$

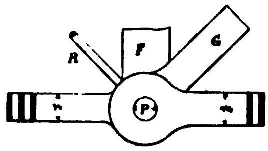
It is usual to select the proper diameter of the pin, when the bending moment M is known, from tables calculated by the two last formulæ.

The bending moment M may be calculated with sufficient accuracy on the assumptions—

(*a*) That the pressure applied to any pin has its centre at the centre of the surface of contact.

(*b*) That the centre of pressure is the centre of such a surface as will reduce the intensity of pressure on the bearing area to its safe maximum value, which for wrought iron is about 12,000 lbs. per square inch, or about one-half of what has been found to be the limit for rivets. The pin does not fit the hole, as there is generally a clearance of $\frac{1}{50}$ of an inch, and the surface over which the pressure is actually exerted increases with the stress in the bar from a line contact to a surface more or less approaching *dt*.

FIG. 280.



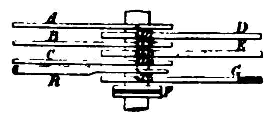


FIG. 281.

Referring to the joint shown in Fig. 281, and denoting the distances between the centres of the eye-bars by x_1, x_2, x_3, etc., and the maximum tensile stress in the various bars by T_a, T_b, T_c, etc., we have—

About the centre of eye-bar D ... $T_a(x_1)$

,, ,, ,, B ... $T_a(x_1 + x_2) - T_d(x_2)$

,, ,, ,, E ... $T_a(x_1 + x_2 + x_3) - T_d(x_2 + x_3) + T_b(x_3)$

,, ,, ,, C ... $T_a(x_1 + x_2 + x_3 + x_4) - T_d(x_2 + x_3 + x_4) + T_b(x_3 + x_4) - T_e(x_4)$, etc.

The stress in the counterbrace R does not usually act when the maximum stresses occur in the pin, and may therefore be neglected. The stress in the diagonal brace G must be resolved both horizontally and vertically, and the moments of the horizontal component about the centre of the post F added to the sum of the moments produced by the eye-bars in the bottom chord about the same point; the result M_h must be added to the moment of the vertical component M_v about the same point, thus—

$$M = \sqrt{M_v^2 + M_h^2}$$

The resultant may also be found thus: Set off Mx to scale representing the moment of the eye-bars in the bottom chord about the centre of the column F, and M_g also to scale in a direction perpendicular to that of the diagonal brace G, representing the moment of the diagonal brace about F. Complete the triangle; then M will represent to scale the magnitude of the greatest moment to which the pin is subjected, and its direction is the axis of the moment. The maximum bending moment in the pins of the lower chord will generally occur when the bridge is loaded completely, but in the upper chord when the bridge is partially loaded.

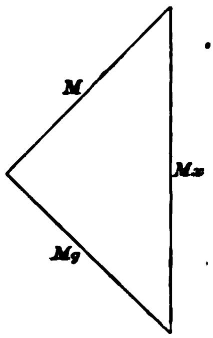


FIG. 282.

The eye-bars should be grouped so that they are symmetrical with reference to the axis of the lower chord, and so that the positive and negative bending moments tend to balance each other as much as possible. It will be observed that if the eye-bar is thickened in order to reduce the pressure on the bearing area, the length of the lever arm is increased with a corresponding increase in the bending moment, necessitating a larger pin. When the size of the pin is sufficient to resist the maximum bending moment, and for bearing area, it will be amply sufficient to resist shearing.

Joints in Iron Roofs.—The student will best understand the method of designing the joints in connection with roof-construction by examining the working drawings of roofs already built, or by inspecting the roofs themselves.

The few examples of joints in roof-construction which can be given in this chapter have been selected from some well-known roofs.

Figs. 283 and 284 illustrate the junction of the tie with the curved bow, and the junction of the strut and tie of the web to the curved bow, in a curved roof over the volunteers' drill-hall at Port Elizabeth. The curved bow and the main tie are each constructed with double angle irons, so that the detail at the ends over the supports consists of a web plate between the angle irons uniting the upper and lower members, and two angle stiffeners uniting the ends to the bed-plate. The junction of the tie and strut to the curved bow is effected by means of a junction-plate inserted between the angles, and extending sufficiently below the bow to allow room for the necessary number of rivets uniting the tie and strut to the junction-plate. These rivets are in single shear, ⅞ of an inch in diameter, but those through the double angles and junction-plate are in double shear.

Figs. 285, 286, 287, and 288 represent the details of two of the joints in the roof principals over the railway station at Penzance. Here the curved upper member is constructed with double angle irons and a stringer plate, and the junction with the main tie is effected by riveting a plate on each side, which extend suffi-

Roof of Volunteer Drill Hall. Port Elizabeth.

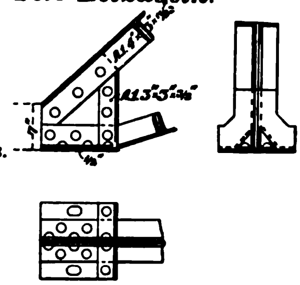


FIG. 283.

Detail at Abutment

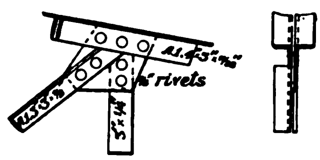


FIG. 284.

Junction of Strut and Tie to Bow.

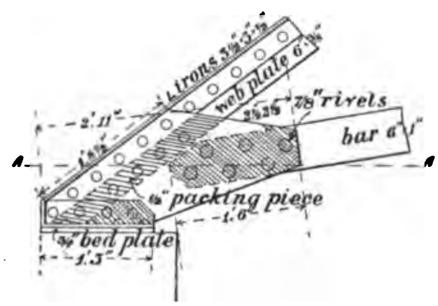


FIG. 285.

Section through AA

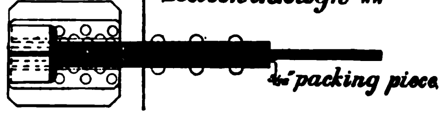


FIG. 286.

Details at Abutment.

Penzance Station Roof, G.W.R.

ciently in the direction of the tie to enclose it between the plates and to give room for six rivets in double shear. It should be noted that packing pieces are used to avoid bending or cranking of the junction-plates; packing pieces are also used at the ends of the bracing at the junction with the main tie, Fig. 288, and in both cases they might have been avoided by riveting reinforcing plates on each side of the tie-rod in Fig. 285, and on each side of the strut and tie in Fig. 287. Packing pieces tend to cause bending in the rivets.

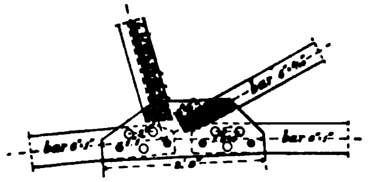


FIG. 287.

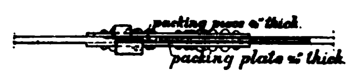


FIG. 288.

Details of Junction of Strut and Tie Rods. Penzance Station Roof. G.W.R.

Fig. 289 represents the attachment of the main tie to the principal rafters in the roof over the railway station at Broad

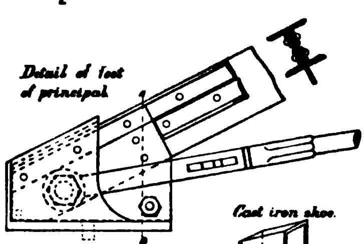


FIG. 289.

FIG. 290.

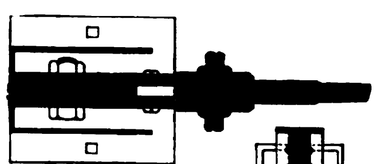
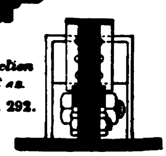


FIG. 291.



Section at aa.

FIG. 292.

Broad St Station, London. N.L.R.

Street, London. In this case a cast-iron shoe is used for receiving the end of the principal, which forms also a bed-plate on the supports. The tie-rod is connected by means of gibs and cotters to the side plates or straps, and the latter are connected to the rafter by means of a turned pin. This adjustable connection allows the tie-bar to be put in initial tension, thereby ensuring its prompt action in resisting the stresses developed in it.

Figs. 293 to 296 illustrate some of the details of the main crescent, or sickle-shaped, truss over the railway station at Lime Street, Liverpool.

The connection of the main tie to the bow, Fig. 293, is effected by means of gibs and cotters, which allow the tie-rod to be initially stressed. The main tie consists of four bars, each

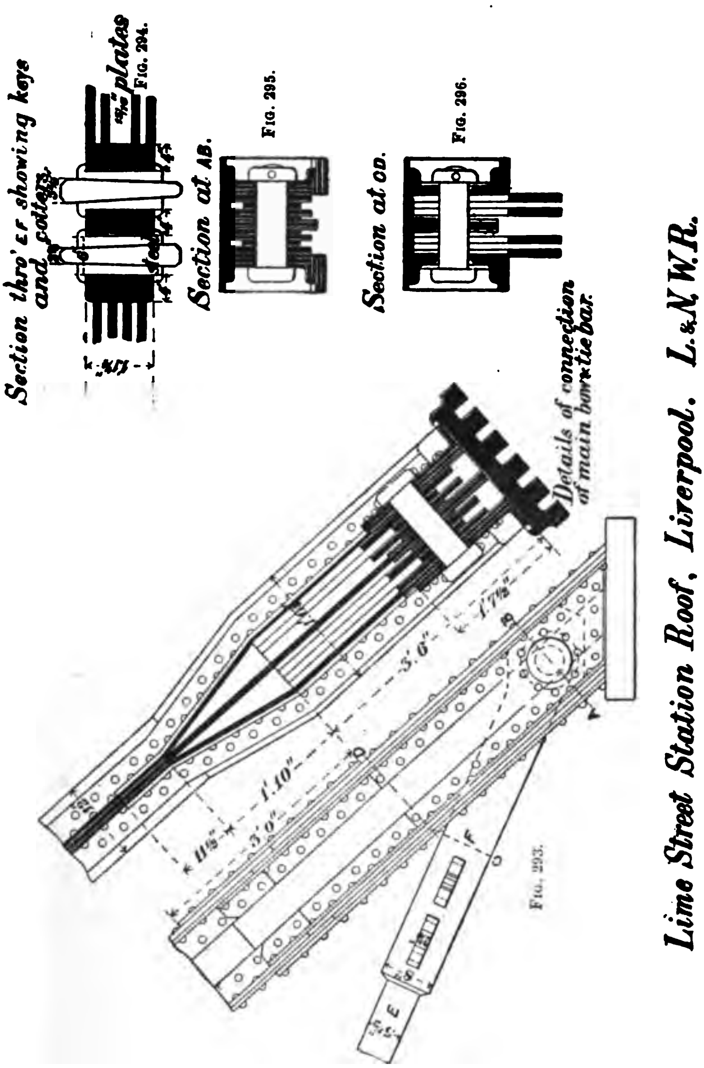


Fig. 294. Fig. 295. Fig. 296. Fig. 293.

$5\frac{1}{2}$ inches by $\frac{15}{16}$ of an inch, and there are four straps 8 inches by $\frac{15}{16}$ of an inch, formed with eye-bars at one end and two rectangular slots at the other. The straps are united to the bow by means of a turned pin, and to the ends of the tie-rods by means of gibs and cotters. The main bow is constructed with four angles, web and flange plates of the ordinary plate web girder section, excepting at the ends, which are widened out in the manner shown in Fig. 293, to give room for the tie-bars and straps.

Lime Street Station Roof.
Liverpool. L.&N.W.R.

Fig. 297.

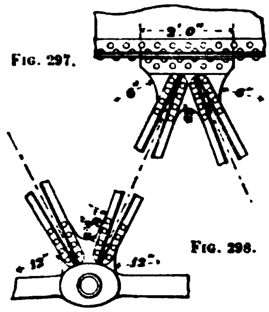


Fig. 298.

Connection of centre braces to bow and tie bars.

The detail is fully illustrated in the various figures, and needs no further explanation. Figs. 297 and 298 illustrate the connection of the centre braces to the bow and tie-bars. The braces are all constructed of four angles, splayed in the centre to a parabolic form, and they are united, as shown, to junction plates, which latter are attached to the bow by means of double angle irons and to the tie-bars by means of turned pins.

Fig. 299. Fig. 300. Fig. 301. Fig. 302.

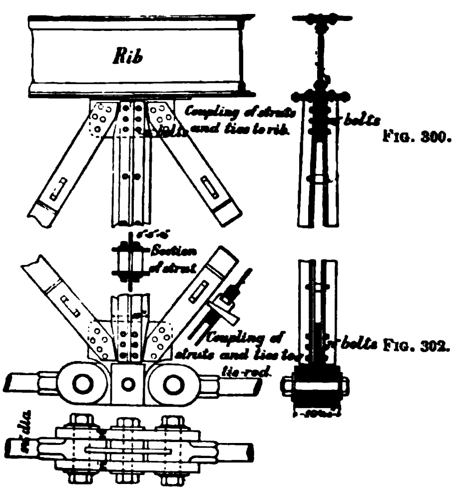


Fig. 303.

Cannon Street Station. S.E.R.

Figs. 299 to 303 illustrate the methods adopted for connecting the bow and main tie to the compression and tension members of the bracing in the roof over the railway station at Cannon Street, London. The roof is similar in form to that at

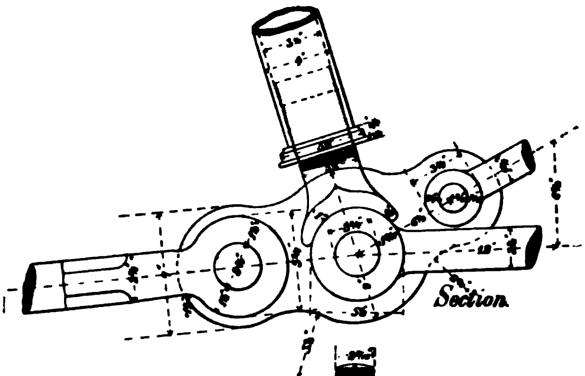


FIG. 304.

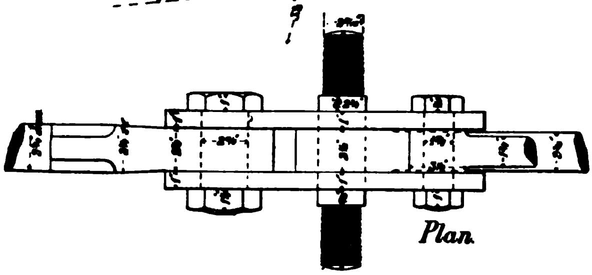


FIG. 305.

Foot of Main Strut.
St David's Station, Exeter.

Lime Street, Liverpool, but the web bracing is redundant and the tension members are made adjustable.

Figs. 304 and 305 represent the detail at the foot of the main strut in the roof over St. David's Station at Exeter. Here the strut is made adjustable by means of the screwed connection with the main tie.

Figs. 306, 307, and 308 represent the junction of strut and tension rods to the upper member of the truss and the main tie in the roof over the Junction Station at Bristol. Here the strut is made adjustable by means of screwed connections at each end, and the main tie and tension braces are provided with union screws.

Joints in Timber.—Figs. 309 and 310 represent the plan and elevation of a tension joint in timber, consisting of two cover-plates notched into the beam at each end and bolted as shown.

The beam is weakened to the extent represented by the reduction of area due to the notching, and the length of the

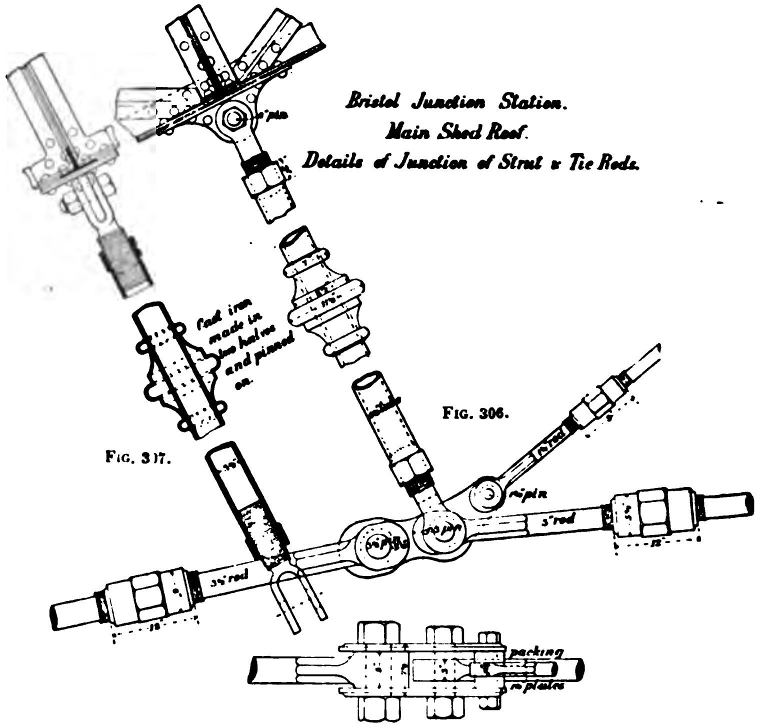


FIG. 317.

FIG. 306.

FIG. 308.

notches on either side of the joint must be such that the shearing resistance will be equal to the tensile resistance of the beam at the reduced section, neglecting the resistance of the bolts.

FIG. 309.

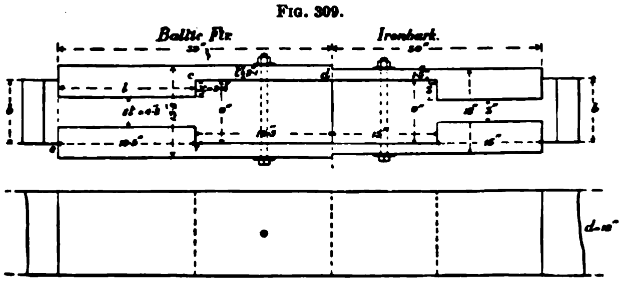


FIG. 310.

Clearly the thickness of the two cover-plates in the centre must equal the reduced thickness of the beam if the same material is used for both, which is indicated in the figure by t and $2t$. Again, the shearing areas *cd* and *ef* must be equal. The depth of the two notches must be sufficient to offer a crushing resistance equal to the tensile resistance of the reduced area of the beam.

Assuming the following data for the intensity of working stress in pounds per square inch for Baltic fir and ironbark, we may dimension the right-hand side of Fig. 309 for the latter material, and the left-hand for the former.

Tension	for Baltic fir,		1300,	for Ironbark,		4000.
Compression	,,	,,	1100,	,,	,,	2000.
Shearing	,,	,,	140,	,,	,,	400.
Bearing	,,	,,	1300,	,,	,,	4000.

Let b = breadth of beam.
d = depth ,, ,,
l = length of notch.
x = depth of notch cut in beam.
t = thickness of cover at the centre.
$2t$ = thickness of reduced section of beam.

If we take the depth of the beam 12 inches, and the breadth 9 inches, we have for Baltic fir—

$$2t = b - 2x$$
$$\therefore 2t = 9 - 2x, \text{ and } t = 4{\cdot}5 - x$$
$$\therefore t = \tfrac{11}{13}x = 4{\cdot}5 - x$$
since $1300t = 1100x$
$$\therefore x = 2{\cdot}4 \text{ inches, and } t = 2{\cdot}1 \text{ inches, and } 1300t = 104l$$
$$\therefore t = \tfrac{14}{130}l = 2{\cdot}1l$$
$$\therefore l = 19{\cdot}5 \text{ inches}$$

For ironbark—

$$t = 4{\cdot}5 - x \text{ as before}$$
but $4000t = 2000x$
$$\therefore t = \frac{x}{2} = 4{\cdot}5 - x$$
$$\therefore x = 3 \text{ inches, and } t = 1{\cdot}5 \text{ inches, } 4000t = 400l$$
$$\therefore t = \frac{l}{10} = 1{\cdot}5$$
$$\therefore l = 15 \text{ inches}$$

In both cases the strength of the joint is much less than the beam, in consequence of the reduction of area due to notching.

The Baltic-fir joint has an efficiency of 47 per cent. nearly.
The Ironbark ,, ,, ,, 34 ,, ,,

The joints shown in Fig. 311 would naturally suggest themselves, in which the crushing and shearing resistances are the same as in Fig. 309.

Neglecting the resistance of the bolts, the dimensions may be as shown—

Efficiency of Baltic-fir joint = 82 per cent.
Efficiency of Ironbark joint = 77·7 per cent.

If the bolts are considered we should not have to cut the

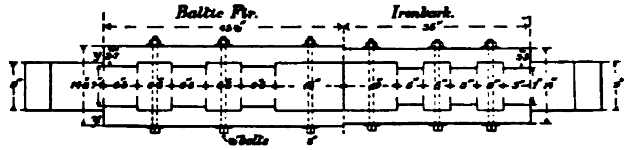


FIG. 311.

notches so deeply, or the efficiency would not be increased, but the length of the notches and spaces between them would not require to be so great.

The objection to this and the preceding joint is that the timber is notched at right angles to its length, rendering it likely to fracture at *yy* with a lower stress than the reduced area would resist under favourable circumstances, so that the efficiencies given above are probably too great.

Fig. 312 shows a modification of the joint shown in Fig. 311, which may be taken to have the efficiencies above stated, as it is not so likely to fail in consequence of sharp corners.

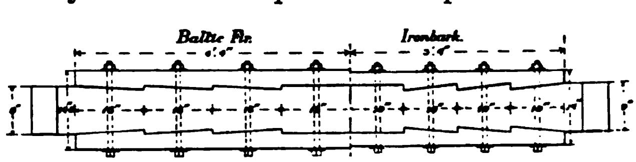
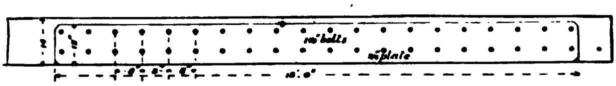


FIG. 312.

Figs. 313 and 314 illustrate a form of joint used in Australia in the bottom chords of timber truss bridges, which are frequently constructed of ironbark timber. The form of the truss is indicated in Fig. 168, Chapter VIII., and the joint under consideration occurs in the third and sixth bays. The chord is constructed with four beams, each 4½ inches wide by 14 inches deep, bolted together with bolts 1 inch in diameter, arranged in a zigzag form, 10 bolts being used in each bay. The apex loads and stresses have been considered in Chapter VIII.

The joint is designed for a maximum tensile stress of 109·1 tons, and the chord for this stress, as well as for the transverse

FIG. 313.



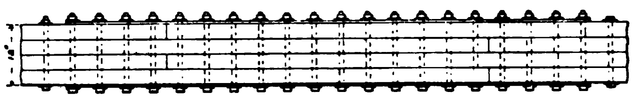


FIG. 314.

stresses due to the loads discharged by the cross-beams between the panel-points. The dimensions and arrangements of the bolts and cover-plates shown in Figs. 313 and 314 have been arrived at after a series of experiments on full-sized joints of similar construction. In all cases the joints experimented upon failed by the bending of the bolts and the crushing of the fibres of the timber on the bearing area, and the following safe working stresses per bolt were deduced :—

Bolts 1⅛ inch in diameter—1·85 ton stress in single and 3·7 tons in double shear.

Bolts 1 inch in diameter—1·34 ton stress in single and 2·68 tons in double shear.

In this way the strength of the joint shown in Figs. 313 and 314 may be estimated by considering the strength of the bolts thus—

$$16 \text{ bolts } 1\tfrac{1}{8} \text{ inch in diameter in single shear} = 16 \times 1{\cdot}85 = 29{\cdot}6 \text{ tons}$$

$$24 \text{ bolts } 1\tfrac{1}{8} \text{ inch in diameter in triple shear} = 3 \times 24 \times 1{\cdot}85 = 133{\cdot}2 \text{ tons}$$

The total strength of the joint, neglecting the ¾-inch cover-plates, is 162·8 tons. The joint is therefore excessively strong.

Figs. 315 and 316 illustrate another form of joint used in the bottom chords of trusses indicated in outline in Fig. 169. In this case the chord is made up of two beams, each 12 inches deep by 6 inches wide, spaced.

There are two strips on each cover-plate, 2 inches long by 1 inch wide by 12 inches deep, which fit into corresponding notches cut in the timber. The maximum tensile stress in this case is 53·04 tons.

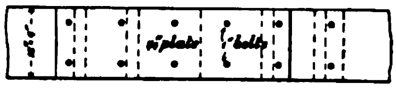


FIG. 315.

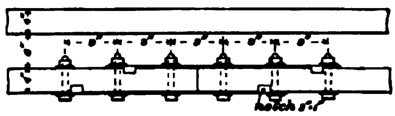


FIG. 316.

The strength of this joint may be considered with reference to the bolts on one side of it, and the crushing strength of the timber at the notches, thus—

Strength of two notches in half-chord = 2 × 1″ × 12 × 2000
= 48,000 lbs. = 21·4 tons
4 bolts in double shear 1 inch diameter = 10·72 ,,
2 ,, single ,, 1 ,, ,, = 2·68 ,,

Total strength of half-chord = 34·80 tons.
Total ,, whole ,, = 69·6 ,,

This joint is therefore abundantly safe, since the maximum tensile stress is only 53·04 tons. There is an advantage in regard to the number of notches; two surfaces may be arranged to bear equally, whereas with a greater number it is practically impossible to obtain the same bearing pressure on each notch. The shearing resistance along the fibre should be at least equal to the crushing resistance, but this does not arise in the joint under consideration, as the area of the timber between the joint and the notch multiplied by the shearing strength is obviously greater than the area subjected to crushing multiplied by the crushing strength.

Again, the area of the timber in tension is only slightly reduced by the notch and the bolts, and need not be considered, as it is obviously excessive.

CHAPTER XIV.

DECKS OF IRON AND STEEL BRIDGES.

THE deck of a bridge usually consists of cross-girders, sometimes called floor beams, and longitudinals, sometimes termed stringers. The cross-girders are spaced at the same distance apart, centre to centre, as the apices of the triangles in lattice girders, or at the panel-points of trusses. In a railway bridge, the sleepers, rails, guard rails or timbers rest directly upon the stringers, and these discharge their load upon the cross-girders, which in like manner discharge their load upon the apices or panel-points of the main girder or truss.

The weight of rails, guard rails, spikes, sleepers, etc., in railway bridges for heavy traffic may be taken as 400 lbs. per lineal foot of bridge for each line of way.

The stringers in truss bridges are usually constructed as plate web girders, having an effective depth of from $\frac{1}{9}$ to $\frac{1}{12}$ of the span; but in lattice girders, where the distance between the apices is much smaller than in truss bridges, rolled iron or steel girders are frequently used. Stringers of iron or steel are spaced from 6 to 8 feet centres transversely to the bridge. Where timber is cheap, timber stringers consisting of two or more groups of beams are spaced 5-feet centres, so that the middle of each group is immediately under the rails.

Timber stringers usually rest on the top flanges of the cross-girders, but iron or steel stringers may rest in a similar manner upon the top flanges of the cross-girders, or be built into them (see Plate II.).

The cross-girders are usually constructed of plate web girders, and are riveted to the vertical compression members in truss bridges, or suspended by means of hangers. In girders with trough-shaped bottom booms, the cross-girders are attached immediately above or below the apices of the triangulation, or panel-points.

The open deck illustrated in Fig. 113 is commonly used in America, but occasionally in England, and more frequently in the colonies. The deck is formed (if of timber) by means of planks from 3 to 4 inches thick, laid diagonally or transversely. Upon this floor is laid the ballast and sleepers as a loose road, which is maintained in the same manner as the permanent way on the rest of the railway. In many English railway bridges, Mallet's buckle plates or cambered plates of iron or steel are riveted to the cross or longitudinal girders, forming a continuous metal floor, upon which the ballast, sleepers, and rails are laid as a loose road as before. There are various systems of flooring suitable for bridges, such as Hobson's

Diagonal Plate System (Steel).

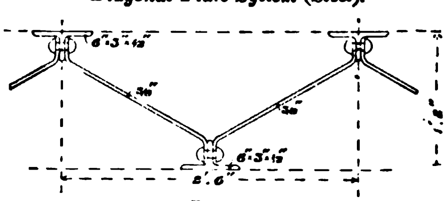


FIG. 317.

TABLE XLVII.

DIAGONAL PLATE SYSTEM (STEEL).

Span in feet.	Breaking weight in tons. Ends supported.	Ends fixed.
20	74	112
22	67	101
24	62	93
26	57	86

Arched Plate System (Steel).

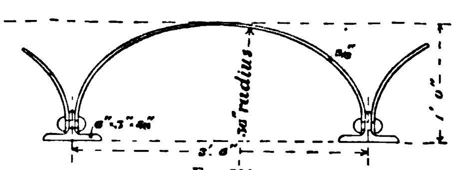


FIG. 318.

TABLE XLVIII.

ARCHED PLATE SYSTEM (STEEL).

Span in feet.	Breaking weight in tons. Ends supported.	Ends fixed.
16	69	104
20	55	83
22	50	76
24	46	69
25	44	67

patent flooring, shown in Figs. 317 and 318. Fig. 319 is a trough section manufactured by Messrs. Braithwaite and Kirk. Lindsay's patent flooring may be used in a similar manner; also Westwood and Baillie's corrugated flooring.

All the forms of bridge floors illustrated in Figs. 317, 318, and 319 may be used without either cross-girders or longitudinals, provided the height and pitch of corrugations or troughs, and the thickness of metal, are proportioned with regard

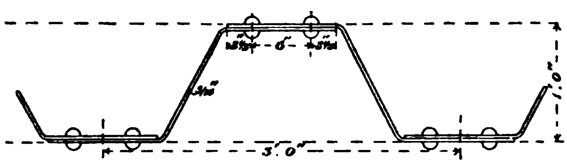


FIG. 319.

to the maximum concentrated wheel loads. The strength of the troughing when connected together in a bridge floor is very great, as each trough acts as a girder; again, a load concentrated on one trough would be partly borne by adjacent troughs.

Figs. 317 to 319 are very suitable for bridge floors where the headway is limited, as the sleepers lie in the troughs, well supported by ballast. All these forms may be obtained of various sizes and thicknesses of metal in iron and steel, and they are largely used for bridge decks. Fig. 319 is shown on Plate V., as applied to a highway bridge.

The decks of bridges are very varied, and must be determined with reference to the locality and nature of the traffic. In towns the standard form of street-paving will influence the design of the decks of the bridges. For country bridges the decks may be formed as illustrated in Fig. 108 and Plates III. and V.

The deck of a bridge forms an invariable portion of the dead load, and is independent of the span of the main girders of trusses.

To design the floor of a bridge consisting of an American deck resting upon stringers 20 feet long, measured from centre to centre of the cross-girders. The cross-girders are attached to the panel-points of the main trusses, and have an effective span of 15 feet. Let the four driving-wheels of the consolidation locomotive, Fig. 326, be arranged symmetrically on the longitudinal stringers, so as to produce the maximum bending moment (Fig. 320).

The dead load, consisting of rails, sleepers, spikes, guard rails, etc., as shown in Fig. 113, may be taken as 400 lbs. per lineal foot of bridge, or 200 lbs. on each stringer. The weight of the stringer itself may be assumed to be 110 lbs. per lineal foot; so that the total dead load is—

$$(110 + 200)20 = 6200 \text{ lbs.} = \text{(say) } 3 \text{ tons}$$

The maximum bending moment due to the dead load is—

$$\frac{Wl}{8} = \frac{3 \times 20}{8} = 7{\cdot}5 \text{ foot-tons}$$

The maximum bending moment due to the live load is—

$$12(2{\cdot}625) + 6(5{\cdot}75) = 66 \text{ foot-tons}$$

The total bending moment is—

$$66 + 7{\cdot}5 = 73{\cdot}5 \text{ foot-tons}$$

If the material is steel, the working stress by the new French formula is—

$$5{\cdot}08\left(1 + \frac{1}{2}\cdot\frac{\text{min.}}{\text{max.}}\right) = 5{\cdot}08\left(1 + \frac{1}{2}\cdot\frac{7{\cdot}5}{73{\cdot}5}\right) = 5{\cdot}33 \text{ tons}$$

This working stress is high for longitudinals, and the formula appears to give greater values than would be allowed in the

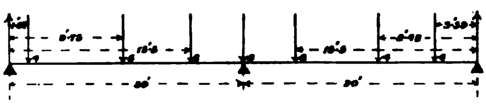


FIG. 320.

best American practice where the impact is considerable. We will therefore adopt 4·5 tons as the working stress.

Let the effective depth be taken as $\frac{1}{10}$ of the span, or 2 feet, then the moment of resistance is—

$$fad = 4{\cdot}5 \times 2 \times a = 9a \text{ foot-tons}$$

$$\therefore 9a = 73{\cdot}5, \text{ and } a = \frac{73{\cdot}5}{9} = 8{\cdot}2 \text{ square inches}$$

Two angles $5 \times 3\frac{1}{2} \times \frac{5}{8}$ may be used both for the top and bottom flanges; thus in the bottom flange the effective area is $2(7\frac{7}{8} - \frac{7}{8})\frac{5}{8} = 8{\cdot}75$ square inches, which gives the necessary area.

Rivets.—Assume that the rivets are $\frac{7}{8}$ inch diameter and 4 inches pitch; there will be, therefore, 3 per foot, each 0·6 square inches area, or a total of 1·8 square inch. The maximum shearing stress will occur at the points of attachment of the longitudinals to the cross-girders when the live load is so distributed that the first driving-wheel is close to the cross-girder; the shearing stress will then be about 14 tons, or 7 tons per foot horizontally and vertically. Let f denote the intensity of stress upon the rivets; then—

$$1{\cdot}8f = 7, \text{ and } f = 3{\cdot}9 \text{ tons per square inch}$$

The rivets in question are in double shear, hence this stress is very safe for shearing.

The pressure on the bearing area may be denoted by p; then, if we have a $\frac{3}{8}$-inch web plate—

$$p \times 3 \times \tfrac{7}{8} \times \tfrac{3}{8} = 7, \text{ and } p = 7 \text{ tons}$$

which is safe for steel, hence $\frac{7}{8}$-inch rivets 4 inches pitch are sufficient.

Thickness of the Web Plate.—The method of determining the thickness of the web plate must take into consideration the maximum intensity of shearing stress and the tendency to buckle. It has been shown that the intensities of shearing stress on two planes at right angles to each other are equal, and it is usually assumed that the web resists the whole of the shearing stress in a plate web girder, and that this shearing stress is equally distributed over the depth of the web.

If a series of planes inclined at 45° be drawn so as to divide the web plate into a series of strips of one inch wide, then each of these strips may be treated as a column, the length of which is equal to the effective depth of the girder multiplied by the secant of 45°. The load upon these elementary columns is the mean shearing stress acting over an area of 1 inch by the thickness of the web.

Let l = the length of the column $= d \sec 45°$.

t = the thickness of the web.

f = a constant for the material = 16 tons for wrought iron or steel.

a = a constant depending on the section and method of fixing $= \frac{1}{3000}$ for rectangular sections.

b = the buckling stress per square inch.

Then, by Rankine's and Gordon's formula—

$$b = \frac{f}{1 + a\left(\frac{l}{t}\right)^2} = \frac{16}{1 + \frac{1}{3000}\left(\frac{l}{t}\right)^2}$$

$$l = 24 \times 1{\cdot}414 = 33{\cdot}936 = 34 \text{ (say)}$$

Let $t = \frac{3}{8}$.

$$\therefore \frac{l}{t} = \frac{34}{\frac{3}{8}} = 90{\cdot}7$$

$$\text{and } \left(\frac{l}{t}\right)^2 = 8226{\cdot}49$$

$$b = \frac{16}{1 + \frac{8226{\cdot}5}{3000}} = \frac{480000}{112265}$$

$$= 4{\cdot}2 \text{ tons per square inch}$$

The intensity of shearing stress per square inch is—

$$\frac{7}{12 \times \frac{3}{8}} = 1{\cdot}56 \text{ ton}$$

Hence the factor of safety against buckling is—

$$\frac{4{\cdot}2}{1{\cdot}56} = 2{\cdot}9 \text{ nearly}$$

In Mr. Theodore Cooper's standard specifications for iron and steel railroad bridges and viaducts, the webs of plate web girders are referred to as follows: "The webs of plate web girders must be stiffened at intervals of about the depth of the girder wherever the shearing strain per square inch exceeds the strain allowed by the following formula:—

$$\text{Allowed shearing stress in pounds} = \frac{12000}{1 + \frac{H^2}{3000}}$$

where H = ratio of depth of web to its thickness; but no web plate shall be less than $\frac{3}{8}$ inch thick.

Applying this formula to the foregoing example—

$$\left.\begin{array}{l}\text{Allowed shearing stress}\\ \text{in pounds}\end{array}\right\} = \frac{12000}{1 + \frac{1}{3000}\left(\frac{24}{\frac{3}{8}}\right)^2}$$

$$= \frac{36000000}{7096} = 5073$$

$$= 2{\cdot}22 \text{ tons per square inch}$$

So that the shearing stress in the $\frac{3}{8}$ web plate is less than this amount, and it is therefore safe.

A common rule in America is to put in stiffeners when the intensity of shearing stress exceeds 4000 lbs. per square inch.

The function of the stiffener is to stiffen the web by supporting the elementary columns and preventing them from buckling. They should be spaced at intervals not exceeding the depth as the girder, and usually consist of double angles riveted to the web plate.

There will be no necessity for stiffeners in the longitudinal stringer.

The weight may now be calculated thus :—

88 feet of angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$ at 16·7 lbs. per lineal foot ...	1470 lbs.
20 feet of web plate $28'' \times \frac{3}{8}''$ at 35 „ „ ...	700 „
Total weight less rivet heads	2170 „
Add 4 per cent. for rivet heads	87 „
Total weight	2257 „

The weight per foot run is $\frac{2257}{20} = 112{\cdot}85$ lbs., which is very slightly above the assumed weight of 110 lbs. per lineal foot, so that no recalculation is necessary. This longitudinal is illustrated on Plate II.

Cross-Girders or Floor Beams.—The dead load upon the cross-girders is the weight of the rails, guard rails, sleepers, and longitudinal stringers, also the weight of the cross-girder. It will be most convenient to consider both the live and dead load as concentrated at the points of attachment of the stringers; hence we have the load discharged by each stringer—

$$(200 + 113)20 = 6260 \text{ lbs.}$$

Assume that the cross-girder weighs 150 lbs. per foot, therefore the load concentrated at the points of attachment of the stringers will be, since the cross-girders are 15 feet span—

$$5\tfrac{1}{2} \times 150 = 825 \text{ lbs.}$$

Therefore the total concentrated dead load is—

$$6260 + 825 = 7085 \text{ lbs.} = 3{\cdot}16 \text{ tons}$$

To find the maximum live load upon the cross-girders, we must find the maximum reaction produced by the engine wheels

R

on the two longitudinal stringers on each side of the cross-girder. It can easily be proved that the maximum reaction will occur when the sum of the loads on the left of the cross-girder is as nearly as possible equal to the sum of the loads on the right. Hence with the consolidation engine, Fig. 326, the fourth wheel will rest immediately over the cross-girder, Fig. 321.

The reaction on the middle cross-girder from the loads on the left is—

$$R_1 = \frac{1}{20}\{4(1·67) + 6(9·75) + 6(15·5)\} = 7 \text{ tons}$$

The reaction from the loads on the right is—

$$R_2 = \frac{1}{20}\{4(3·59) + 4(8·42) + 6(15·5)\} = 7·9 \text{ tons}$$

The total reaction which gives the maximum live load on the cross-girder is—

$$R = R_1 + R_2 + 6 = 7 + 7·9 + 6 = 20·9 \text{ tons}$$

Hence the girder is loaded as shown in Fig. 321.

The maximum bending moment is—

$$24 \times 4 = 96 \text{ foot-tons}$$

The working stress may be taken the same as in the longitudinal, viz. 4·5 tons per square inch, as the same remarks apply here as in the longitudinals with regard to impact.

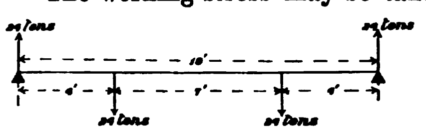


FIG. 321.

The total depth should be arranged so as to reduce the flange section, and leave room for the attachment of the stringers to the web plate direct without packing pieces; 3 feet effective depth will be convenient.

The moment of resistance is—

$$fad = 4·5 \times 3 \times a = 13·5a$$
$$\therefore 13·5a = 96$$
$$\therefore a = \frac{96}{13·5} = 7·1 \text{ square inches}$$

Hence the flanges may be made in a similar manner to those of the longitudinal stringer, but with angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$, thus—

$$\text{Two angles } 2(8 - \tfrac{7}{8})\tfrac{1}{2} = 7·125 \text{ square inches}$$

Rivets.—Assume that the rivets are $\frac{7}{8}$ of an inch in diameter and 4 inches pitch as before, then—

$$1{\cdot}8f = \tfrac{24}{3}$$
$$\therefore f = 4{\cdot}5 \text{ tons per square inch}$$

which is safe, since the rivets are in double shear. The pressure on the bearing area is—

$$p \times 3 \times \tfrac{7}{8} \times \tfrac{3}{8} = \tfrac{24}{3}$$
$$\therefore p = 8{\cdot}1 \text{ tons per square inch}$$

which is safe for steel.

Thickness of Web.—It now remains to be seen whether the web is thick enough to resist buckling.

The maximum intensity of shearing stress is—

$$\frac{24}{3 \times 12 \times \frac{3}{8}} = 1{\cdot}77 \text{ ton per square inch}$$

which is safe, according to Mr. Cooper's rule, without stiffeners; the attachment of the stringers will, however, considerably stiffen the web over the 5-feet panels, while the middle panel is hardly stressed at all.

If we apply Rankine's formula, the factor of safety against buckling is about 1·7. This factor is very small, but when the assumptions are considered upon which the buckling stress is calculated it is clear that the factor of safety is understated, and therefore the $\frac{3}{8}$-inch web may be adopted.

The weight of the cross-girders may now be calculated—

72 feet of angles 5 × $3\frac{1}{2}$ × $\frac{1}{2}$ at 13·9 lbs. per foot	1001 lbs.
15 „ web plate $39\frac{1}{2}''$ × $\frac{3}{8}''$ at 50 „ „	750 „
4 „ angles to support stringer, at 50 lbs.	50 „
	1801 „
Add 4 per cent. for rivets	72 „
Total weight of cross-girder	1873 „

or 125 lbs. per foot run, which is considerably below the assumed weight of 150 lbs. per foot, so that no recalculation is necessary.

The weight of the stringers and cross-girders per lineal foot of truss is 314 lbs.

The web plate between the stringers may be replaced with two angles 3 × 3 × $\frac{3}{8}$, as the shearing stress is practically nothing. If this is done, the weight per foot run of truss is 801 lbs.

In the cross-girder shown on Plate II., for which the above calculation was used, the thickness of the web is increased, as the cross-girders act as compression members in the lower lateral system of wind bracing. For the same reason the plate is retained throughout the web, the central 7 feet not being replaced with angles.

The open deck which we have investigated would not be suitable for a bridge over a street in a town, and some form of continuous deck would have to be adopted; this would, however, add considerably to the dead load to be carried.

In the case of a floor of ⅜ plates riveted to the top flanges of the cross-girders and longitudinals, if these are used, we should have for the plate floor, ballast, sleepers, and rails about 1400 lbs. per lineal foot of bridge instead of 400 with the American deck.

Sir B. Baker estimates[1] the weight of the deck and wind bracing for a double-line railway bridge as follows:—

Table XLIX.

Span in feet.	Dead load per foot run exclusive of main girders.		
	When there are two main girders.	When there are three main girders.	When there are four main girders.
	cwt.	cwt.	cwt.
10 to 100	14	12	10
100 to 150	15	13	11
150 to 200	16	14	12
200 to 250	17	15	13
250 to 300	18	16	14

The weight of the deck in a road bridge is easily estimated when the design and live load to be carried is decided upon.

[1] "Long Span Bridges," by Sir B. Baker.

CHAPTER XV.

WEIGHT OF MAIN TRUSSES AND GIRDERS. LIVE LOADS UPON BRIDGES.

In a single-line through bridge of from 100 to 250 feet span with an open deck, as shown in Plate II., the tension members consisting of eye-bars with pin connections, the weight per lineal foot may be assumed, for the purposes of calculation, to be two and a half times the span in feet for each truss.

In a double-line the weight of each truss in pounds per lineal foot will be about five times the span in feet.

If the bridge is a deck bridge, carrying the load upon the top of the main trusses, with no cross-girders or stringers, the weight per lineal foot will be about 120 lbs. less than a through bridge of the same span.

The weights of stringers and cross-girders may generally be assumed, for the purposes of calculation, with sufficient accuracy after a little experience; but the following formula, proposed by Professor Unwin, may be used both for these girders and the main girders constructed of plate webs or lattice bars :—

Let W_1 = the weight of the girder.
W = the total equivalent uniformly distributed load.
l = the effective span.
r = the ratio of span to effective depth.
c = a constant.
s = the intensity of working stress per square inch.
a = the mean gross area of both booms or flanges in square inches.

Then—

$$W_1 = \frac{Wlr}{cs - lr}$$

To find c from girders of known weight—

$$c = \frac{4al}{W_1}$$

Examples of this formula are given in connection with the design of plate web-girder bridges.

This formula is based upon the assumption that two-thirds the weight of a well-designed girder is in the flanges, and that the weight is proportional to its length and the area of the central cross-section of the girder.

The tables on pp. 247, 248 have been compiled from the quantities of material in some highway bridges, which have been designed to carry a live load of 84 lbs. per square foot of deck as well as the traction engine shown in Figs. 322 and 323.

Live Loads upon Bridges.—In a highway bridge, the greatest live load is generally produced by a dense crowd of people, which may be taken to weigh about 140 lbs. per square foot. Such a dense crowd will rarely ever extend over the whole of the bridge excepting in towns, and the live load usually provided is somewhat as follows :—

TABLE L.

Span in feet.	Live load in pounds per square foot.
0 to 30	100 to 120
30 „ 50	90 „ 110
50 „ 100	80 „ 100
100 „ 200	60 „ 80
200 „ 500	40 „ 75

Mr. Theodore Cooper recommends[1] that for city and suburban bridges.

Class A.—" A load upon each square foot of floor, including foot-walks, of 100 lbs. for all spans up to 100 feet; of 80 lbs. for all spans over 200 feet; and proportionately for intermediate spans.

" Or a steam-roller load of 15 tons, arranged as follows :— 6 tons on forward axle, and 9 tons on rear axle; axles 11 feet apart; rollers 20 inches wide, the two on forward axle placed 2 feet 6 inches centre to centre, and the two on rear axle 6 feet centre to centre."

Class B.—" Bridges liable to the passage of excessive loads from quarries or special manufactories.

[1] "General Specifications for Iron and Steel Highway Bridges."

TABLE LI.

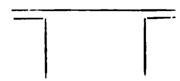
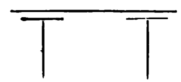
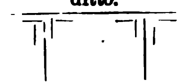
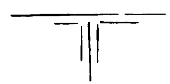
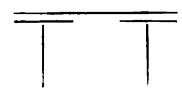
Description of bridge.	Effective span of main girders in feet.	Effective depth of main girder in feet.	Transverse distance between centres of main girders in feet.	Clear width of deck in feet.	Section of booms or flanges.	Total weight of one main girder in tons.	Weight of main girder per foot run in tons.	Value of the constant C in formula $c = \frac{4dl}{W_1}$	Total dead load per square foot of deck in tons.	Spacing of cross-girders in feet.	Weight of one cross-girder in tons.
Independent lattice girders.	60	5·0	19·0	18·0		6·0	0·100	1344	0·6850	5·0	0·500
"	80	8·0	21·7	18·3		13·3	0·166	1313	0·8870	8·0	0·981
"	90	10·0	21·7	18·3	ditto.	14·3	0·159	1329	0·9410	10·0	1·070
"	110	10·0	21·7	18·3	ditto.	21·7	0·197	1362	0·0907	10·0	1·066
"	126	10·6	21·0	18·0		26·0	0·206	1403	0·0820	5·0	1·190
Plate web girders	58	5·0	20·7	18·0		6·6	0·113	1076	0·0680	Troughing.	—
"	40	3·0	22·0	20·0	ditto.	4·0	0·100	1392	0·0826	4·0	0·890
"	70	5·0	20·7	18·0	ditto.	9·8	0·140	980	0·0453	7·0	0·910

TABLE LII.

Description of bridge.	Effective span of main girders in feet.	Effective depth of main girders in feet.	Transverse distance between centres of main girders in feet.	Clear width of deck in feet.	Section of booms or flanges.	Weight of main girder per foot run in tons.	Total dead load per square foot of deck in tons.	Spacing of cross-girders in feet.	Weight of one cross-girder in tons.
Continuous lattice girders of three spans averaging 154 ft. each.	140, 182, 140	10·0	21·0	18·0		0·193	0·0516	7·0	0·932
Continuous lattice girders of three spans averaging 121·3 feet.	112, 140, 112	9·0	21·0	18·0	ditto.	0·175	0·0497	7·0	0·964
Continuous lattice girders of three spans averaging 100·3 feet.	91, 119, 91	8·0	23·0	19·5	ditto.	0·176	0·0869	7·0	1·011

"A load upon each square foot of floor, including foot-walks, of 80 lbs. for all spans up to 100 feet; of 60 lbs. for all spans over 200 feet, and proportionately for intermediate spans.

"Or a concentrated load of 8 tons on two pairs of wheels, 8 feet centres."

Class C.—"Country highway bridges.

"The same loads per square foot of floor as for Class B., or a concentrated load of 5 tons on two pairs of wheels 8 feet centres."

Figs. 322 and 323 show the wheel loads produced by a traction engine, which highway bridges may have to carry, and it should

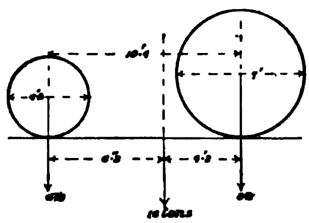


FIG. 322.

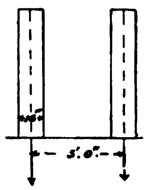


FIG. 323.

be considered as passing over the bridge close to the kerb for stresses in the main girder, and in the centre for the stresses in the cross-girders.

In railway bridges it is the common practice in England to estimate the live load in tons per foot run, which would produce the same stresses as those occurring when the heaviest engines and trains are passing over the structure.

The same practice obtains to a large extent in America, where the standard train used consists of two consolidation engines of the heaviest type, followed by the heaviest train in use on the line.

The concentrated-load system consists in finding the position of the wheel loads of the two engines and the uniform load behind them which produce the maximum stress in the particular member of the structure under consideration, and then to calculate the stress on the member for this position of the wheel loads and train. The process is repeated for each member of the structure.

Another system in use in America consists in using the uniform train load over the structure, with an engine excess so disposed as to produce the maximum stress on the member of the structure under consideration.

The first and third methods are approximate, the second is exact for the particular engine and train loads assumed.

The advocates of the concentrated-load system maintain that a uniform load, or equal loads concentrated at the panel-points, does not represent what actually occurs when two coupled engines followed by a train pass over the bridge, and the same applies to a less extent with an engine excess.

The advocates of the uniform load contend that the concentrated-load system is unnecessarily complicated, as it is possible to assume such uniformly distributed loads as shall cover the maxima stresses which actually occur without producing errors which are excessive on the side of safety or danger. Moreover, that the actual wheel loads and uniform loads used in calculating the stresses may be considerably modified by the use of heavier engines and trains, or different wheel spacings, in the future.

With regard to the uniform-load system. Sir B. Baker[1] estimates the equivalent uniform load for heavily engined lines as follows :—

TABLE LIII.

Span in feet.	Equivalent uniform load per foot run in tons.
10	3·000
20	2·400
30	2·100
60	1·500
100	1·375
150	1·250
200	1·125
300	1·000

The loads given in the foregoing table are not now sufficient to provide for the traffic on heavily engined lines, although they were ample at the time the table was compiled.

The weight of engines and the trains hauled by them have been considerably increased during the last ten years; the use of heavy gradients of 1 in 40 in America and the colonies necessitates the use of heavy engines. Figs. 324 and 325 illustrate two types of consolidation goods engines in use on the New South Wales Government Railways, where grades as steep as 1 in 30 occur. Fig. 329 shows one of Mr. Cooper's wheel-load diagrams known as Class A, which has been very largely

[1] "Short-Span Railway Bridges."

used in calculating the stresses on American bridges; much heavier engines are, however, in use in America.

Consolidation engines on New South Wales Government Railways.

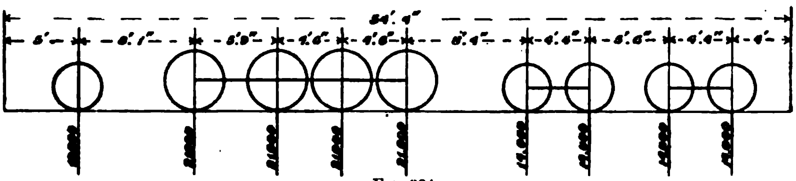


FIG. 324.

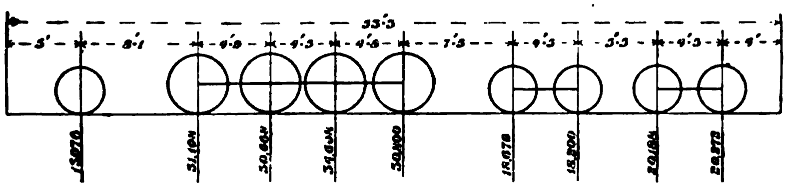


FIG. 325.

Fig. 326 represents a diagram of half-engine loads, which will be made use of in connection with an example of the concentrated-load system.

In estimating the equivalent uniform load per foot run for the purpose of calculating the maxima stresses in a structure, due consideration should be given to the probable increase in the engine and train loads which may be required to pass over the structure in the future.

Half-engine loads used in tabulations for maxima stresses in example on the concentrated-load system.

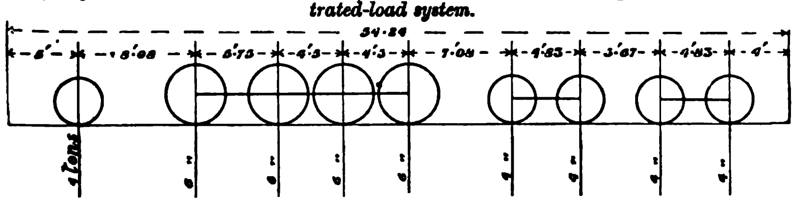


FIG. 326.

The same uniform load will not generally be suitable for both trusses and plate web girders, being slightly greater in the former than in the latter. The uniform load producing maxima bending moments is generally less than that required to produce the maxima shearing stresses.

This diagram might be still further simplified by using the nearest foot in all cases, or by using the first place of decimals only in the spacing of the driving wheels.

Mr. Theodore Cooper, in the following table, gives the equivalent uniform loads for the 101 American tons consolidation engine (Fig. 327), which is not very different from the New South Wales consolidation engine (Fig. 325); also for a still heavier engine known as the Lehigh Heavy Grade engine (Fig. 328).

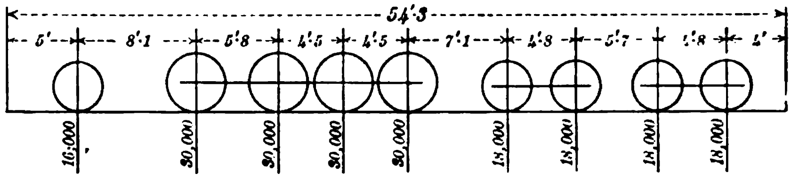


Fig. 327.

Lehigh Heavy Grade engine.

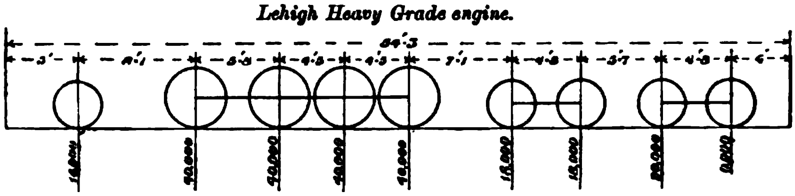


Fig. 328.

TABLE LIV.

Span in feet.	Two 101 American tons consolidations, followed by 3000 lbs. 1884.	Two Heavy Grade Lehighs, followed by 4000 lbs. 1889.
20	6190	8560
40	4760	6690
60	4110	5480
100	3910	4520
150	3740	4760
200	3590	4640
250	3480	4470
300	3390	4290
400	3180	4200
500	3150	4120

Mr. Cooper specifies for the extra heavy Class A service:—Two engines like Fig. 327 followed by a train of 3000 pounds per foot run, or 80,000 pounds equally distributed on two pairs of drivers spaced 7 feet centres, as shown in Fig. 329.

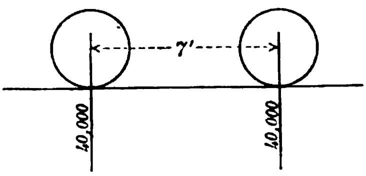


Fig. 329.

Professor Waddell has prepared the following table of equivalent uniformly distributed live loads which produce practically the same bending moments in plate web girders as the concentrated wheel-load diagram shown in Fig. 330:—

TABLE LV.

Span in feet.	Equivalent distributed load in pounds.	Span in feet.	Equivalent distributed load in pounds.	Span in feet.	Equivalent distributed load in pounds.
15·0	5760	35	4327	60	3652
17·5	5407	40	4164	70	3490
20·0	5040	45	3995	80	3379
22·5	4993	50	3860	90	3310
25·0	4804	55	3726	100	3226
30·0	4573				

He further recommends that, in order to produce the maximum shearing stress, an amount be added to the total

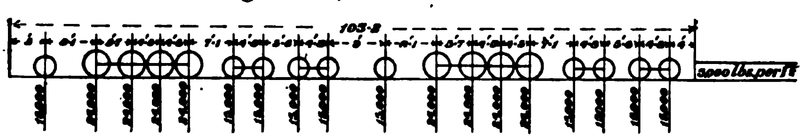


FIG. 330.

distributed load divided by two, which is given by the following formula:—

$$W = A + Bl$$

where $W =$ the additional load, $l =$ the span; A and B are constants which, for the engine loads shown in Fig. 330, are 8000 and 100 respectively.

Other constants may be determined for different engine loads. Some engineers use two different sets of tables, one for maxima moments, the other for maxima shears.

In order to find the maxima reactions of longitudinal girders upon the cross-girders, Professor Waddell gives the following simple rule, which may be used instead of the method explained in the design for a deck of a railway bridge, Chapter XIV., Fig. 320. Multiply the uniformly distributed live load per foot run for a span of two panel lengths by the length of one panel.[1]

Thus for a 20-foot panel the maximum cross-girder reaction is—

$$\frac{4164 \times 20}{2240} = 37·16 \text{ tons}$$

[1] *Trans. of the Amer. Soc. of C.E.*, February and March, 1892.

or 18·58 tons at each point of attachment. With the half-engine loads shown in Fig. 326, the equivalent distributed load would be 4700 lbs. per foot for a span of 40 feet, and the reaction is—

$$\frac{4700 \times 20}{2240} = 42 \text{ tons (nearly)}$$

or 21 tons at each point of attachment, agreeing very well with the reaction calculated from the wheel concentrations, Fig. 320.

The rule is, however, strictly true, and is proved thus—

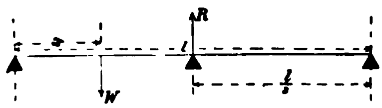


FIG. 331.

In Fig. 331 we have $R = \frac{2Wx}{l}$.

In a girder of length l the central bending moment is—

$$M = \frac{Wx}{2} = \frac{wl^2}{8}$$

$$\frac{wl}{2} = \frac{l}{2}\left(\frac{4Wx}{l^2}\right) = \frac{2Wx}{l} = R$$

Therefore, as the rule is true for a single load, it is true for any summation of loads.

In simple Pratt trusses, with panel length of 25 feet, Professor Waddell gives the following equivalent distributed loads to cover the stresses produced by the engine diagram, Fig. 330.

TABLE LVI.

Span of truss in feet.	Equivalent distributed load in pounds.
100	3308
150	3215
200	3151
250	3112
300	3085

In using this table instead of the actual concentrations for bending moments, the greatest error on the side of safety occurs in the 100-feet truss, and is 4·77 per cent. The greatest error on the side of danger is 2·21 per cent. In the shearing stresses, the loads given in the table produce stresses the error of which on the side of danger is never above 2½ per cent. and affects the heavier members, while the maximum error on the side of safety is 12·66 per cent. The safety errors, although the larger, occur principally in the counterbraces, which is a good thing, because of impact and adjustment.

It appears, therefore, that equivalent uniform loads may be so chosen for a structure that the errors in the stresses calculated from these loads are not greater than would be provided for in the process of designing the various members.

The concentrated-load system is also used in America, and the methods of applying it have been investigated by Mr. Theodore Cooper,[1] Professors Burr,[2] Eddy,[3] and Dubois.[4] Only a brief outline of the method will be given in this work, just sufficient to illustrate its application in a common truss bridge in the manner explained more fully in the works by the authors quoted.

Concentrated-Load System.—Let W_1, W_2, W_3 . . . W_n denote a series of loads advancing from the right abutment corresponding with the loads on the driving wheels of a locomotive.

Let x_1, x_2, x_3 . . . x_n denote the distances between the loads corresponding with the distances between the wheel centres.

Let W_1, W_2, W_3 . . . W_{n1} denote the loads which act in the panel AB (W_{n1} is not shown in the figure), in which we require to find the maximum shearing stress, and let b', b'', b''', etc., denote the distances measured from B. The length of the span

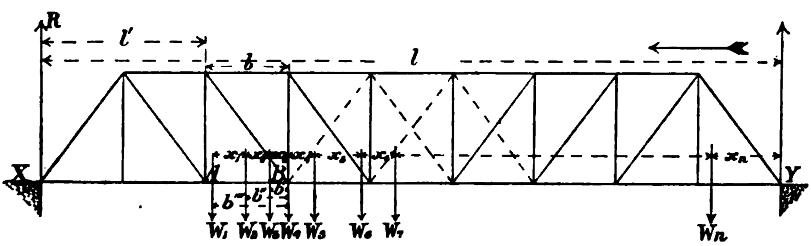


FIG. 332.

is denoted by l, and the length of each panel by b, so that $l = nb$. The reaction R at the left abutment may be expressed thus—

[1] Mr. Theodore Cooper's "Standard Specifications."

[2] "The Stresses in Bridge and Roof Trusses," by Professor Burr.

[3] Paper in the *Trans. of the Amer. Soc. C.E.*, on "A Graphical Solution of the Concentrated Load Problem," by Professor Eddy.

[4] "The Strains in Framed Structures," by Professor Dubois.

$$R = \frac{W_1}{l}(x_1 + x_2 + \ldots + x_n) + \frac{W_2}{l}(x_2 + x_3 + \ldots + x_n)$$
$$+ \frac{W_3}{l}(x_3 + x_4 + \ldots + x_n) + \ldots + \frac{W_n}{l}(x_n)$$

The reaction at A from the loads W_1, W_2, W_3, etc., acting in the panel AB is—

$$\frac{W_1}{b}b' + \frac{W_2}{b}b'' + \frac{W_3}{b}b''' + \ldots$$

Hence the shear in the panel AB is—

$$S = R - \frac{1}{b}(W_1b' + W_2b'' + W_3b''' + \ldots)$$

Suppose that the train advances by the distance dx, and let R_1 denote the new reaction, then—

$$R_1 = R + (W_1 + W_2 + \ldots + W_n)\frac{dx}{l}$$

and the new shear is—

$$S_1 = R_1 - \frac{1}{b}(W_1b' + W_2b'' + \ldots) - (W_1 + W_2 + \ldots)\frac{dx}{b}$$

$$\therefore S_1 = S + (W_1 + W_2 + \ldots + W_n)\frac{dx}{l} - (W_1 + W_2 \ldots)\frac{dx}{b}$$

$$\therefore S_1 - S = \frac{dx}{l}(W_1 + W_2 + \ldots + W_n) - \frac{dx}{l}\left(\frac{l}{b}\right)(W_1 + W_2 + \ldots)$$

When $S_1 - S = 0$, S_1 will be a maximum, provided that $S_1 - S$ is positive just before it equals zero.

$$\therefore n(W_1 + W_2 + \ldots) = W_1 + W_2 + \ldots + W_n$$

Or the maximum shear in any panel will occur when n times the moving load which it contains is equal to, or most nearly equal to, the total live load on the bridge. We observe that the position of the load in the panel is of no consequence, therefore we may always consider one of the wheel loads concentrated at the panel-point under consideration.

The above equation enables the position of the wheel loads on the bridge to be fixed, and x_n determined for the maximum shearing stress.

We may write the equation of shearing stress in a convenient form for tabulation, since—

$$R = \frac{1}{l}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_n)x_n\}$$

and
$$\frac{W_1b'}{b} + \frac{W_2b''}{b} + \frac{W_3b'''}{b} + \ldots = \frac{1}{b}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + \ldots + W_{n'-1})\theta\}$$

where θ = the distance between $W_{n'-1}$ and $W_{n'}$.

Hence the general equation of shearing stress may be written thus—

$$S = \frac{1}{l}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_n)x_n\} - \frac{1}{b}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_{n'-1})\theta\}$$

This equation is most easily solved in any particular case by tabulating the wheel loads, leaving only the term involving x_n to be added afterwards. The above equations may be shown to hold when W_1, W_2, W_3, etc., advance beyond the panel AB.

To find the position of the engine and train loads which will produce the maximum bending moment in the top and bottom chords.

Let $W_{n'}$ act at a distance x' in front of A, and let there be n' wheel loads between A and X; then, taking moments about the point A, we have—

$$M = Rl' - \{W_1(x_1 + x_2 + \ldots + x'_n) + W_2(x_2 + x_3 + \ldots + x'_n)_1 + \ldots + W_nx'\}$$

$$= \frac{l'}{l}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_n)x_n\} - \{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_{n1})x'\}$$

If the loads advance through a distance dx as before—

$$M_1 = M + \frac{l'}{l}(W_1 + W_2 + \ldots + W_n)dx - (W_1 + W_2 + \ldots + W_{n'})dx$$

Hence for a maximum $M_1 - M = 0$, and—

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_{n'}}{W_1 + W_2 + \ldots + W_n}$$

If W_n act at a panel-point such as B, this ratio will seldom or never exist exactly, so that $W_{n'}$ should be considered as that portion of a wheel load concentrated at B in order that the above ratio may exist exactly, and x' may always be put equal to zero.

Hence the general equation of moments may be expressed in a form suitable for tabulation, thus—

$$M = \frac{l'}{l}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_n)x_n\}$$
$$- \{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_{n'-1})\theta\}$$

where θ denotes the distance between $W_{n'-1}$ and $W_{n'}$.

Since at the point where the maximum moment occurs the shearing stress is zero, we may write—

$$\frac{l'}{l} = \frac{R}{W_1 + W_2 + \ldots + W_n}$$

or the distance between the point where the maximum moment occurs and the centre of gravity of the load is bisected by the centre of the span. The foregoing investigation is general so far as the web members are concerned in any truss or girder; but in the case of the chords, they apply for both top and bottom members only in a truss with vertical members. The equations may easily be modified so as to include every kind of truss with parallel chords.[1]

In the case of a girder with curved booms or chords, such as, for instance, a hog-back lattice girder with two systems of web bracing and 20 feet panel lengths. It will in general be most convenient, for chord stresses, to consider all the panel points equally loaded with the equivalent uniform train load, and also with four concentrated loads corresponding with the excess of the two engines over the train load. The four engine excesses may be considered as divided over the two systems of web bracing, *i.e.* two to each, acting one on each side of the chord length under consideration.

For the web stresses, the four concentrations representing the locomotive excesses should be considered at the head of the train, with the panel points in front unloaded, in a similar manner to that explained in Chapter IX.

[1] "Stresses in Bridge and Roof Trusses," by W. H. Burr.

CHAPTER XVI.

TO DESIGN A PLATE WEB-GIRDER DECK BRIDGE FOR A SINGLE LINE OF RAILWAY (FIGS. 333 TO 337).

THE bridge to consist of two main girders, each 42 feet long over all, and 40 feet between the centres of bearings on supports at end.

The end supports, or abutments, to be built of brick, with stone girder beds under ends of main girders.

The deck to be of the open American type, with pine sleepers, 10 feet long × 8 inches × 8 inches, spaced 16 inches centre to centre, with guard rails and timber kerbs as shown in Figs. 333 to 337.

The weight of rails to be 75 lbs. per yard. The bridge to be designed for the consolidation engine shown in Fig. 330, which is assumed to be the heaviest on the railway.

Table LV., on p. 253, shows that the equivalent uniformly distributed load is 4164 lbs. per foot run, or 1·86 ton. The total live load is therefore—

$$1{\cdot}86 \times 40 = 74{\cdot}4 \text{ tons}$$

Dead load.—The dead load consists of the rails, guard rails, kerb, and sleepers, and fastenings, which will weigh 400 lbs. per lineal foot,[1] and the weight of the main girders and wind bracing.

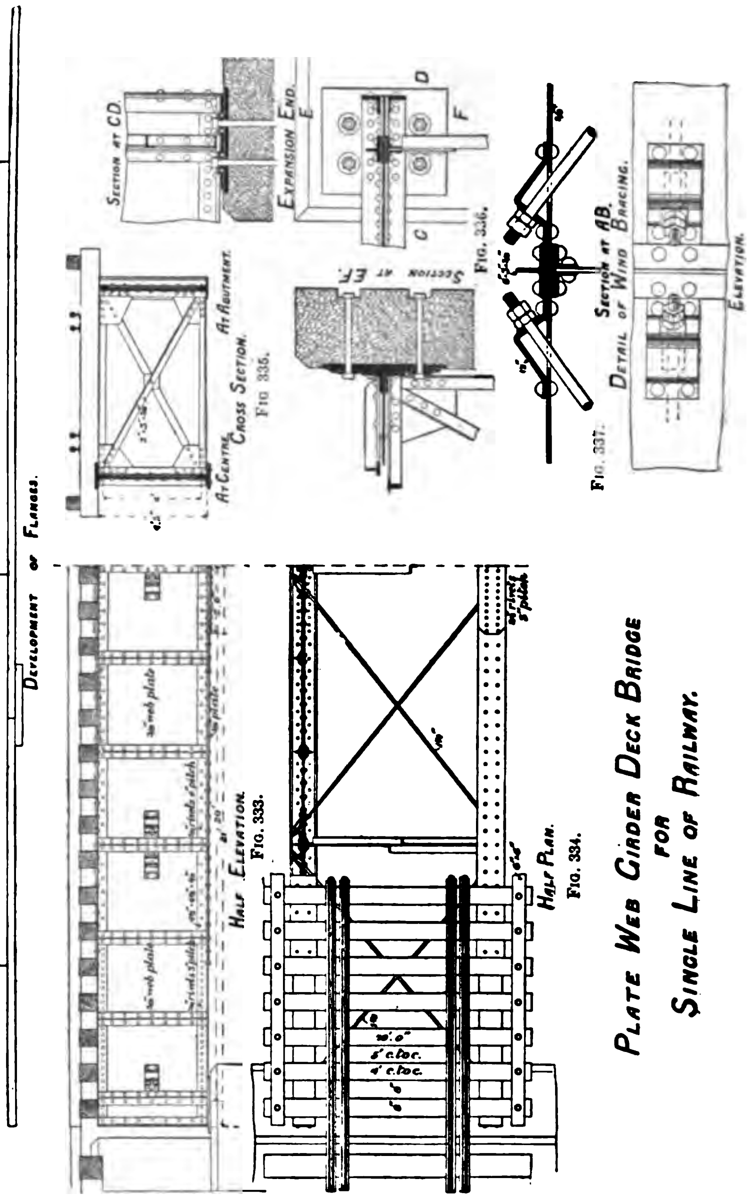
The dead load, less the weight of the main girders, is—

$$400 \times 40 = 16{,}000 \text{ lbs.} = 7{\cdot}14 \text{ tons}$$

[1] With heavier timber for the sleepers this load per foot run will be correspondingly increased, as for ironbark sleepers. The guard rail shown in the figures is frequently omitted in practice; it may, however, be used advantageously as a re-railing arrangement by bringing the two rails together at each end of the bridge.

The total dead and live load, less the weight of main girders, is—

$$74 \cdot 4 + 7 \cdot 14 = 81 \cdot 54 \text{ tons}$$



The weight of the main girders may be found approximately from the formula, p. 245.

$$W_1 = \frac{Wlr}{Cs - lr}, \quad W = 82 \text{ tons}, \ C = 1200, \ r = \frac{40}{4} = 10$$

$$s = 3{\cdot}5\left(1 + \tfrac{1}{2}\frac{\text{min.}}{\text{max.}}\right) = 3{\cdot}5\left(1 + \frac{7{\cdot}14}{2 \times 81{\cdot}54}\right) = 3{\cdot}65 \text{ tons}$$

Take $s = 3{\cdot}6$ tons per square inch.

$$W_1 = \frac{82 \times 40 \times 10}{1200 \times 3{\cdot}6 - 40 \times 10} = 8{\cdot}36 \text{ tons}$$

The total dead load is therefore $8{\cdot}36 + 7{\cdot}14$	15·5	tons.
The total live load	74·0	„
The total live and dead load	89·5	„
The total live and dead load per foot run	2·24	„
The total live and dead load on each girder	1·12	„

The equation of bending moments is—

$$y = {\cdot}56(400 - x^2)$$

The curve of bending moments may be plotted from this equation for different values of x measured from the centre. The maximum moment occurs when $x = 0$, and is 224 foot-tons.

The moment of resistance is—

$$3{\cdot}6 \times 4 \times a = 14{\cdot}4a$$

$$a = \frac{224}{14{\cdot}4} = 15{\cdot}6 \text{ square inches}$$

Adopting two angles $4\frac{1}{2} \times 4\frac{1}{2} \times \frac{5}{8} = 9{\cdot}44$ square inches in tension, we have left 6·16 square inches to be made up by the cover-plate. Hence we may use a plate 12 inches wide by $\frac{5}{8}$ inch thick, making the total area in tension 16 square inches.

We may plot the moments of resistance of the angles and plate on the bending-moment diagram, and find the length of the top plate; or we may find where the bending moment equals the moment of resistance of the angles from the equation of moments.

A slightly longer length should be given to the plate than obtained by either of the foregoing methods, and we may adopt 13 feet as the length from the centre, or a total length of 26 feet. As this length may be obtained in one piece, no cover will be necessary, excepting for shipment, in which case it will be more convenient to have a single cover with the rivets in single shear; therefore a length of 5 feet will be sufficient, with $\frac{3}{4}$ rivets, 3 inches pitch.

The compression flange should be made similar to the tension flange; the excess in area will give extra lateral stiffness.

Shearing Stress.—The maximum shearing stress occurs at the ends, and is half the total load, or 22·5 tons. The shearing stress per foot run horizontally and vertically is—

$$\frac{22 \cdot 5}{4} = 5 \cdot 625 \text{ tons}$$

Assume that the web is $\frac{3}{8}$ inch thick, and apply Mr. Theodore Cooper's rule for the safe intensity of working stress per square inch—

$$\frac{12{,}000}{1 + \frac{H^2}{3000}} = \frac{12{,}000}{1 + \frac{128^2}{3000}} = 1857 \text{ lbs.} = 0 \cdot 83 \text{ ton}$$

$$H = \frac{48}{\frac{3}{8}} = 128$$

The actual shearing stress per square inch in the section provided in the web is—

$$\frac{5 \cdot 625}{12 \times \frac{3}{8}} = 1 \cdot 02 \text{ ton}$$

Hence the web may be used with stiffeners as shown in Fig. 333.

Let n = the number of rivets $\frac{3}{4}$ inch diameter per foot run uniting the angles with the web; then, since the rivets are in double shear—

$$1 \cdot 75n \times 0 \cdot 44 \times 3 \cdot 6 = 1 \cdot 625$$
$$\therefore n = \text{about } 2, \text{ or } 6 \text{ inches pitch.}$$

The pressure on the bearing area will be excessive with this pitch.

Let the pitch be 3 inches, or 4 rivets per foot, then—

$$4 \times \tfrac{3}{4} \times \tfrac{3}{8} \times p = 5 \cdot 5 \quad \therefore p = 4 \cdot 9 \text{ tons}$$

Hence $\frac{3}{4}$ rivets, 3 inches pitch, may be adopted at the ends. Since there are 8 rivets per foot in the flanges in single shear, it is clear that they need not be further considered.

The pitch of the rivets need not be made 3 inches throughout excepting in the angle wrappers, and in the cover-plate of the flange, if the latter is used.

The maximum shearing stress at a distance of 10·5 feet from the centre is—

$$s = 1{\cdot}94x + \frac{0{\cdot}93}{80}(x + 20)^2 = 1{\cdot}94 \times 10{\cdot}5 + \frac{0{\cdot}93}{80}(30{\cdot}5)^2$$
$$= 12{\cdot}85 \text{ tons}$$

The maximum shear per foot is—

$$\frac{12{\cdot}85}{4} = 3{\cdot}21 \text{ tons}$$

Comparing this value with the corresponding value of the shearing stress at the ends, we see that $\frac{3}{4}$ rivets, 6 inches pitch, may be used in the central half of the girder. The web plate may be reduced to $\frac{5}{16}$ in the central 14 feet of the girder, but it will be better in this case to make the thickness uniform throughout, as the extra strength in the centre will fully compensate for the slight saving in the $\frac{5}{16}$-inch plate.

The web may be conveniently jointed in the centre, and at distances of 10 feet 6 inches on either side of the centre, and the stiffners may be spaced conveniently so as to act as covers to the web joints.

The joints in the angle irons may be conveniently arranged in the centre for shipment or carriage in two sections. The length of the wrappers and number of rivets are similar to Fig. 267, Chapter XIII.

Lateral System.—The girders must be braced together laterally, so as to resist vibrations caused by rolling loads; the dimensions of the frames which occur at the ends, and the three intermediate points where the web plate is jointed, are more a matter of judgment than calculation.

The horizontal reactions at the ends of the girders are for a wind pressure on the girders and train of 459 lbs. per lineal foot.[1]

$$\frac{459 \times 42}{2} \times 9639 \text{ lbs.} = 4{\cdot}3 \text{ tons}$$

[1] The wind pressure on the exposed area of the bridge and on the train will be about 459 lbs. per foot run for a pressure of 30 lbs. per square foot.

Only one tie-rod in each bay will be in action at a time, so that the stress in the tie-rods of end bays is equal to the reaction resolved along the bar, to which must be added about 1 ton for initial tension. The total stress is therefore about 8·4 tons.

Taking the working stress at 7 tons per square inch, we require—

$$\frac{8\cdot4}{7} = 1\cdot2 \text{ square inch}$$

The rods shown in Fig. 337 are not upset at the ends, so that they must be made 1½ inch in diameter; at the bottom of the thread the diameter is 1·28 inch.

It may be shown that tie-rods 1¼ inch in diameter will be sufficient for the other bays.

The main girders are fixed at one end and allowed to expand at the other, a gun-metal plate being provided to slide upon a planed cast-iron surface, as shown in the detail, Fig. 336. At the fixed end the cast-iron bed-plate is used with an iron bearing plate. The area of the cast-iron plate bearing on the abutments is 4·6 square feet, so that the pressure per square foot is about 5 tons. The pressure per square foot on good sandstone may be 12 tons, and on good brickwork 4 tons.

The weight of the girders and bracing may now be calculated.

TABLE LVII.

Description.	Lineal feet.	Weight in pounds per lineal foot.	Weight in pounds.
Angle iron in flanges, 4 × 42 ft. of $4\frac{1}{2}'' \times 4\frac{1}{2}'' \times \frac{5}{8}''$	168	17·45	2932
„ „ at ends, 4 × $4\frac{1}{2}$ ft. of $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$	$17\frac{1}{2}$	8·3	144
„ „ wrappers (double), $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$...	12	8·3	100
Flange plates, 13′ × 12″ × $\frac{5}{8}''$	26	25·0	650
Web plate, 42′ × 4′ 5″ × $\frac{3}{8}''$	42	66·3	2785
End plates, 4′ 5″ × 10″ × $\frac{3}{8}''$	8·9	12·5	112
T-iron stiffeners and covers } 26′ × 4′ 4″ × 6″ × 3″ × $\frac{3}{8}$ ...	113	10·8	1220
			7948
Add 4 per cent for rivets			318
Total weight of one girder			8256
„ „ two girders			16512 = 7·38 tons.
Transverse bracing.			
5 frames containing 29 feet of L iron 3″ × 3″ × $\frac{3}{8}$	145	7·0	1015
20 plates $\frac{3}{8}$ in. thick at 20 lbs. each	20	20·0	400
5 „ $\frac{3}{8}$ „ „ 4 „ „	5	4·0	20
4 tie-rods, 13·5 feet long by $1\frac{1}{2}$ in. diameter ...	54	5·9	319
4 „ 13·5 feet „ by $1\frac{1}{4}$ in. „	54	4·1	521
6 double and 4 single bent attachment plates, each 3 feet long by $\frac{3}{8}$ inch	24	12·5	300
16 nuts and washers for $1\frac{1}{2}$-inch rods	16	$2\frac{1}{4}$	36
16 „ „ „ $1\frac{1}{4}$ „ „	16	$1\frac{1}{4}$	20
Total weight of bracing			2631 = 1·31 ton.

Total weight of two girders and bracing = 8·51 tons. If the top flange plate is in two lengths, we shall have a total weight of 8·74 tons.

The details of attachment of the wind bracing rods is shown in Fig. 337. The remaining details are simple, and need not be further illustrated. This is a most economical form of girder for railway traffic up to spans of 60 feet.

CHAPTER XVII.

TO DESIGN A WROUGHT-IRON PLATE WEB GIRDER BRIDGE FOR A DOUBLE LINE OF RAILWAY.

Description (see Plate I.).—The bridge to consist of two main girders 70 feet long over all, 66 feet between centres of bearings on piers or abutments, 26 feet wide between centres of main girders.

Cross-girders to be spaced 5 feet 4 inches centre to centre, and longitudinal stringers arranged under each rail. Cambered iron plates, ⅜ inch thick, to be riveted to top flanges of longitudinals and to angle irons on main girders, forming a continuous floor, upon which are laid the ballast rails and sleepers as a loose road. The main girders to be fixed at one end and allowed to expand at the other.

The longitudinal stringers should be first designed thus:—

The permanent load on longitudinals consists of the weight of rails, guard rails, sleepers, fastenings, ballast, iron plates, and the weight of the longitudinal itself. The permanent load will vary with the weights of the materials used in the permanent way; with ordinary creosoted larch sleepers, 9″ 6″ × 10″ × 5″, and rails 80 lbs. per yard, chairs and fastenings, guard rail, etc., it will weigh 190 lbs. per lineal foot for each line of way, or 380 lbs. per lineal foot for the two lines of way. Ironbark sleepers weighing 240 lbs. each without chairs, with the rails and fastenings as before, would weigh 404 lbs. per foot. Adopting the former weight, we have—

Permanent way	380 lbs. per foot.
Cambered plates, 21 square feet per foot at 15 lbs. per square foot	315 " "
Ballast averaging 4 inches deep at 93 lbs. per cubic foot	651 " "
Total load exclusive of weight of longitudinals ...	1346 " "

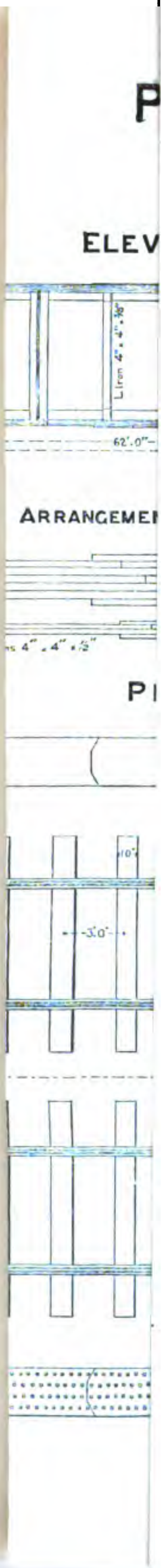
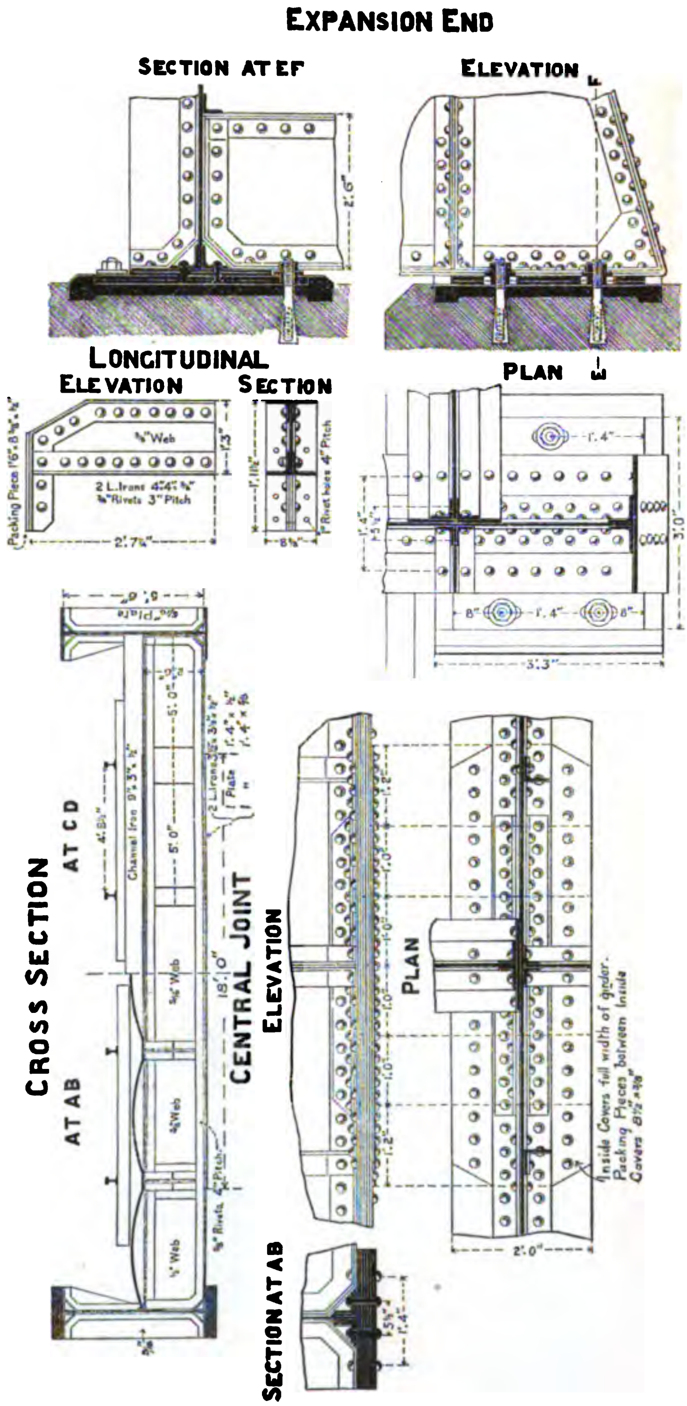

ELEV
ARRANGEMEI
P
PI

PLATE I.


EXPANSION END
SECTION AT EF
ELEVATION
LONGITUDINAL
ELEVATION
SECTION
PLAN
2 L. Irons 4"x4"x½"
¾" Rivets 3" Pitch
2'.7½"
CROSS SECTION
AT CD
AT AB
CENTRAL JOINT
ELEVATION
PLAN
SECTION AT AB
3'.3"
2'.0"

Load on each longitudinal $= \dfrac{5'' 4'' \times 1346}{4} = 1795$ lbs. $= 0\cdot8$ ton

Live Load on Each Longitudinal.—This will consist of half the greatest load on a pair of driving wheels, which will be assumed as 18 tons; since the load of 9 tons may be in the centre of the longitudinal, we have the equivalent distributed live load 18 tons, or a total load of 18·8 tons exclusive of the weight of the longitudinal.

To calculate the weight of the longitudinal, we may use the formula—

$$W_1 = \frac{Wlr}{cs - lr}$$

c may be taken at 1500; s must be taken low, on account of impact and range of stress. Mr. Theodore Cooper allows 8000 lbs. per square inch.

The depth may be made one foot effective—

$$W_1 = \frac{18\cdot8 \times 5\cdot3 \times 5\cdot3}{1500 \times 3\cdot5 - 5\cdot3 \times 5\cdot3} = \cdot096 \text{ ton}$$

The total load $= 18\cdot8 + 0\cdot096 = 18\cdot896$ tons, say 18·9

Shearing-Stress.—The maximum shear at each end is—

$$\frac{18\cdot9}{2} = 9\cdot45 \text{ tons}$$

Rivets.—Let $n =$ number of rivets per foot run. Assume the diameter to be $\frac{3}{4}$ inch, area 0·44 square inch. Then—

$$9\cdot45 = n \times 1\cdot75 \times 0\cdot44 \times 3\cdot5$$
$$\therefore n = 3\cdot5, \text{ or } 3 \text{ inches pitch}$$

Assume the thickness of the web $\frac{3}{8}$ inch, and let p denote the pressure per square inch on the bearing area; then—

$$p \times \tfrac{3}{4} \times \tfrac{3}{8} \times 4 = 9\cdot45$$
$$p = 8\cdot4 \text{ tons}$$

This pressure may be allowed, although some engineers would consider it excessive, in which case $\frac{7}{8}$ of an inch diameter may be used.

Intensity of Shearing Stress in Web.—The sectional area of one foot in length of the web, horizontally or vertically, through a line of rivets is—

$$(12 - 4 \times \tfrac{3}{4})\tfrac{3}{8} = 3{\cdot}3 \text{ square inches}$$

The shearing stress on this line $= \dfrac{9{\cdot}45}{3{\cdot}3}$

$= 2{\cdot}9$ tons per square inch

By applying Mr. Cooper's formula,[1] we can show that the safe intensity of shearing stress may be 4 tons per square inch; or applying Rankine's formula for buckling, we obtain a factor of safety of 6; hence the thickness of web may be $\frac{3}{8}$ inch.

Bending moment—

$$\frac{Wl}{8} = \frac{18{\cdot}9 \times 5{\cdot}3}{8} = 12{\cdot}5 \text{ foot-tons}$$

Moment of resistance—

$$fad = 3{\cdot}5 \times a \times 1 = 3{\cdot}5a \text{ foot-tons}$$
$$\therefore 3{\cdot}5a = 12{\cdot}5$$
$$a = 3{\cdot}6 \text{ square inches}$$

The area of two angle irons, $4'' \times 4'' \times \frac{3}{8}''$, is 3·75 square inches in tension, and 5·25 inches in compression. The elevation and the section of the girder is shown on Plate I.

The weight may now be calculated.

26 feet of L iron at 9·2 lbs. per foot	...	...	239·0 lbs.
5·3 feet of web 15 inches by $\frac{3}{8}$ inch	...	...	103·5 „
			342·5
Add 5 per cent. for rivets	...	...	17·1 „
Total weight of longitudinal	...	...	359·6 lbs. = 0·16 ton.

The calculated weight was 0·096; the difference is due partly to the angle irons at ends being carried down for attachment to the cross-girders. It will not be necessary to recalculate.

The total dead load on the longitudinal is—

$$0{\cdot}8 + 0{\cdot}15 = 0{\cdot}95 \text{ ton}$$

Cross-Girders.—The dead load will consist of 0·95 ton concentrated at four points immediately under the rails, and also of the weight of the cross-girder.

[1] P. 240.

The maximum reactions of the longitudinal girders upon the cross-girder will be taken at 9 tons, which will not be exceeded when a mogul or consolidation engine, with 12 tons upon each axle, passes over the bridge.

The bending moment with the four lines loaded with the live and dead load, viz. 9·95 at each of four points, say 10 tons, is—

$$20 \times 10 - 10 \times 5 = 150 \text{ foot-tons}$$

The equivalent uniform load per foot run is therefore—

$$\frac{wl^2}{8} = \frac{w \times \overline{26}^2}{8} = 150$$
$$\therefore w = 1{\cdot}77 \text{ tons, say } 1{\cdot}8 \text{ ton}$$

The depth of the cross-girder may be made $\frac{1}{12}$ of the span, and the same unit stress taken as in the longitudinals, viz. 3·5 tons per square inch. The weight of the cross-girder is—

$$\frac{1{\cdot}8 \times 26 \times 26 \times 12}{1500 \times 3{\cdot}5 - 26 \times 12} = 2{\cdot}9 \text{ tons}$$

This load may be included in the load concentrated at the four points by adding 0·6 tons to each, making a total of—

$$4 \times 10{\cdot}6 = 42{\cdot}4 \text{ tons}$$

The maximum shearing stress at the end of the cross-girder is 21·2 tons.

The shear per foot—

$$\frac{21{\cdot}2}{2{\cdot}16} = 9{\cdot}8 \text{ tons}$$

Assume that the rivets through the angles uniting the flanges to the web are $\frac{7}{8}$ inch diameter, and 0·6 square inch in area. Each rivet will resist, since it is in double shear—

$$1{\cdot}75 \times 3{\cdot}5 \times 0{\cdot}6 = 3{\cdot}675 \text{ tons}$$

Let n = the number of rivets required per foot run, then—

$$3{\cdot}675n = 9{\cdot}8$$
$$\therefore n = 3 \text{ nearly}$$

Assume that the thickness of the web is $\frac{1}{2}$ inch. The pressure on the bearing area is—

$$p \times 3 \times \tfrac{7}{8} \times \tfrac{1}{2} = 9{\cdot}8$$
$$\therefore p = 7{\cdot}46 \text{ tons}$$

Hence $\frac{7}{8}$-inch rivets, 4 inches pitch, may be adopted.

The intensity of shearing stress on the web per square inch is—

$$\frac{9{\cdot}8}{(12 - 3 \times \frac{7}{8})\frac{1}{2}} = \text{about 2 tons}$$

It can be shown, by applying Cooper's or Rankine's formulæ, that there is no danger of buckling the web.

The maximum shear at the centre of the cross-girder occurs when one line only is loaded with the live load; thus—

$$\text{Maximum shear at centre} = 16 - 21{\cdot}2 = - 5{\cdot}2 \text{ tons}$$
$$\frac{5{\cdot}2}{2{\cdot}16} = 2{\cdot}4 \text{ tons per foot}$$

Using $\frac{5}{16}$-inch plate, the thinnest allowable in bridge-work, it can be shown, as before, that the pressure on the bearing area is moderate, and that there is no danger of the web buckling.

In a similar manner, it can be shown that a $\frac{3}{8}$-inch plate between the longitudinals is sufficient.

Bending Moments.—The ordinates of the bending-moment polygon at the points of application of the longitudinals on each side of the centre of the cross-girder can be shown to be 106 and 159 foot-tons respectively.

Moment of resistance—

$$3{\cdot}5\text{A} \times 2{\cdot}16 = 159$$
$$7{\cdot}56\text{A} = 159$$
$$\therefore \text{A} = 21 \text{ square inches}$$

The angles uniting the flanges to the web may be $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$, having an area in tension of say 5 square inches. The plates may be 16 inches wide, hence one plate $\frac{1}{2}$ inch thick and one $\frac{3}{8}$ inch thick may be shown to be sufficient. The $\frac{3}{8}$-inch plate should be on the outside, giving a total area in tension of 21·03 square inches; and the $\frac{3}{8}$-inch plate need not be made more than 18 feet long. The moments of resistance of the angles and plates, 7·56*a*, should be plotted to the same scale as the bending moments, and the length of the top plate may be scaled off the diagram.

The compression flange should be made similar to the tension flange. The weight of the cross-girder may now be calculated.

Two flange plates, each 26′ × 16″ × $\frac{1}{2}$″ at 26·6 lbs. per foot	1383·2 lbs.
„ „ „ 18′ × 16″ × $\frac{5}{8}$″ at 33·3 „ „	1198·8 „
Web, 10′ × 2′ 5″ × $\frac{1}{2}$″ at 48·3 lbs. per foot	483·0 „
„ 10′ × 2′ 5″ × $\frac{3}{8}$″ at 36·3 „ „	363·0 „
„ 6′ × 2′ 5″ × $\frac{5}{16}$″ at 30·2 „ „	181·2 „
Angle iron, 113′ × $3\frac{1}{2}$″ × $3\frac{1}{2}$″ × $\frac{1}{2}$″ at 10·8 lbs. per foot	1220·4 „
	4829·6
Add 5 per cent. for rivets	241·4 „
Total weight of cross-girder	5071 lbs. = 2·26 tons.

The calculated weight was 2·9 tons, which is in excess, as it should be, so that no recalculation is necessary.

Main Girders. Dead Load.—The dead load consists of the cross and longitudinal girders, deck, ballast, sleepers, etc., as well as the weight of the main girder itself.

11 cross-girders at 2·26 tons each	24·86 tons.
44 longitudinal stringers, allowing for 8 at ends resting partly on the end cross-girders, at 0·16 ton each	7·04 „
Deck = 44 times the load on stringers, at 0·9 ton	39·60 „
Total dead load exclusive of the weight of the main girders ...	71·50 „

Live Load.—This will generally consist of one complete engine and tender. We will assume in this example that the equivalent uniformly distributed live load is 1·5 ton per foot run. This will fairly represent English traffic, but will be too small for American and colonial traffic.

$$66 \times 1{\cdot}5 = 99 \text{ tons on each line of way}$$

The total dead and live load exclusive of the weight of the main girders is—

$$198 + 71{\cdot}5 = 269{\cdot}5 \text{ tons, say } 270 \text{ tons}$$

The weight of the main girders is approximately—

$$\frac{270 \times 66 \times 12}{1700 \times 4 - 66 \times 12} = 35{\cdot}5 \text{ tons}$$

The effective depth $= \frac{66}{12} = 5{\cdot}5$ feet, and $s = 4$ tons per square inch.

The total load is therefore 305 tons, or 152·5 tons on each girder.

The intensity of working stress per square inch may be found from the formula—

$$s = 3{\cdot}8\left(1 + \tfrac{1}{2}\frac{\text{min.}}{\text{max.}}\right) = 3{\cdot}8\left(1 + \tfrac{1}{2}\frac{53{\cdot}5}{152{\cdot}5}\right) = 4{\cdot}5 \text{ tons}$$

4 tons per square inch will be adopted.

Rivets.—The maximum shearing stress at ends is 76·3 tons, or $\frac{76{\cdot}3}{5{\cdot}5} = 13{\cdot}9$ tons per foot horizontally and vertically.

It can easily be shown that 1-inch rivets, 4 inches pitch, will be necessary. Even with this size the pressure on the bearing area is 7·4 tons per square inch with a $\frac{5}{8}$-inch web and a single row of rivets.

The dead load per foot run is—

$$\frac{53{\cdot}5}{66} = 0{\cdot}81 \text{ ton}$$

Shearing Stress.—The equation of shearing stress is—

$$s = w_1 x + \frac{w_2}{4c}(c + x)^2 = 0{\cdot}81x + \frac{1{\cdot}5}{132}(33 + x)^2$$

It will be convenient to tabulate the results obtained by substituting $x = 5' \, 4''$, $10' \, 8''$, etc., in this equation.

TABLE LVIII.

x.	s.	x.	s.
—	12·40	21·3	50·75
5·7	20·96	26·7	62·15
10·3	30·36	33·0	76·30
16·0	40·16		

The maximum intensity of shearing stress at the ends, with a web plate $\frac{5}{8}$ inch thick on the gross section, is—

$$\frac{13{\cdot}9}{12 \times \frac{5}{8}} = 1{\cdot}85 \text{ ton per square inch nearly}$$

The safe intensity by Cooper's rule is—

$$\frac{12{,}000}{1 + \frac{H^2}{3000}} = \frac{12{,}000}{1 + \frac{1}{3000}\left(\frac{65}{\frac{5}{8}}\right)^2} = 2605 \text{ lbs.}$$

Hence stiffeners must be introduced in order to shorten the elementary columns into which the web is assumed to be divided. Let the stiffeners be spaced every 32 inches horizontally, at the attachment of cross-girders and midway between them, in which case the horizontal distance between the centres of rivets in stiffeners is 30 inches; and therefore $H = \frac{30}{\frac{5}{8}} = 48$, and the safe intensity of stress by Cooper's rule is 6787 lbs., or about 3 tons.

Rankine's rule for the buckling stress gives $\frac{16}{1 + \frac{1}{3000}(68)^2}$ = 6·3 tons per square inch nearly; hence the factor of safety against buckling is—

$$\frac{6{\cdot}3}{1{\cdot}85} = 3{\cdot}4$$

Hence the $\frac{5}{8}$ web may be adopted at the ends.

Assume that the web is $\frac{1}{2}$ inch thick in the second bay from the end, then the intensity of shearing stress is $\frac{11{\cdot}3}{2 \times \frac{1}{2}} = 1{\cdot}9$; the pressure on the bearing area of the rivets is 7·5 tons per square inch.

Cooper's rule gives for the web 5454 lbs., as the safe intensity of shearing stress per square inch; hence $\frac{1}{2}$ inch may be adopted.

In a similar manner it may be shown that the other thickness shown on Plate I. may be adopted.

The equation of bending moments is—

$$y = 1{\cdot}16(\overline{33}^2 - x^2) = 1{\cdot}16(1089 - x^2)$$

We may tabulate the moments and stresses as follows :—

TABLE LIX.

x.	y.	Stress in tons.	Area required in square inches.	Section provided.	Area provided in tension.
0	1263	230	57·5	5 plates 24″ × ½″, 2 angles 4″ × 4″ × ½″	60·25
5	1234	224	56·0		
10	1148	209	52·2		
15	1002	182	45·5	4 „ 24″ × ½″, 2 „ 4″ × 4″ × ½″	49·25
20	799	145	36·4	3 „ 24″ × ½″, 2 „ 4″ × 4″ × ½″	38·40
25	538	98	24·5	3 „ 24″ × ½″, 2 „ 4″ × 4″ × ½″	38·40
30	219	39	9·8	2 „ 24″ × ½″, 2 „ 4″ × 4″ × ½″	27·25
33	—	—	—	1 „ 24″ × ½″, 2 „ 4″ × 4″ × ½″	16·25

The bending moments and moments of resistance may now be plotted as in Fig. 338.

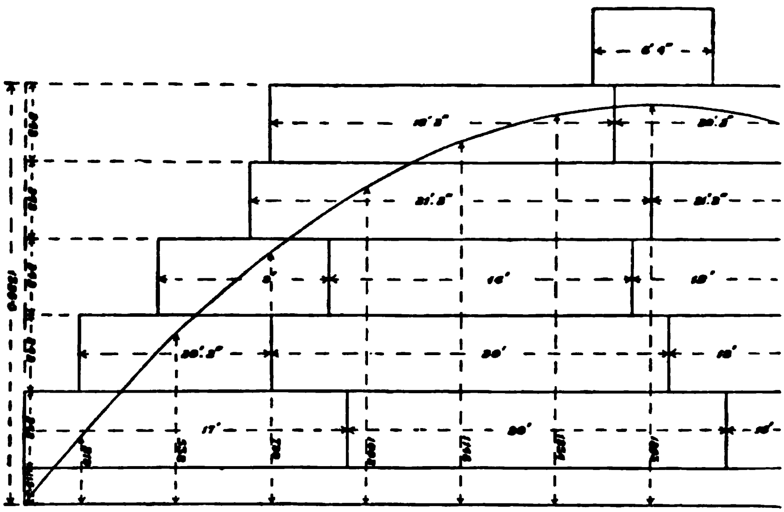


FIG. 338.

The moment of resistance of the angle irons in tension is—

$$5{\cdot}25 \times 4 \times 5{\cdot}5 = 115{\cdot}5 \text{ foot-tons}$$

The moment of resistance of each plate in tension is—

$$(24 - 2)\tfrac{1}{2} \times 4 \times 5{\cdot}5 = 242 \text{ foot-tons}$$

The details of the bridge are sufficiently illustrated in Plate I. The design of the group joints in the booms, and angle wrappers, have been fully considered in Chapter XIII.

The weight of the girder may now be found.

TABLE LX.

	Weight in pounds per lineal foot.	Weight in pounds.
Booms.		
5347 feet of plates in top and bottom booms, including top covers 2' wide by ½" thick	40·0	21,388
56 feet of plates in inside cover strips 7" × ½" ...	11·7	655·2
2 × 5' 5" × 2' × ⅜" = 11 feet, end plates	30·0	330·0
280 feet of angle iron, 4" × 4" × ½"	12·5	3,500 0
22 „ „ at ends, 3" × 3" × ½" ...	9·2	202·4
12 „ „ in covers, 3½" × 3½" × ⅜" ...	7·0	84·0
8 corner covers at ends, at 4 lbs. each	—	32·0
Web.		
10⅜ feet of web plate, 5' 5" × 5/16 = 57·7 feet ...	12·5	721·3
21⅓ „ „ 5' 5" × ⅜ = 115·4 „ ...	15·0	1,731·0
10⅓ „ „ 5' 5" × 7/16 = 57·7 „ ...	17·5	1,009·8
10⅓ „ „ 5' 5" × ½ = 57·7 „ ...	20·0	1,154·0
16 „ „ 5' 5" × ⅝ = 86·5 „ ...	25·0	2,162·5
247 feet angle iron in stiffeners, 3" × 3" × ⅜" ...	7 0	1,729·0
13 plates in stiffeners, 5' 5" × 11 × 5/16 = 71·5 feet...	11·5	822·0
13 „ „ 3' 0" × 8" × 5/16 = 39 „ ...	8·4	327·6
120 feet of angle iron in bent plate attachment, 3" × 3" × ⅜"	7·0	840·0
		36,688·8
Add 5 per cent. for rivets		1,834·4
Total weight of one main girder		38,523·2 = 17·2 tons.

Total weight of two main girders = 34·4 tons. The approximate weight assumed in the calculations was 35·5 tons, so that no recalculation is necessary.

The weight of the flanges in one girder is 26,191 pounds, and of the web 10,497 pounds; so that more than ⅔ of the total weight is in the flanges.

The main girders may be riveted up in two lengths, leaving the rivets in the central covers and web joint to be put in on the site. There would be no difficulty in introducing two more joints in the angle wrappers similar to the central joint, in which case the girder could be riveted up in four lengths, leaving the rivets in the three group joints, in the covers, and in the three web joints to be put in on the site.

CHAPTER XVIII.

EXAMPLE OF THE DESIGN OF A STEEL AMERICAN TRUSS BRIDGE, WITH PIN CONNECTIONS, FOR A SINGLE LINE OF RAILWAY.

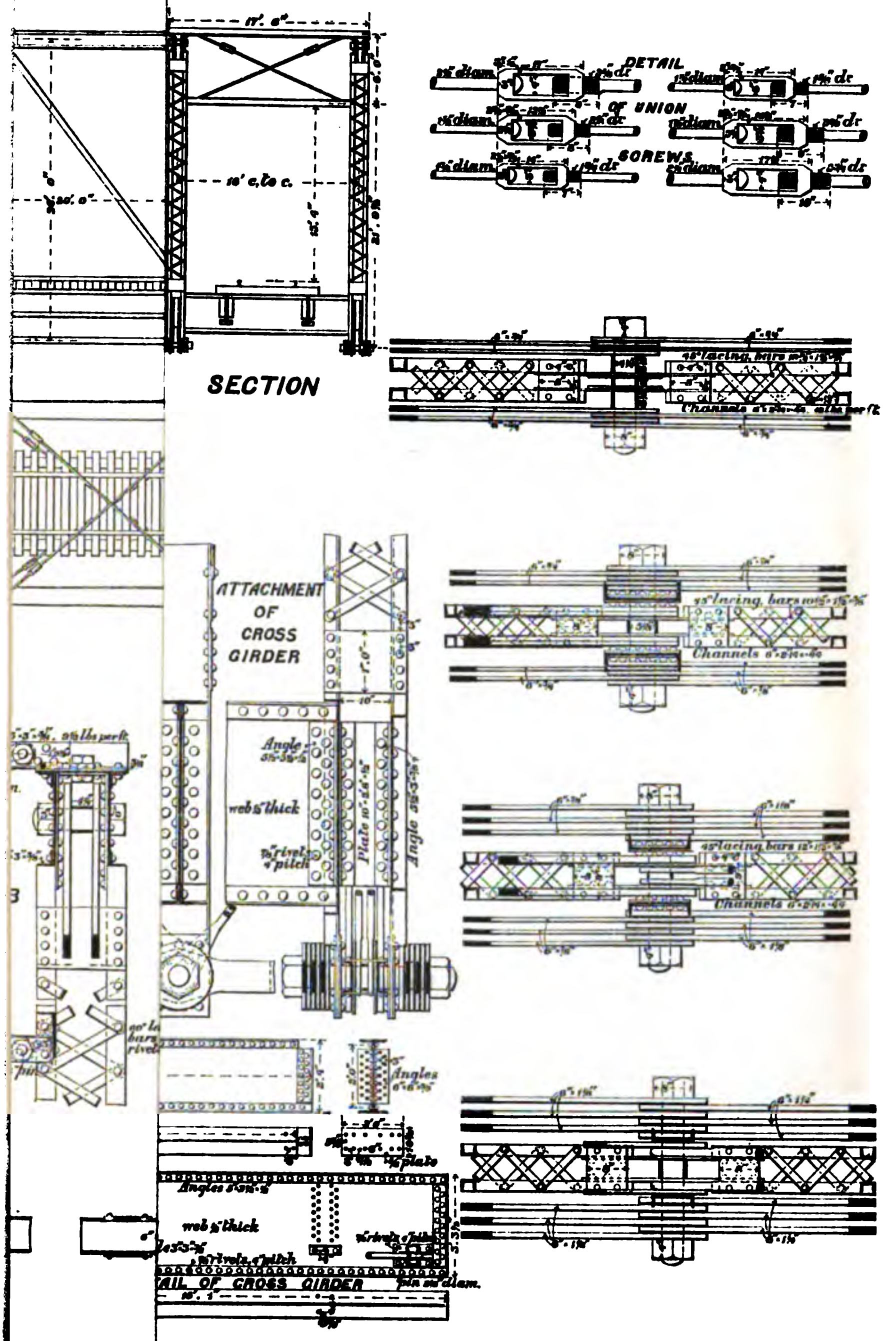
THE order of procedure which should be followed in the design of a bridge of this class is to first consider the deck and the floor beams in the manner explained in Chapter XIV. The deck and floor beams designed in Chapter XIV. will be used in this bridge. The panel loads due to the weight of the structure should next be determined in the manner explained in Chapter XV., and a skeleton diagram prepared for calculating the stresses due to the dead load of the structure, and those due to the live load. In this bridge the concentrated-load system explained in Chapter XV. will be used to determine the stresses due to the live load. The total maximum stresses should be then determined and written on the skeleton diagram of the truss, from which the sectional areas of the various members and the details of construction may be provisionally considered, to be afterwards reconsidered in regard to the additional stresses due to wind.

It will probably be most convenient in this example to describe briefly the bridge as actually designed before considering the stresses, in order that the functions of the various members may be more clearly understood.

Fig. 339 shows a skeleton diagram of half a span of the bridge in isometric projection, with the names of the various members written on the diagram; and Plate II. shows the details of the truss, with the more important joints and connections.

It will be seen, by referring to Plate II., that the two trusses forming one span are 180 feet between centres of bearings on the supports or piers, and are spaced 16 feet apart from centre

DESIGN PLATE II.



to centre. The effective depth of the truss is 26 feet, measured between the centres of the pins in the top and bottom chords. The bottom chord, hip verticals, and the diagonal members of the web are constructed with eye-bars of the form shown in Fig. 278. The top chord and batter braces are constructed in

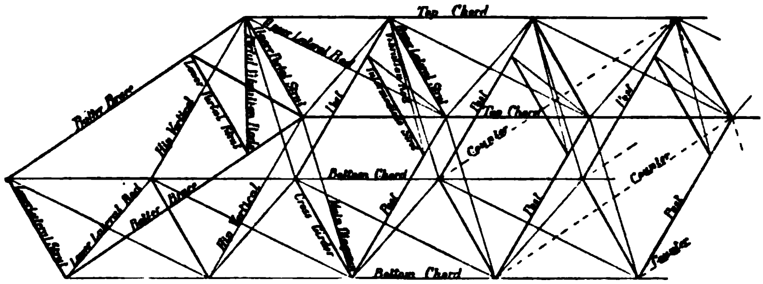


FIG. 339.

the form of an inverted-trough section, with the material concentrated as much as possible equally on each side of the line passing through the centres of the pins. The top flange plate is 18 inches by $\frac{3}{8}$ inch throughout, and the vertical plates are 18 inches deep, united to the flange plates with angles. The vertical plates are stiffened at the bottom with angles, and laced with bars inclined at 60 degrees. The joints in the top chord are planed so as to butt truly, the covers and rivets being merely necessary to hold the pieces in position until they are permanently fixed by the pins; reinforcing plates are also used for limiting the pressure on the bearing area of the pins. These joints occur at a distance of eight inches from the centre of the pins in each panel, excepting at the top hips, where the joint is made at the intersection of the top chord and batter brace, passing also through the centre of the pin, extra reinforcing plates and cover-plates being used as shown in Plate II.

The pins are $4\frac{1}{4}$ inches in diameter excepting at the hips, and joints in the bottom chord, where they are $5\frac{1}{8}$ inches in diameter.

The vertical members of the web, or compression posts, are constructed with two channel bars arranged back to back, 10 inches apart, and laced on each side with bars inclined at 60 degrees. The vertical members butt against the lower portion of the top chord, and are connected to the chord by means of plates $\frac{1}{2}$ inch thick, through which pass the pins.

The deck, longitudinal, and cross floor beams are shown on plate II., and have been fully considered in Chapter XIV. The cross floor beams are riveted to the vertical members above the pins in the bottom chord, and the web is made extra thick in order to resist the wind stresses developed in the bottom lateral system of wind bracing, in which the cross floor beams act as struts. There is a top and bottom system of wind bracing shown in Plate II., Figs. 2 and 3, in which the tension members are provided with union screws for putting them in initial tension. The compression members in the top system consist of double angles, and in the bottom system of the cross floor beams, excepting at the ends of the truss and at the hip, where double channel bar struts are used similar in construction to the vertical compression posts.

The vertical sway bracing is shown in Figs. 4 and 5, and consists of a horizontal strut formed with double angle bars, with diagonal vibration rods, provided with union screws for initial tension. The portal bracing lies in the plane of the batter braces, and is constructed in a similar manner to the sway bracing, but with larger struts and vibration rods. The wind pressure may reverse the stresses in the eye-bars of the bottom chord, so that a double channel-iron strut is used throughout the length of the chord, which is designed to resist the excess of the compressive stress due to wind over the tensile stress due to the dead load.

The trusses are provided with saddle bearings at each end, with expansion rollers at one end.[1]

Plate II. should be carefully studied by the student, and the function of each member of the truss, and details of joints and connections, thoroughly understood before commencing the detail calculations.

Let w_1 denote the load acting at each panel-point of the top chord; then, as explained in Chapter XV.,

$$w_1 = \frac{5 \times 180}{2 \times 2} \times 20 = 4500 \text{ lbs.} = 2{\cdot}1 \text{ tons}$$

Let w_2 denote the load acting at each panel-point of the bottom chord, including the half-panel load of the truss, the dead load of the deck discharged by the cross floor beams, which

[1] The expansion rollers used were designed by Mr. J. A. MacDonald, M.Inst.C.E. Engineer for Bridges, N.S. Wales, and have been used extensively in Australia.

will be about 200 lbs. per foot for the deck, and 173 lbs. for the weight of the longitudinals and cross floor beams, thus—

$$w_2 = \left\{\frac{5 \times 180}{2 \times 2} + 173 + 200\right\}20 = 11{,}960 \text{ lbs.} = 5{\cdot}4 \text{ tons}$$

$$\tan\theta = 0{\cdot}769$$

$$\sec\theta = 1{\cdot}26$$

$$w_1 + w_2 = 7{\cdot}5 \text{ tons}$$

The live load is carried on the bottom chord; the vertical members are in compression, and the inclined members of the web in tension. The dotted lines indicate the counterbraces.

The tabulation for the half-engine loads for two locomotives as shown in Fig. 326 is given in Table No. LXI., and with the aid of this table there is no difficulty in finding the position of the engines and the train when the maximum shearing stress occurs. In this case the maximum shearing stress in any panel will occur when nine times the load on it is as nearly as possible equal to, and not less than, the total load on the bridge. In panel 11–8, for example, the leading wheel W_1 advancing from the right must be at the point 11, while in panel 14–5 the first driving wheel W_2 must be at the point 14.

TABLE LXI.

	Number of wheels on the bridge n.	Total load on the bridge, $W_1 + W_2 + \ldots + W_n$.	Distance between the centres of wheels.	Total distances measured from the first or leading wheel $x_1 + x_2 + \ldots + x_n$.	Product of the load on each wheel into the distance from the next following wheel.	Sum of the products of the loads upon each wheel multiplied by the distances between the wheels $W_1x + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_n)x_n$.
First engine.	1	4	8·08	8·08	32·32	32·32
	2	10	5·75	13·83	57·50	89·82
	3	16	4·50	18·33	72·00	161·82
	4	22	4·50	22·83	99·00	260·82
	5	28	7·08	29·91	198·24	459·06
	6	32	4·83	34·74	154·56	613·62
	7	36	5·67	40·41	204·12	817·74
	8	40	4·83	45·24	193·20	1010·94
	9	44	9·00	54·24	396·00	1406·94
Second engine.	10	48	8·08	62·32	387·84	1794·78
	11	54	5·75	68·07	310·50	2105·28
	12	60	4·50	72·57	270·00	2375·28
	13	66	4·50	77·07	297·00	2672·28
	14	72	7·08	84·15	509·76	3182·04
	15	76	4·83	88·98	367·08	3549·12
	16	80	5·67	94·65	453·60	4002·72
	17	84	4·83	99·48	405·72	4408·44
	18	88	4·00	103·48	352·00	4760·44

The position of the wheel loads producing the maximum shearing stress in the various panels of the truss is shown in the following table:—

TABLE LXII.

Panel.	Number of engine wheel at the right of panel.	Total load on the truss in tons.	n.	x_n.
11–8	W_1 at 11	22	4	1·67
12–7	W_1 „ 12	36	7	5·26
13–6	W_2 „ 13	60	12	0·01
14–5	W_2 „ 14	76	16	3·93
15–4	W_3 „ 15	88 + 0·75(10·35)	Uniform load.	5·175
16–3	W_3 „ 16	88 + 0·75(30·35)	„	15·175
17–2	W_3 „ 17	88 + 0·75(50·35)	„	25·175
1–2	W_3 „ 18	88 + 0·75(70·35)	„	35·175

The maximum shearing stresses in the panels due to the position of the loads indicated in the above table are—

TABLE LXIII.

Panel.	$\frac{1}{l}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_n)x_n\} - \frac{1}{b}\{W_1x_1 + (W_1 + W_2)x_2 + \ldots + (W_1 + W_2 + \ldots + W_{n'-1})\theta\}$	Shear in tons.
11–8	$\frac{1}{180}\{22(1·67) + 161·82\}$	1·10
12–7	$\frac{1}{180}\{36(5·26) + 613·62\}$	4·46
13–6	$\frac{1}{180}\{60(0·01) + 2105·28\} - \frac{32·32}{20}$	10·08
14–5	$\frac{1}{180}\{76(3·93) + 3182·04\} - \frac{32·32}{20}$	17·70
15–4	$\frac{1}{180}\{88(10·35) + 0·75(10·35)(5·175) + 4760·44\} - \frac{89·82}{20}$	27·24
16–3	$\frac{1}{180}\{88(30·35) + 0·75(30·35)(15·175) + 4760·44\} - \frac{89·82}{20}$	38·71
17–2	$\frac{1}{180}\{88(50·35) + 0·75(50·35)(25·175) + 4760·44\} - \frac{89·82}{20}$	51·85
1–2	$\frac{1}{180}\{88(70·35) + 0·75(70·35)(35·175) + 4760·44\} - \frac{89·82}{20}$	66·66

The stresses in the inclined bars may be found by combining the shear in the panel due to live load with that due to the dead load, and multiplying by sec θ.

TABLE LXIV.

Inclined bars.	Maximum shearing stress due to live and dead load in tons.	Total.
13–6 and 16–5	(10·08)1·26	+ 12·7
14–5 „ 15–6	(17·70)1·26	+ 22·3
15–4 „ 14–7	(27·24 + 7·5)1·26	+ 43·77
16–3 „ 13–8	(38·71 + 15·0)1·26	+ 67·67
17–2 „ 12–9	(51·85 + 22·5)1·26	+ 93·68
1–2 „ 10–9	− (66·66 + 30·0)1·26	− 121·79

The stresses in all the vertical bars, excepting 11–9 and 18–2, may be written down in a similar manner, thus—

TABLE LXV.

Vertical bars.	Maximum shearing stress due to live and dead loads in tons.	Total.
15–5 and 14–6	17·70 + 2·1	− 19·81
16–4 „ 13–7	27·24 + 5·4 + 4·2	− 36·84
17–3 „ 12–8	38·71 + 10·8 + 6·3	− 55·81

The maximum stress in the bars 11–9 and 18–2 will occur when the reaction at the cross-girder is a maximum, and this, we have seen in Chapter XIV., is + 20·9 tons for the live load, and the dead-load stress is + 5·4, so that the total stress is 26·3 tons.

The maximum stresses in the horizontal members may be found when the position of the engine-wheels and uniform load is known. We have seen that the maximum bending moment occurs when—

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_{n'}}{W_1 + W_2 + \ldots + W_n} = \frac{R}{W_1 + W_2 + \ldots + W''}$$

where l' = the horizontal distance from the left abutment to the point about which the moments are taken. The following table shows the position of the engine loads and train which produce the maximum stresses in the top and bottom horizontal members, when one load is at the panel-point l' from the left abutment:—

TABLE LXVI.

Horizontal members.	Point about which moments are taken.	Ratio $\frac{l'}{l}$	$W_{n'}$.	$W_{n'-1}$.	Uniform load W_n.	Half length of uniform load.	Total load on l.	Load on l'.
4–5 and 5–6	15	$\frac{4}{9}$	12	11	48·5	$\frac{65·59}{2}$	136·5	60
3–4	16	$\frac{3}{9}$	9	8	46·32	$\frac{61·76}{2}$	134·32	44
2–3	17	$\frac{2}{9}$	6	5	49·82	$\frac{66·43}{2}$	137·84	32

Horizontal members top and bottom.	$\frac{M}{d}=\frac{l'}{ld}\{W_1x_1+(W_1+W_2)x_2+\dots+(W_1+W_2+\dots+W^n)x_n\}$ $-\frac{1}{d}\{W_1x_1+(W_1+W_2)x_2+\dots+(W_1+W_2+\dots+W_{n'-1})\theta\}$	Live load stresses in tons.
4–5, 5–6, 6-7 ...	$\frac{4}{9\times 26}\{4760·44+[(88)2+48·5]32·295\}-(2105·28)\frac{1}{26}$	– 124·40
3–4, 7–8 ...	$\frac{3}{9\times 26}\{4760·44+[(88)2+46·32]30·88\}-(1010·94)\frac{1}{26}$	– 110·16
2–3, 8–9 ...	$\frac{2}{9\times 26}\{4760·44+[(88)2+49·82]33·215\}-(459·06)\frac{1}{26}$	– 87·15
1–18, 18–17 12–11, and 11–10	Shear × tan θ = 66·66 × 0·769	51·26
15–14	" "	124·44
16–15, 14–13 ...	" "	110·16
17–16, 13–12 ...	" "	87·15

TABLE LXVII.

TOTAL STRESSES IN TONS.

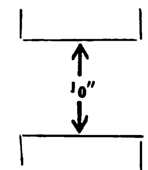
Panel.	Stresses.	Total.
4–5, 5–6, 6–7	$-\{124·40+10(w_1+w_2)\tan\theta\}$	– 182·08
15–14	+ { " " " }	+ 182·08
3–4, 7–8	$-\{110·16+9(w_1+w_2)\tan\theta\}$	– 162·1
16–15, 14–13	+ { " " " }	+ 162·1
2–3, 8–9	$-\{87·15+7(w_1+w_2)\tan\theta\}$	– 127·5
17–16, 13–12	+ { " " " }	+ 127·5
1–18, 18–17, 12–11, 11–10	$+\{51·26+4(w_1+w_2)\tan\theta\}$	+ 74·3

The stresses may now be written on the various members of the truss, Plate II., Fig. 1.

The wind stresses in this truss will be considered in Chapter XIX.

Table LXVIII.

Number of member.	Stresses in tons due to — Dead load.	Live load.	Total load.	Working stress in tons per sq. in. for — Dead load.	Live load.	Sectional area required in square inches.	Section provided in bridge.
Vertical columns—							
6–14	2·1	17·70	19·8	4	2·0	9·38	Two channels 8 inches deep at 17¼ lbs. per foot.
5–15	2·1	17·70	19·8	4	2·0	9·38	Ditto. Ditto.
4–16	9·6	27·24	36·9	4	2·0	16·02	Two channels 12 inches deep at 28 lbs. per foot.
3–17	17·1	38·71	55·8	4	2·0	23·64	Two „ 15 „ „ 41 „ „
Hip verticals ...	5·4	20·90	26·3	7	3·5	6·74	Two bars, each 7″ × ½″.
Diagonals—							
2–17	28·35	65·4	92·9	7	3·5	22·8	Four bars, each 6″ × 1′.
3–16	18·90	48·8	67·2	7	3·5	16·7	Two „ „ 6¼″ × 1⅜″.
4–15	9·45	34·3	43·5	7	3·5	11·2	Two „ „ 6″ × 1″.
5–14	—	22·3	22·3	7	3·5	6·4	Two „ „ 4″ × ⅞″.
6–13	—	12·7	12·7	7	3·5	3·7	Two round rods screwed 1⅝″ diameter.
Bottom chord—							
15–14	57·68	124·4	182·1	7	3·5	43·8	Three pairs of bars 6″ × 1¼″.
16–15	51·91	110·16	162·1	7	3·5	38·9	Three „ „ 6″ × 1⅛″.
17–16	40·38	87·15	127·5	7	3·5	30·7	Three „ „ 6″ × ⅞″.
18–17	23·07	51·26	74·4	7	3·5	18·0	Two „ „ 6″ × ¾″.
1–18	23·07	51·26	74·4	7	3·5	18·0	Two „ „ 6″ × ¾″.
Top chord—							
1–2	23·07	51·26	74·3	6	3·0	20·9	Four angles 3″ × 3″ × ⅜″, one cover 18″ × ⅜″, two plates 18″ × ⅜″.
2–3	40·38	87·15	127·5	6	3·0	35·8	Four angles 3″ × 3″ × ⅜″, one cover 18″ × ⅜″, two plates 18″ × ⅝″.
3–4	51·91	110·16	162·1	6	3·0	45·4	Four angles 3″ × 3″ × ⅜, one cover 18″ × ⅜″, two plates 18″ × ⅞″.
4–5	57·68	124·4	182·1	6	3·0	51·1	Four angles 3″ × 3″ × ⅜″, one cover 18″ × ⅜″, two plates 18″ × 1″.



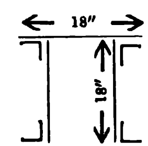


Table LXVIII. may now be prepared, and the sectional areas of the various members determined. In this case Mr. Theodore Cooper's rules have been used for finding the safe intensity of working stress instead of the Launhardt and Weyrauch formulæ given in Chapter I. The figures explain themselves for the tensile stresses, but the straight-line formulæ given in Chapter XII. have been used for determining the working stress in compression.

The actual sections adopted have been taken from Carnegie's list of rolled sections.

The details may be further studied by referring to Plate II.

CHAPTER XIX.

WIND PRESSURE.

THE published information on this subject which may be consulted for more complete information includes the following:—

Papers by Mr. C. B. Bender and Professor Gaudard, published in *Proceedings of the Institution of Civil Engineers* (Vol. LXIX. p. 80), with discussions by Sir B. Baker and other well-known authorities.

"Experiments by Sir B. Baker at the site of the Forth Bridge," published in *Engineering*, February 28, 1890.

Experiments by Mr. O. T. Crosby, published in *Engineering*, May 30, June 6, and June 13, 1890.

Paper by Mr. C. Shaler Smith, in *Proceedings of the American Society of Civil Engineers*, Vol. 10, p. 139.

When a horizontal force of wind strikes an inclined surface it causes a normal pressure upon that surface (Fig. 340), the intensity of which is expressed by the following formula, deduced from experiments made by Mr. Hutton for gunnery purposes.

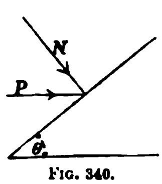


FIG. 340.

Let P denote the horizontal pressure on an inclined surface, such as a roof inclined at an angle θ, Fig. 340.

Let N denote the normal pressure. Then—

$$N = P(\sin\theta)^{1\cdot84\cos\theta - 1}$$

Professor Unwin states that this formula gives almost the same results as Duchemin's, which is—

$$N = P\frac{2\sin\theta}{1+\sin^2\theta}$$

The following table gives the values of N for different values of θ when P = 40 lbs. per square foot.

TABLE LXIX.

Angle of roof, θ.	Normal pressure, N. Hutton.	Normal pressure, N. Duchemin.	Angle of roof, θ.	Normal pressure, N. Hutton.	Normal pressure, N. Duchemin.
5°	5·1	6·89	50°	38·1	38·64
10°	9·7	13·59	60°	40·0	39·74
20°	18·4	24·24	70°	40·0	39·91
30°	26·5	32·00	80°	40·5	40·00
40°	33·3	36·40	90°	40·0	40·00

The experiments of Sir B. Baker show that the average pressure over a large surface is less than over a small surface. The large gauge used for these experiments was 20 feet long by 15 feet deep, exposing 300 square feet of surface; a circular gauge 18 inches in diameter was also arranged in the centre of the large gauge.

The small gauge consisted of a circular plate of 1·5 square feet area.

The revolving gauge also exposed a surface of 1·5 square feet, but was so arranged that it turned round and always faced the direction of the strongest wind.

The large and small fixed gauges were arranged to face the direction from which the strongest winds blow. Some of the results are recorded in the following table:—

TABLE LXX.

Year.	Month and day.	Pressure in pounds per square foot. Revolving gauge.	Small fixed gauge.	Large fixed gauge.	Circular gauge 18″ in diameter in centre of large gauge.	Direction of the wind.
1884	Oct. 27	29	23	18		S.W.
"	" 28	26	29	19		S.W.
1885	March 20	30	25	17		W.
"	Dec. 4	25	27	19		W.
1886	March 31	26	31	19		S.W.
1887	Feb. 4	26	41	15		S.W.
1888	Jan. 5	27	16	7		S.E.
"	Nov. 17	35	41	27		W.
1889	" 2	27	34	12		S.W.
1890	Jan. 19	27	28	16		S.W.
"	" 21	26	38	15		W.
"	" 22	27	24	18	23½	S.W. by W.

The British Board of Trade Rules require that bridges in exposed situations should be calculated for 56 lbs. per square foot, and this pressure to be considered to act over twice the area exposed by the girder; the resistance to be provided by the dead weight of the structure.

In order to deduce the pressure from the velocity of the wind, measured by means of an anemometer, we may use the following formula:—

$$P = \frac{V^2}{200}$$

where P = pressure in pounds per square foot, and V = velocity in miles per hour.

This formula is due to Smeaton, and represents the pressure on small surfaces of about one square foot.

In the present state of our knowledge of wind pressures on structures, it is only possible to estimate roughly the loads which may be expected. The experiments of Sir B. Baker, although probably the most valuable for our purpose, require to be extended considerably before the wind loads on structures can be estimated as accurately as the ordinary live and dead loads. The importance of the subject in connection with the design of large structures may be realized by a consideration of Sir B. Baker's estimate of the stresses in the Forth Bridge, which are as follows:—

Stresses due to dead load	2228 tons.
" " live "	1022 "
" " wind "	2920 "
Total	6224 "

The pressures deduced from anemometer experiments are for an instant only, and do not represent the pressure at the same instant over a large structure such as a bridge, the maximum pressures being due to gusts in advance of the main body of the moving mass of air.

It is known, however, that the velocity of the wind is greater the higher the surface exposed is from the ground, and Rankine has deduced the greatest average pressure which could be realized on tall chimneys as 55 lbs. per square foot.

By calculating the overturning moment of railway carriages and trucks, it is clear that the average wind pressure on the surface exposed in such cases but rarely exceeds 30 lbs. per square foot, as only in very exposed situations and under exceptionally strong winds have railway vehicles been overturned. Since 30 lbs. would overturn most of the rolling stock on existing railways, it is not necessary to estimate a greater pressure on the exposed surfaces of a bridge and train, although a greater pressure may be allowed on the bridge itself without the train. The correct estimate of the exposed surface is very important, and the amount of shelter afforded by the truss on the wind side to the truss on the lee side.

As the pressure of wind on structures which rise a considerable distance above the ground is greater in proportion to the height above the ground, tall chimneys, piers of viaducts, Eiffel towers, etc., may be calculated for from 50 to 56 lbs. per square foot of exposed surface for heights above 100 feet; the first 100 feet may be calculated for 40 lbs. In a circular chimney, the surface exposed is equivalent to a flat surface the width of which is half the diameter of the chimney. In braced piers, towers, and bridges, in order to allow for the effect of the wind on the lee side, the area on the exposed side should be multiplied by two.

The American practice in estimating the force of wind over bridges is as follows:—

The train surface is taken as 10 square feet for every foot in length, and the pressure of wind at 30 lbs. per square foot, or 300 lbs. per lineal foot of train, which must be added to the exposed area of the truss in square feet multiplied by 30 lbs. The former is treated as a moving load, and the latter as a fixed load.

The bridge is also calculated for the case when the train is not on the bridge, by multiplying the exposed surface by 50 lbs. per square foot. The maximum stresses produced by either method of loading is used for designing the wind bracing. The exposed surface of the trusses is sometimes estimated by multiplying the actual area of the vertical projection of chords, verticals, and diagonals by two, and adding the result to the exposed area of the deck, *i.e.* to the span of the truss multiplied by the distance from the bottom of the stringer to the rail level.

The following rules are sometimes used for finding the wind pressure on trusses:—

Let A = the actual areas of the upper chord and verticals.
B = the area of the diagonal ties.
C = the area of the bottom chord.

Then the exposed surface is estimated thus—

$$A + \frac{3B}{2} + 2C$$

This quantity must be added to the area of the floor system as before. To find the fixed load, the exposed area in square feet, found by either of the foregoing methods, is multiplied by 30 lbs., and two-thirds of the result is assumed to act on the bottom lateral system, and the remaining one-third on the top system.[1]

The moving load of 300 lbs. per lineal foot of truss is assumed to act entirely on the bottom lateral system in through bridges, and on the upper lateral system in deck bridges.

The intensity of working stress allowed in tension braces is 15,000 lbs. per square inch for iron, and 18,000 lbs. may be allowed in the case of steel.

The intensity of working stresses is proportionately increased on the compression members (see pp. 197 and 198), as the maximum wind stresses will only occur occasionally, or may never occur.

The foregoing principles may now be applied to the calculations of the stresses due to wind in the truss illustrated in Plate II., which has been already considered for the live and dead load in Chapter XVIII.

Figs. 2 and 3 illustrate the plan of the top and bottom lateral systems, and Figs. 4 and 5 the sway and portal bracing.

$$\text{Length of batter brace} = 33 \text{ feet}$$
$$\text{Length of } pz = 19{\cdot}8 \text{ feet}$$
$$\text{,, } mn' = 25{\cdot}6 \text{ feet}$$
$$\text{Sec } a = 1{\cdot}6$$

The area of the vertical projection of any truss may be

[1] Mr. Theodore Cooper specifies: "To provide for wind stresses and vibrations, the top lateral bracing in deck bridges and the bottom lateral bracing in through bridges shall be proportioned to resist a lateral force of 450 lbs. for each foot of span; 300 lbs. of this to be treated as a moving load." "The bottom lateral bracing in deck bridges and the top lateral bracing in through bridges shall be proportioned to resist a lateral force of 150 lbs. for each foot of span." Mr. Cooper also prefers to make the lateral bracing in the floor system to resist compression as well as tension, so that he does not approve of the practice of considering the cross-girders as struts, and designing the diagonals as ties.

calculated when the sizes of the various members have been determined; in this case they have been found (p. 283).

Area of the upper chord $= \dfrac{(140 + 66) \times 12 \times 18}{144} = 309$ sq. feet.

Area of the six vertical posts $= \dfrac{6 \times 26 \times 12 \times 12}{144} = 156$ „

Area of the two hip verticals $= \dfrac{2 \times 26 \times 12 \times 7}{144} = 30$ „

Area of the bottom chord, averaging 6 inches in width $= \dfrac{180 \times 12 \times 6}{144} = 90$ „

Area of the eight diagonals, averaging 6·6 inches in width $= \dfrac{8 \times 33 \times 12 \times 6{\cdot}5}{144} = 143$ „

Total = 728 „

The effective area $= 728 \times 2 = 1456$ sq. feet.
The area of the floor system $= 180 \times 3{\cdot}5 = 630$ „

Total exposed area = 2086 „

The total fixed load on the apices of the top lateral system $= \dfrac{2086 \times 30}{3} = 20{,}860$ lbs.

Load per lineal foot $= \dfrac{20860}{180} = 116$ lbs.

Apex load $= 116 \times 20 = 2320$ lbs. $= 1{\cdot}04$ tons

The total fixed load on the apices of the bottom lateral system $= \dfrac{2 \times 2086 \times 30}{3} = 41{,}720$ lbs.

Load per lineal foot $= \dfrac{41720}{180} = 232$ lbs.

Apex load $= 232 \times 20 = 4640$ lbs. $= 2{\cdot}08$ tons

Total moving load on the bottom lateral system $= 180 \times 300 = 54{,}000$ lbs.

Apex load $= 20 \times 300 = 6000$ lbs. $= 2{\cdot}68$ tons

The following is the tabulation of the stresses :—
Top lateral system ; fixed load only—

$pp' = -1{\cdot}04$ tons $pq' = 0$
$oo' = -2{\cdot}08$,, $op' = +2{\cdot}08 \sec a = +3{\cdot}22$ tons.
$nn' = -3{\cdot}12$,, $no' = +3{\cdot}12 \sec a = +4{\cdot}99$,,
$mm' = -4{\cdot}16$,, $mn' = +4{\cdot}16 \sec a = +6{\cdot}66$,,

Bottom lateral system; fixed and moving load—

$$ee' = -(2{\cdot}08 + \tfrac{10}{9} \times 2{\cdot}68) = -5{\cdot}06 \text{ tons}$$
$$ef' = 5{\cdot}06 \sec a = +8{\cdot}09 \text{ tons}$$
$$dd' = -(4{\cdot}16 + \tfrac{15}{9} \times 2{\cdot}68) = -8{\cdot}63 \text{ tons}$$
$$de' = 8{\cdot}63 \sec a = +13{\cdot}81 \text{ tons}$$
$$cc' = -(6{\cdot}24 + \tfrac{21}{9} \times 2{\cdot}68) = -12{\cdot}49 \text{ tons}$$
$$cd' = 12{\cdot}49 \sec a = +19{\cdot}98 \text{ tons}$$
$$bb' = -(8{\cdot}32 + \tfrac{28}{9} \times 2{\cdot}68) = -16{\cdot}66 \text{ tons}$$
$$bc' = 16{\cdot}66 \sec a = +26{\cdot}66 \text{ tons}$$
$$aa' = -(10{\cdot}40 + \tfrac{36}{9} \times 2{\cdot}68) = -21{\cdot}12 \text{ tons}$$
$$ab' = 21{\cdot}12 \sec a = +33{\cdot}79 \text{ tons}$$

Let the bridge be empty and subjected to a wind pressure of 50 lbs. per square foot.

The stresses in the lower lateral system will not be increased, and need not be considered.

The pressure at the upper panel-points is—

$$\frac{2086 \times 50 \times 20}{180 \times 3 \times 2240} = 1{\cdot}73 \text{ ton}$$

The stresses are therefore—

$pp' = -1{\cdot}73$ tons $pq' = 0$
$oo' = -3{\cdot}46$,, $op' = 3{\cdot}46 \times \sec a = +5{\cdot}53$ tons
$nn' = -5{\cdot}19$,, $no' = 5{\cdot}19 \times \sec a = +8{\cdot}30$,,
$mm' = -6{\cdot}92$,, $mn' = 6{\cdot}92 \times \sec a = +11{\cdot}07$,,

2·5 tons must be added to all the foregoing stresses for initial tension, and the resolved part of this initial tension, viz. 2·2 tons, in the compression members.

Vertical Sway Bracing.—The form of bracing shown in Figs. 4 and 5 is very convenient when there is sufficient headway to allow for the insertion of the intermediate lateral strut. The stresses due to the lateral oscillations caused by the passage of the live load cannot be well determined, but the wind stresses may be investigated by assuming that the total horizontal pressure acting on the wind side of the truss causes it to bend in a vertical plane towards the lee side, relieving the wind side of part of its load, and transferring the same to the lee side.

The forces acting at the upper and intermediate lateral struts (Fig. 4) may be estimated in the following manner:—

The exposed area of the two trusses = 1456 square feet. The total pressure, assuming the bridge to be empty, is 1456 × 50 = 72,800 lbs. At each panel the pressure is—

$$\frac{72.800 \times 20}{180} = 8088 \text{ lbs.} = 3·61 \text{ tons}$$

At the two top panel-points, pp', we have—

$$\frac{3 \times 3·61}{2 \times 26} = 0·208 \text{ ton}$$

At the two intermediate points, yz—

$$\frac{13 \times 3·61}{2 \times 26} = 0·903 \text{ ton}$$

The pressure of the wind on the floor system must be considered if the cross-girders are attached to the vertical compression posts, as the wind pressure acting on the vertical projection of the deck will also tend to relieve the wind side of a portion of its load, transferring the same to the lee side.

The total wind pressure on the deck is 630 × 50 = 31,500 lbs.

The pressure per panel is—

$$\frac{31500 \times 20}{180} = 3500 \text{ lbs.} = 1·5 \text{ ton}$$

This force acts at the centre of the group of rivets uniting the cross-girders to the vertical posts, is 2·4 feet above the centre of the pins in the bottom chord.

If the cross-girders are suspended by means of hangers, the wind pressure on the deck will not affect the vertical sway bracing.

In the present case the effect may be estimated by assuming a force to be concentrated at the points, $pp'yz$, which would produce the same moment about e as the wind pressure on the deck, and therefore relieving the wind side of the same portion of its load; thus—

$$\frac{2·4 \times 1·5}{4 \times 23} = 0·04 \text{ ton}$$

This must be added to the pressures at the top and intermediate panel-points already obtained, thus—

$$\text{at } pp' = 0{\cdot}208 + 0{\cdot}040 = 0{\cdot}248 \text{ ton}$$
$$\text{at } yz = 0{\cdot}903 + 0{\cdot}040 = 0{\cdot}943 \quad ,,$$

The released weight, found by taking moments about e, is—

$$\frac{2(0{\cdot}943 \times 20) + 2(0{\cdot}248 \times 26)}{16} = 3{\cdot}16 \text{ tons}$$

The stress in the bar $p'y = 3{\cdot}16 \sec \theta$
$= 3{\cdot}16 \times 2{\cdot}85 = 9$ tons

Add 2½ tons for initial tension,[1] making the total stress 11·5 tons. We may take the working stress as 8 tons for steel, and 6½ tons for iron in such bars, thus the area required in a steel bar is $\frac{11{\cdot}5}{8} = 1{\cdot}44$ square inch.

The bar pz will be stressed to a similar amount when the wind acts on the right side instead of the left.

The stress in the strut yz may be found by the method of sections, thus: Take moments about p', and, considering the forces on the lee side, we obtain—

$$\frac{-1{\cdot}191^{2} \times 26 + 0{\cdot}943 \times 6}{6} = -4{\cdot}22 \text{ tons}$$

The horizontal component of the initial tension must be added, which will increase the stress to 6·44 tons.

The stress in the bar pp', considered as part of the vertical sway bracing, will also be 6·44 tons; so that wherever the stresses in the top lateral posts are less than this amount, considering them as part of the top lateral system, they must be increased. In this case the struts oo', pp', $q'q'$, and rr' must be designed for

[1] Considerable difference of opinion exists as to the proper amount to allow for initial tension. Professor Burr allows 500 lbs. for all cases of stresses in bridge and roof trusses. Mr. Theodore Cooper states in his specifications, "The lateral struts shall be proportioned by the formula $I = 9000 - 50\frac{l}{r}$, to resist only the resultant due to an assumed initial stress of 10,000 lbs. per square inch upon all the rods attaching to them, assumed to be produced by adjusting the bridge or towers." Here l = length of strut, r = the least radius of gyration. Others allow about 1½ ton per square inch of the section of the bar for initial tension.

[2] In Fig. 4, Plate II., this force has been incorrectly engraved as 1·291 instead of 1·191.

6·44 tons. We may apply either Rankine's or the straight-line formula to find the necessary area.

For angle-iron struts the following formula may be used for finding the working stress, f—

$$f = \frac{9300}{1 + \frac{l^2}{30000r^2}}$$

l = length of strut
r = least radius of gyration

Hence the sectional area will require to be about 3·22 square inches, and two angles 5″ × 3″ × ⅜″ at 9·5 lbs. per foot may be used.

The bending moment at y should be considered when the bridge is loaded with the live load and 30 lbs. wind pressure. The post should be considered as fixed at the ends.

In this case the increase in the sectional area of the channel iron post is about 1¼ square inch. The transferred load from the wind to the lee side will also slightly increase the compression on the post.

The stresses in the portal bracing are found in a similar manner to the foregoing by making the following substitutions:—

(*a*) For panel pressures at top points, the sum of the panel pressures concentrated on one-half of the top lateral system; *i.e.* in example—

$$\frac{6 \cdot 92}{2} = 3 \cdot 46 \text{ tons}$$

(*b*) For the distance py, Fig. 4, the perpendicular distance between the end strut of the top lateral system and the intermediate portal strut.

(*c*) For the depth of the girder the length of the batter brace.

The released load is—

$$\frac{6 \cdot 92 \times 33 + 1 \cdot 89 \times 25 \cdot 4}{16} = 16 \cdot 023 \text{ tons}$$

The stress in the bar lm', Fig. 5 $= 16 \cdot 023 \sec \theta = 37 \cdot 34$ tons

Allowing 5 tons for initial tension in two rods, the total stress is 42·34 tons. Two bars 1⅞ inch diameter will be sufficient.

The stresses in the intermediate portal strut is—

$$\frac{-4{\cdot}405 \times 33 + 0{\cdot}94 \times 7{\cdot}6}{7{\cdot}6} = 19{\cdot}5 \text{ tons}$$

Increase for component of initial tension = 4·44 tons
∴ Total stress = 23·94 tons

The top strut must also be designed for this stress. The section may consist of two channel bars 7 inches deep, at 14½ lbs. per foot, spaced 7 inches apart.

With a wind pressure of 50 lbs., the bottom chord undergoes a compressive stress which exceeds the tensile stress produced by the dead load, hence a compression member must be introduced to take the difference of these stresses. The excess of compressive over tensile stress is shown in the following table:—

TABLE LXXI.

Bar.	Tension due to dead load in tons.	Compression due to wind pressure in tons.	Difference of stresses in tons.
1–18	+ 23·07	− 26·4	− 3·33
18–17	+ 23·07	− 47·23	− 24·16
17–16	+ 40·38	− 62·84	− 22·46
16–15	+ 51·91	− 73·63	− 21·72
15–14	+ 57·68	− 79·95	− 22·27

The details of the wind bracing are sufficiently illustrated on Plate II.

The wind pressure produces an increase in the stresses of the bottom chord, which will necessitate an increase in the sectional area if the stress is 25 per cent. greater than that for which the eye-bars were designed.

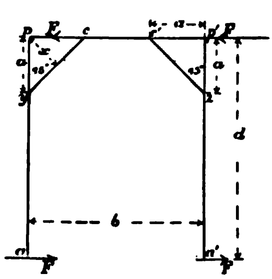


FIG. 341.

When the depth of the truss is not sufficient to allow for the vertical sway bracing shown in Figs. 4 and 5, Plate II., angle brackets or knee pieces are used, as shown in Fig. 341.

Let F denote the wind force at the panel-points p and p'—

$$\text{The released weight} = \frac{2Fd}{b} = w'$$

$$\text{The bending moment at } c = \frac{2Fd}{b}(b - a) - Fd$$

The bending moment at $y = F(d - a)$, considering the post as merely supported at each end; really it is fixed, and the bending moment at y is—

$$\frac{F}{2}(d - a)$$

The stress in cy is found by taking moments about p, and denoting the perpendicular on cy by x—

$$\text{Stress in } cy = \frac{Fd}{x}$$

The stresses in the portal bracing may be found in a similar manner, substituting for d, a, and x their actual lengths as before.

Professor Waddell[1] gives an investigation for determining the stresses in the vertical sway bracing of a double-track bridge, which may be briefly described as follows:—

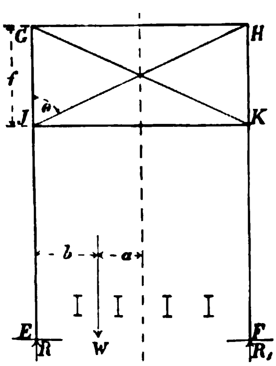


FIG. 342.

When only one track of a double-track bridge without vertical sway bracing is covered by the moving load, the trusses are loaded unequally, the load being divided according to the law of the lever.

If vertical sway bracing is used, we may assume that the load is equally divided between the trusses, the difference between the loads being taken up by the sway bracing, thus (Fig. 342):—

Let R and R_1 = the reactions produced by a load on one track.

W = the load.

G = the load transferred by the bracing; then—

$$G = R - \frac{W}{2} = W\left(\frac{2a + b}{2(a + b)}\right) - \frac{W}{2} = \frac{Wa}{2(a + b)}$$

The stress in the vibration rod JH is—

$$G \sec \theta = \frac{Wa \sec \theta}{2(a + b)}$$

[1] "Memoirs of the Tôkiô Daigaku," by Professor J. A. L. Waddell.

The stress in JK is—

$$\frac{Wa}{2(a+b)} \times \frac{2(a+b)}{f} = \frac{Wa}{f}$$

The stress in GH can be shown to be zero.

The bending moment at K is—

$$M' = \frac{Wa}{2(a+b)}\{2(a+b)\} = Wa$$

This bending moment does not exist at the same time as the maximum stress in the post HF, and need not generally be considered.

The bending moment produced in the long vertical posts about the centre of the trusses in double-line railway bridges will in general require the section of the post to be increased in area beyond that found for the ordinary live and dead loads. It should be remembered that this bending moment has been calculated on the assumption that the reactions R and R_1 have been exactly equalized by the bracing; again, no account has been taken of the fact that the vertical post acts as a beam fixed at the ends in resisting the bending moment at K. Taking these matters into consideration, it follows that the actual bending moment is probably not greater than $\frac{Wa}{8}$.

CHAPTER XX.

STRESSES IN BRACED PIERS.

Braced piers must be designed not only to carry the dead and live load discharged by the main girders or trusses, but to resist the maximum horizontal force due to wind.

The stresses due to wind are considerable in lofty viaducts of this class. One of the best examples occurs on the New York, Lake Erie, and Western Coal and Railway Company's Railway across the Kinzua Valley. The superstructure consists of lattice deck girders designed for a single line, spaced 10 feet apart transversely, and alternatively of 61 and 38½ feet spans.

The towers or piers vary in height from 20 to 280 feet; the total height from rail level to the water level in the Kinzua Creek is 301 feet. The piers are composed of Phœnix columns for the main compression members and transverse struts, with longitudinal struts of four latticed angles and diagonal tension rods arranged in pairs. Very high winds sweep through the gorge, which, taken in conjunction with the vibrations caused by passing trains on such a lofty viaduct, necessitate careful consideration.

The width of the piers at the top is 10 feet, and at the widest part of the base 103 feet.

The method of obtaining the stress in braced piers will be illustrated in the following example :—

Figs. 343 and 344 show two views of a braced pier of similar proportions to those adopted for the Kinzua viaduct. The wind is supposed to act on the right side, and the stresses on one of the frames of the pier will be investigated.

Let P_1 = the pressure on the train taken over one-half the spans r and s.

P_2 = the pressure on the main girders taken over the same distance as P_1.

w_1, w_2, w_3, and w_4 = the panel pressures acting at the four corners of the pier at each story.

H = the total horizontal force acting at the top of the pier.

W = the dead load of half the spans r and s, in addition to the live load distributed over them, considered as acting at the points u v, Fig. 343.

W_1, W_2, W_3, and W_4 = the panel weights of the pier itself acting at the joints as shown in Fig. 343.

The remaining quantities are sufficiently explained in Fig. 343.

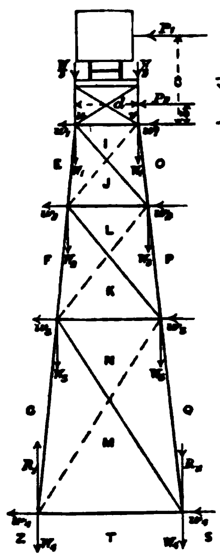


Fig. 343.

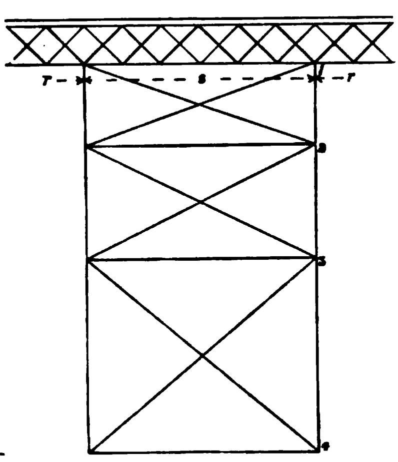


Fig. 344.

The wind pressures P_1 and P_2 on the train and main girders will relieve the pressure at v, and transfer an equal amount to u.

Let x denote this amount; then—

$$x = \frac{P_1 a + P_2 b}{d}$$

Therefore the resultant reactions at u and v will be respectively—

$$\frac{W}{2} + x, \text{ and } \frac{W}{2} - x$$

The horizontal component of x must be added to the horizontal forces P_1 and P_2, and to the panel pressures w_1, to find the total force acting at the top of the pier—

$$H = P_1 + P_2 + x \tan \theta + 2w_1$$

We can set off H and the panel pressures w_2, w_3, and w_4 on a horizontal line as a force polygon, and obtain the reciprocal polygon in the usual way, thus determining the wind stresses, Fig. 345.

The post O1 and the diagonals indicated by the dotted lines are unstressed when the direction of the wind is as indicated. If the wind acts in the opposite direction, the post EJ will be unstressed as well as the bars IJ, LK, and MN. The force Y acting at the base of the pier is represented in magnitude by the length of the line TT in the reciprocal polygon, Fig. 345. It tends to produce sliding at the bed-plates, and must be resisted by the frictions developed by the total reactions due to the live and dead load added to the reactions measured from Fig. 345, or by the horizontal resistance at the feet of the posts due to these reactions added to the shearing resistance of the bolts.

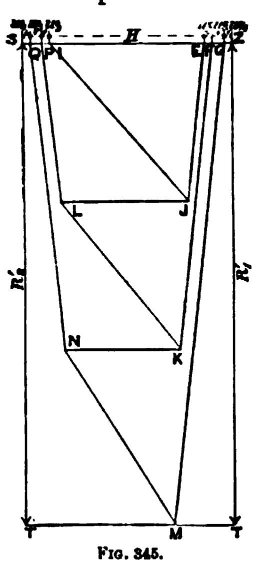


FIG. 345.

The stresses obtained by measuring the lines in Fig. 345 must be added to those produced by the live and dead loads, thus—

TABLE LXXII.

Member of structure, Fig. 343.	Stress due to wind pressure obtained from Fig. 345.	Stresses due to live and dead loads.
JE	− JE	$-\left(\frac{W}{2} + x + W_1\right) \sec \theta$
KF	− KF	$-\left(\frac{W}{2} + x + W_1 + W_2\right) \sec \theta$
MG	− MG	$-\left(\frac{W}{2} + x + W_1 + W_2 + W_3\right) \sec \theta$
IO	—	$-\left(\frac{W}{2} - x + W_1\right) \sec \theta$
LP	+ LP	$-\left(\frac{W}{2} - x + W_1 + W_2\right) \sec \theta$
NQ	+ NQ	$-\left(\frac{W}{2} - x + W_1 + W_2 + W_3\right) \sec \theta$
IE	− IE	$-\left(\frac{W}{2} + W_1\right) \tan \theta$
JL	− JL	$- W_2 \tan \theta$
KN	− KN	$- W_3 \tan \theta$
TM	− TM	$+\left(\frac{W}{2} - x + W_1 + W_2 + W_3\right) \tan \theta$
IJ	+ IJ	
LK	+ LK	
NM	+ NM	

The total reactions R_1 and R_2 may also be found by adding the vertical reactions due to wind pressure as measured from Fig. 345, to the total live and dead load on each side of the frame, thus—

$$R_1 = \frac{W}{2} - x + W_1 + W_2 + W_3 + W_4 + R_1' \text{ (Fig. 345)}$$

$$R_2 = \frac{W}{2} + x + W_1 + W_2 + W_3 + W_4 + R_2' \text{ (Fig. 345)}$$

It will generally be necessary to investigate the stresses in the pier when the live load is not on the main girders, as it may happen, for instance, that the tension in the diagonal members will be increased; 50 lbs. per square foot of exposed area, estimated as in the case of trusses, should be allowed when the train is off the structure, and 30 lbs. when the train is on the structure. The maxima stresses due to either case should be used in designing the structure. The same remarks as to initial tension and intensities of working stress in the lateral systems of main girders and trusses apply without modification to braced piers.

In a braced pier carrying a double line of railway, the

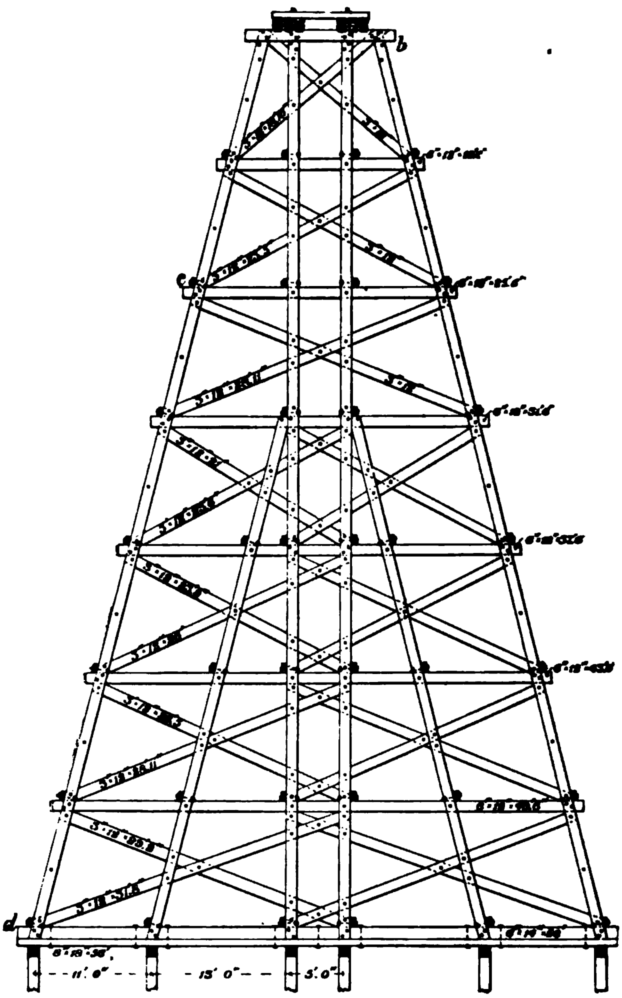
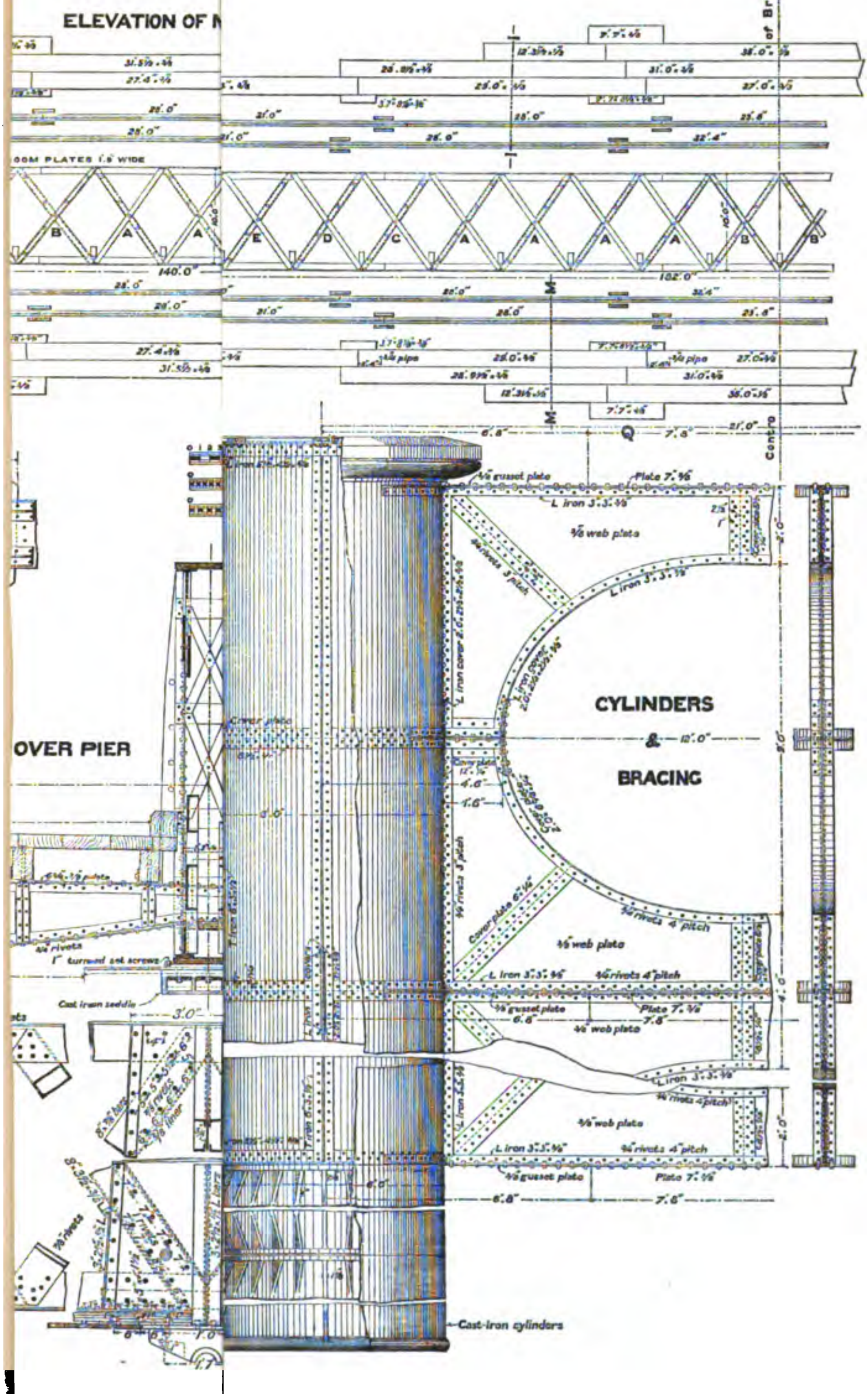


FIG. 346.

maxima stresses in the diagonal members of the pier will occur when the live load is on the same side as the wind, the lee side line of rails being empty.

PLATE III.



DESIG
ELEVATION OF M
CYLINDERS
&
BRACING
OVER PIER
Cast-iron cylinders
Centre of Bridge

The stresses due to the horizontal traction of the train should be added to those already found. If the line is curved, the force acting along the radius of the curve due to the motion of the train in the curve should also be considered.

The stresses in Eiffel towers may be investigated in a similar

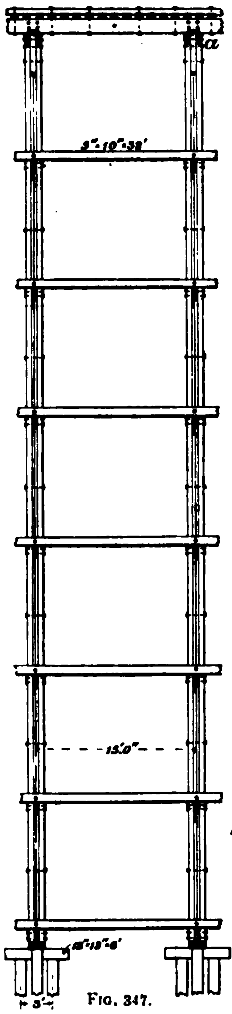


Fig. 347.

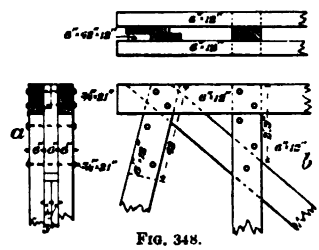


Fig. 348.

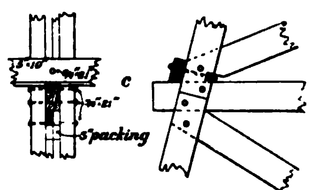


Fig. 349.

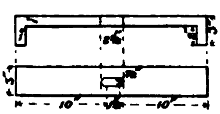


Fig. 350.

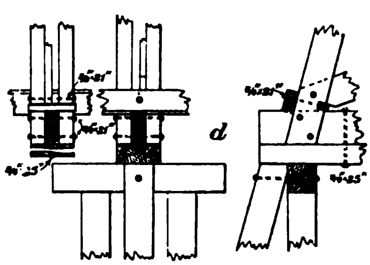


Fig. 351.

manner to that indicated for a braced pier, the only vertical forces being the weight of the structure and the loads on the various floors which may be considered as acting vertically at the panel-points. The stresses due to these loads should be added to those obtained by drawing a reciprocal polygon for the horizontal forces due to wind. The inclination of NQ and MG may easily be adjusted from the diagram of wind pressure, so that the diagonal members MN are not necessary.

Timber Trestle Piers.—Figs. 109 and 114 illustrate the method of constructing trestle piers or bents of timber in viaducts of moderate height. When the height of the trestle pier is under 10 feet it is not usual to provide sway bracing, and from 10 to 20 feet sway braces are arranged as shown in Fig. 114. The braces should be bolted to the cap, to each pile or post, and to the sill with bolts $\frac{3}{4}$ inch in diameter, with cast-iron washers under both the head and the nut.

Where the height of the pier exceeds 20 feet, the upper panel should be made from 15 to 20 feet, and the odd lengths put in the lower panel.

Figs. 346 and 347 illustrate a high timber trestle pier used on the Californian and Central Railway, United States, America. These high piers require the greatest care and attention on the part of the designer. High trestle piers of timber present great varieties in design. They should be thoroughly sway-braced, each story having one set of braces. Counter-posts are also used to stiffen the bent, and longitudinal bracing between the bents, either in every bay or in every third or fourth bay, arranged horizontally and diagonally. A good plan is to brace every third bay diagonally so as to form a tower, and to provide simply horizontal longitudinal braces between the bents in the intermediate bays at each story.

CHAPTER XXI.

CONTINUOUS GIRDER ROAD BRIDGE.

THE calculation of the stresses in continuous girder bridges may be illustrated by the following example of a road bridge.

The bridge is illustrated in Plate III., and consists of two wrought-iron continuous lattice main girders 464 feet long over all, supported upon iron cylinder piers, forming three spans of 140 feet 3 inches, 182 feet, and 140 feet 3 inches respectively, measured from the centres of the piers.

The main girders are spaced 21 feet centres across the bridge, and are 10 feet deep, excepting at the ends, which are curved to an elliptical outline. The top and bottom booms or chords are trough-shaped, formed with horizontal plates and T-irons riveted together.

The web consists of vertical struts over river piers and plates over land piers, with flat lattice bars for the tension members, and channel bar compression members braced together, with ladder bracing set at an angle, the tangent of which is $\frac{7}{10}$.

The cross-girders are attached to saddles riveted to the bottom boom immediately over the apices of the triangulation, dividing the girder into panel lengths, each 7 feet long measured from apex to apex, or centre to centre of cross-girder.

Longitudinal timber beams are attached to the top flanges of the cross-girders, upon which is laid a continuous floor of planks 4 inches thick. There is a timber kerb on each side of the roadway having a clear width of 18 feet.

The piers are each constructed with two cylinders filled with cement concrete and braced together. The diameter of the cylinders of the river piers is 6 feet, and of the land piers 4 feet 6 inches.

X

The upper portion of each cylinder consists of wrought-iron plates ⅜ inch thick, riveted together with single butt joints.

The lower portion consists of cast iron in lengths of 6 feet and 1⅛ inch thick, with making-up piece of a length depending on the depth of foundation. Only two lengths are shown on the plate; the bottom length is 1¼ inch thick, and is provided with a cutting edge to facilitate sinking. The wrought-iron cylinders are secured together with diaphragm bracing having elliptica openings, and consists of ⅜-inch plates and 3 × 3 × ⅜ inch angle bars arranged as shown.

The cylinders are sunk by excavating the material from the interior and loading until a satisfactory foundation is obtained.

The dimensions of the plates, bars, angles, channels, etc., as well as the principal details are shown on Plate III.

Cross-Girders.—The dead load on one cross-girder consists of the weight of the floor, timber stringers, and the weight of the cross-girder itself. The dimensions of the stringers and the thickness of the floor may easily be determined from the data given, as explained in Chapter XIV. The weight on the driving wheels of the traction engine in the worst position must be taken. The sizes actually adopted are shown on Plate III.,[1] and the weight of timber is 827 lbs. per foot run of bridge. The dead load on one cross-girder is therefore—

$$7 \times 827 = 5789 \text{ lbs.} = 2{\cdot}6 \text{ tons distributed}$$

$$\text{Assumed weight of cross-girder} = 1{\cdot}0 \quad ,, \quad ,,$$

$$\text{Total dead load} = 3{\cdot}6 \quad ,, \quad ,,$$

The distributed live load is—

$$84 \times 18 \times 7 = 10{,}584 \text{ lbs.} = 4{\cdot}75 \text{ tons distributed}$$

Concentrated load from traction engine shown in Figs. 322 and 323, which may be in the centre of the bridge, = 9·5 tons.

Hence we must design the cross-girder for the 9·5 tons in the centre plus the distributed dead load of 3·6 tons. This will be equivalent to a uniformly distributed load of 22·8 tons.

The bending moment in the centre is, since the girder is 21 feet span—

$$\frac{22{\cdot}8 \times 21}{8} = 59{\cdot}85 \text{ foot-tons}$$

[1] The floor in this bridge consists of New South Wales tallow-wood, and the stringers of ironbark. The weight is therefore greater than would be the case with pine timbers.

The effective depth may be 1·83 feet, and the working stress, considering that the maximum load occurs very seldom, and then without much impact, may be found by the following formula:—

$$4{\cdot}7\left(1 + \frac{1}{2}\frac{\text{min.}}{\text{max.}}\right) = 4{\cdot}7\left(1 + \frac{3{\cdot}6}{2 \times 22{\cdot}8}\right) = 5{\cdot}076, \text{ say 5 tons per square inch}$$

The moment of resistance of the bottom flange is—

$$fad = 5 \times 1{\cdot}83 \times a = 9{\cdot}15a$$
$$\therefore 9{\cdot}15a = 59{\cdot}85$$
$$\therefore a = 6{\cdot}54 \text{ square inches}$$

The section provided consists of—

2 angles $3'' \times 3'' \times \frac{1}{2}'' = 4{\cdot}7$ square inches
1 plate $5\frac{1}{4}'' \times \frac{3}{8}'' = 2$,, ,,
Total area $= 6{\cdot}7$,, ,,

The diameter and pitch of the rivets, and the thickness of web plate with regard to the intensity of shearing stress, may be investigated as in the other examples given.

The rivets are $\frac{3}{4}$ inch diameter and 4 inches pitch.

The web is $\frac{5}{16}$ inch thick.

The weight of the cross-girder may be calculated in the manner already sufficiently illustrated, and will be found to weigh 2238 lbs.

The total weight of the deck, including cross-girders, per foot run is = 0·51 ton.

Main Girders.—The dead load on one main girder is half the weight of the deck and the main girder itself.

Total weight of deck per foot run = 0·256 ton.

The live load per foot run is—

$$\frac{84 \times 18}{2 \times 2240} = 0{\cdot}337 \text{ ton}$$

The weight of the main girder per foot run may be approximately estimated by means of the formula—

$$\frac{W_1}{l} = \frac{Wr}{Cs - lr}, \text{ where } l = \text{the mean span} \times 0{\cdot}8, \text{ and } C = 1200$$

$$l = 154 \times 0{\cdot}8 = 123{\cdot}2$$

$$\therefore \frac{W_1}{l} = \frac{123 \times 0{\cdot}6 \times 12{\cdot}3}{1200 \times 5 - 123 \times 12{\cdot}3} = 0{\cdot}2 \text{ ton per foot run}$$

The total dead and live load distributed per foot run $= 0{\cdot}793$ ton

The dead load per foot run used in the following calculations $= 10$ cwt.

The live load per foot run $= 6{\cdot}75$ cwt.

Case I.—Span II. fully loaded. $w_1 = 10$ cwt., $w_2 = 16{\cdot}75$ cwt., $w_3 = 10$ cwt.

$$2m_1(140 + 182) + 182m_2 = \tfrac{1}{4}(10 \times 140^3 + 16{\cdot}75 \times 182^3)$$
$$\therefore m_1 = m_2 = 38{,}867{\cdot}6 \text{ foot-cwt.} = 1943{\cdot}4, \text{ say } 1944 \text{ foot-tons}$$

$$\therefore \quad {}_1 = \frac{\cdot 10}{2}(140x - x^2) - \frac{38867}{140}x = 0$$
$$\therefore x = 84{\cdot}5 \text{ feet, say } 84 \text{ feet}$$
$$y_2 = \frac{16{\cdot}75}{2}(182x - x^2) - 38867 = 0$$
$$x = 30{\cdot}67 \text{ feet from piers I. and II.}$$

Taking the nearest apex, say 28 feet.

Case II.—Spans I. and II. fully loaded. $w_1 = w_2 = 16{\cdot}75$ cwt., $w_3 = 10$ cwt.

$$\therefore 644m_1 + 182m_2 = \tfrac{1}{4}(16{\cdot}75 \times \overline{140}^3 + 16{\cdot}75 \times \overline{182}^3)$$
$$\text{and } 182m_1 + 644m_2 = \tfrac{1}{4}(16{\cdot}75 \times \overline{182}^3 + 10 \times \overline{140}^3)$$
$$m_2 = 36{,}659 \text{ foot-cwt.} = 1833 \text{ foot-tons}$$
$$m_1 = 2334 \text{ foot-tons}$$
$$y_1 = \frac{16{\cdot}75}{2}(140x - x^2) - \frac{46681}{140}x = 0$$
$$\therefore x = 100{\cdot}2 \text{ feet from abutment I.}$$

Taking the nearest apex, say 98 feet.

$$y_2 = \frac{16{\cdot}75}{2}(182x - x^2) - 36{,}659 - \frac{46681 - 36659}{182}x = 0$$

$\therefore x = 30$ feet and 145 feet from Pier II., say 28 and 49 feet, from Piers II. and III. respectively

$$y_3 = \frac{10}{2}(140x - x^2) - \frac{36659x}{140} = 0$$
$$\therefore x = 87{\cdot}6 \text{ feet from abutment II., say } 91 \text{ feet}$$

Case III.—Spans I. and III. fully loaded. $w_1 = w_3 = 16{\cdot}75$ cwt., $w_2 = 10$ cwt.

$$644m_1 + 182m_2 = \tfrac{1}{4}(16{\cdot}75 \times \overline{140}^3 + 10 \times \overline{182}^3), \text{ and } m_1 = m_2$$

$$826m_1 = 26{,}561{,}920$$

$$\therefore m_1 = m_2 = 32{,}157 \text{ foot-cwt.} = 1608 \text{ foot-tons}$$

$$y_1 = \frac{16{\cdot}75}{2}(140 - x^2) - \frac{32157}{140}x = 0$$

$\therefore x = 112{\cdot}6$ feet from abutments I. and II., say 112 feet

$$y_2 = \frac{10}{2}(182x - x^2) - 32157 = 0$$

$\therefore x = 48$ and 134 feet from piers I. and II., say 49 and 133 feet

The points of contra-flexure having been determined, the shearing stresses over piers and abutments should be calculated. The results are given for the various cases in the following table:—

TABLE LXXIII.

Case.	Shear at	Due to loading with the full load on	Shear in tons.	Pressure on supports in tons.	Maximum shear in tons.
I.	Abuts. I. and II. Piers I. and I.	Spans I. and III.	21·125 48·88	125·08	On abutment due to Case III. = 47·15
	Piers I. and II. Point of contra-flexure	Span II.	76·20 50·52		On piers due to Case II. = 154·33
II.	Abutment I. Pier I.	Span I.	41·86 75·19	154·33	
	Pier I. Point of contra-flexure Pier II.	Span II.	79·14 48·16 73·28	121·38	
	Pier II. Abutment II.	Span III.	48·10 21·90		
	Abuts. I. and II. Piers I. and II.	Spans I. and III.	47·15 70·10	115·60	
	Piers I. and III. Point of contra-flexure	Span II.	45·50 21·50		

The bending moments in the centre of each span considered as a detached girder are—

For the 140 feet spans fully loaded—

$$m = \frac{16{\cdot}75 \times \overline{140}^2}{20 \times 8} = 2052 \text{ foot-tons}$$

For the dead load only—

$$m = \frac{10 \times 140^2}{20 \times 8} = 1225 \text{ foot-tons}$$

For the 182 feet span fully loaded—

$$m = \frac{16{\cdot}75 \times 182^2}{8 \times 20} = 3468 \text{ foot-tons}$$

For the dead load only—

$$m = \frac{10 \times 182^2}{8 \times 20} = 2070 \text{ foot-tons}$$

The diagram of upward and downward bending moments may now be plotted to scale, and the positions of the points of contra-flexure determined as a check on the values given by the equations. The diagrams of bending moments may, however, be plotted from the results of solving the equations of bending moments for different values of x, which may be taken as 1, 2, 3, 4, etc., times the panel length, viz. 7 feet; i.e. $x = 7$, 14, 21, etc.

If the latter method is adopted, it will be correct for a plate web girder, and very nearly correct for such a lattice girder as the one under consideration; these diagrams are left for the student to draw for himself. The method adopted in this example will be similar to that explained for lattice girders in Chapter VIII.

The shearing-stress diagrams can be drawn in the manner shown in Fig. 215, using the results given in the foregoing table, and the stresses in the lattice bars may be deduced without difficulty.

Here also the method adopted will be the same as explained in Chapter VIII., as the results will be more accurate.

For a plate web girder, however, the shearing stresses may be scaled from the diagram. The stresses in the booms or chords, and the lattice bars, will now be considered by the method referred to.

Dead Load.—Let w_1 denote the load at each apex. $\theta = 35°$, $\therefore \sec \theta = 1{\cdot}22077$, and $\tan \theta = 0{\cdot}7$.

$$\therefore w_1 = \frac{10}{20} \times 7 = 3{\cdot}5 \text{ tons}$$

$$w_1 \sec \theta = 3{\cdot}5 \times 1{\cdot}22077 = 4{\cdot}27 \text{ tons}$$

Live Load.—Let w_2 denote the live load at each apex.

$$\therefore w_2 = \frac{6{\cdot}75}{20} \times 7 = 2{\cdot}3625 \text{ tons}$$
$$w_2 \sec\theta = 2{\cdot}3625 \times 1{\cdot}22077 = 2{\cdot}84 \text{ tons}$$

Dead and Live Load.—Let w_3 denote the total load at each apex—

$$\therefore w_3 = \frac{16{\cdot}75}{20} \times 7 = 5{\cdot}8625 \text{ tons}$$
$$w_3 \sec\theta = 5{\cdot}8625 \times 1{\cdot}22077 = 7{\cdot}15676 \text{ tons}$$
$$w_3 \tan\theta = 5{\cdot}8625 \times 0{\cdot}7 = 4{\cdot}10375 \text{ tons}$$

The values of $w_1 \sec\theta$ and $w_2 \sec\theta$ will be used in tabulating the stresses in the lattice bars in the manner illustrated in Chapter VIII.

The concentrated moving load of the traction engine will generally increase the stresses on some of the lattice bars towards the middle of the spans, and should always be considered. In this example the most natural place for the engine is in the centre of the bridge, and $w_4 = 8$ tons therefore has been taken at each apex, giving a value for $w_4 \sec\theta = 9{\cdot}76616$ tons. In a bridge with a wider deck, the traction engine should be assumed to be close to the kerb on one side, and the load divided between the two girders by the law of the lever, giving a correspondingly greater apex load on one of the girders, which should be used in tabulating the maxima stresses near the centre of the effective spans. The values of $w_3 \tan\theta$ will produce the maxima stresses in the chords or booms, and should be used in tabulating these stresses in the manner illustrated in Chapter VIII. (Figs. 147, 149, and 150).

It will now be most convenient to make outline diagrams of the effective spans and the cantilever portions of the main girder for the three cases of loading considered, and to write index numbers on the lattice bars as in Figs. 147, 149, Chapter VIII.

The stresses in both the lattice bars and booms can then be tabulated. Only the results of these tabulations will be given, as the methods are fully illustrated in the chapter referred to; but it will facilitate the work if a table is first prepared giving the values of $w_1 \sec\theta$ and $w_3 \tan\theta$ when multiplied by the index numbers, such as the following:—

TABLE LXXIV.

For the dead-load stresses in the lattice bars.

Index number.	$w_1 \sec \theta =$ 4·27 multiplied by the index.
0·5	2·14
1·0	4·27
1·5	6·41
2·0	8·55
2·5	10·68
3·0	12·82
3·5	14·95
4·0	17·09
4·5	19·23
5·0	21·36
5·5	23·50
6·0	25·63
6·5	27·77
7·0	29·91

TABLE LXXV.

For the live and dead load stresses in the chords or booms.

Index number.	$w_3 \tan \theta =$ 4·1 multiplied by the index.	Index number.	$w_3 \tan \theta =$ 4·1 multiplied by the index.
0·5	2·05	6·0	24·62
1·0	4·10	6·5	26·67
2·0	8·21	7·0	28·73
3·0	12·31	8·0	32·84
3·5	14·36	9·0	36·93
4·0	16·42	10·0	41·04
4·5	18·47	11·0	45·14
5·0	20·52	12·0	49·25
5·5	22·57	13·0	53·35

The live stresses in the lattice bars may be found as in Chapter VIII., p. 127, using the following values :—

For the centre span—

$$\frac{w_2 \sec \theta}{l} = \frac{2·8843}{18} = 0·16023 \text{ ton}$$

$$\text{and } \frac{w_2 \sec \theta}{l} = \frac{2·8843}{17} = 0·16965 \text{ ton}$$

For the side spans—

$$\frac{w_2 \sec \theta}{l} = \frac{2·884}{14} = 0·206 \text{ ton}$$

$$\text{also } \frac{w_2 \sec \theta}{l} = \frac{2·884}{16} = 0·18025 \text{ ton}$$

This will necessitate four separate tabulations if worked out completely.

For the concentrated load due to the traction engine :—

For the centre span—

$$\frac{w_4 \sec \theta}{l} = \frac{9·76616}{18} = 0·54256 \text{ ton}$$

$$\text{also } \frac{w_4 \sec \theta}{l} = \frac{9·76616}{17} = 0·57448 \text{ ton}$$

For the side spans—

$$\frac{w_4 \sec\theta}{l} = \frac{9{\cdot}76616}{14} = 0{\cdot}69758 \text{ ton}$$

$$\text{also } \frac{w_4 \sec\theta}{l} = \frac{9{\cdot}76616}{16} = 0{\cdot}61038 \text{ ton}$$

These will require, if worked out completely, four more separate tabulations.

It is not generally necessary to work out the tabulations for all these cases completely, as a little experience will enable the computor to decide which cases produce maxima stresses in the effective spans and cantilevers.

A careful consideration of the diagrams of shearing stress, which should generally be made before commencing the detailed tabulations, will help the computor to decide which cases require to be fully worked out, and where the cases not fully worked out are likely to increase those tabulated. By referring to the following tables of stresses in lattice bars, it will be seen how far the traction engine increases the stresses due to the dead and moving loads. If the wheels of the traction engine had been taken as close as possible to the kerb, the effect would have been greater. Again, if the spans were smaller, a greater number of lattice bars would be governed by the load on the wheels of the traction engine. In any span, if the extreme positions of the point of contra-flexure is known, there will be no difficulty in determining the maxima stresses in the two effective spans and the two cantilevers, either for the dead and moving load, or for the dead load and the traction engine.

TABLE LXXVI.

CENTRE SPAN.—TABLE OF STRESSES IN CHORDS OR BOOMS IN TONS.

Number of chord or boom.	Case I. Span II. fully loaded.	Case II. Spans I. and II. fully loaded.	Case III. Spans I. and III. fully loaded.	Maxima.	Minima.
1	− 166·20	− 147·74	− 44·10	− 166·20	− 44·10
2	+ 164·15	+ 143·63	+ 42·88	+ 166·20	+ 42·88
3	− 162·10	− 139·53	− 41·65	− 162·10	− 41·65
4	+ 160·05	+ 139·53	+ 40·43	+ 160·05	+ 40·43
5	− 153·89	− 131·32	− 36·75	− 153·89	− 36·75
6	+ 151·84	+ 127·22	+ 35·53	+ 151·84	+ 35·53
7	− 141·58	− 114·91	− 29·40	− 141·58	− 29·40
8	+ 139·53	+ 114·91	+ 28·18	+ 139·53	+ 28·18
9	− 125·06	− 98·49	− 19·60	− 125·06	− 19·60
10	+ 123·11	+ 94·39	+ 18·38	+ 123·11	+ 18·38
11	− 104·65	− 73·87	− 7·35	− 104·65	− 7·35
12	+ 102·59	+ 73·87	+ 6·13	+ 102·59	+ 6·13
			— · — ·		
13	− 80·02	− 49·25	+ 7·35	− 80·02	+ 7·35
14	+ 77·97	+ 45·14	− 8·58	+ 77·97	− 8·58
15	− 51·30	− 16·42	+ 24·50	− 51·30	+ 24·50
16	+ 49·25	+ 16·42	− 25·73	+ 49·25	− 25·73
		— · — ·			
17	− 18·47	+ 16·42	+ 44·10	+ 44·10	− 18·47
18	+ 16·42	− 20·52	− 45·33	− 45·33	+ 16·42
	— · — ·				
19	+ 18·47	+ 57·45	+ 66·15	+ 66·15	+ 18·47
20	− 20·52	− 57·45	− 67·38	− 67·38	− 20·52
21	+ 59·50	+ 98·49	+ 90·65	+ 98·49	+ 59·50
22	− 61·56	− 102·59	− 91·88	− 102·59	− 61·56
23	+ 104·65	+ 147·74	+ 117·60	+ 147·74	+ 104·65
24	− 106·70	− 147·74	− 118·83	− 147·74	− 106·70
25	+ 153·89	+ 196·98	+ 147·00	+ 196·98	+ 147·00
26	− 155·94	− 201·08	− 148·23	− 201·08	− 148·23
25–27	+ 180·57	+ 225·71	+ 162·93	+ 225·71	+ 162·93
26–28	− 180·57	− 225·71	− 162·93	− 225·71	− 162·93
B. mt. at centre	1525 ft.-tons	1310 ft.-tons	462 ft.-tons		
B. mt. at piers	1944 ,,	2334 ,,	1608 ,,		

— · — · signifies point of contra-flexure. + Tension. − Compression.

Table LXXVII.

Centre Span.—Summary of Stresses in Lattice Bars in Tons.

Number of bar.	Evenly distributed live and dead load stresses due to unequal loading of spans.		Stresses due to dead load and to distributed moving load.		Stresses due to the dead load and concentrated load of the traction engine.		Total final stresses.	
	Maxima.	Minima.	Compression.	Tension.	Compression.	Tension.	Maxima.	Minima.
1	+ 7·16	+ 2·14	− 0·42	+ 8·52	− 1·66	+ 9·44	+ 9·44	− 1·66
2	− 0	− 0	− 3·39	+ 3·20	− 4·34	+ 4·60	+ 4·60	− 4·34
3	+ 7·16	+ 4·27		+ 9 37		+ 10·01	+ 10·01	+ 4·27
4	− 7·16	− 2·14	− 8·52	+ 0·42	− 9·44	+ 1·66	− 9·44	+ 1·66
5	+ 14·31	+ 6·14		+ 14·67		+ 14·87	+ 14·87	+ 6·14
6	− 7·16	− 4·27	− 9·37		− 10·01		− 10·01	− 4·27
7	+ 14·31	+ 8·55		+ 15·69		+ 15·44	+ 15·69	+ 8·55
8	− 14·31	− 6·41	− 14·67		− 14·87		− 14·87	− 6·41
9	+ 21·47	+ 10·68		+ 21·15		+ 20·29	+ 21·47	+ 10·68
10	− 14·31	− 8·55	− 15·69		− 15·44		− 15·69	− 8·55
11	+ 21·47	+ 12·82		+ 22·32		+ 20·86	+ 22·32	+ 12·82
12	− 21·47	− 10·68	− 21·15		− 20·29		− 21·47	− 10·68
13	+ 28·63	+ 14·95		+ 27·97		+ 25·71	+ 28·63	+ 14·95
14	− 21·47	− 12·82	− 22·32		− 20·86		− 22·32	− 12·82
15	+ 28·63	+ 17·09		+ 29·32		+ 26·28	+ 29·32	+ 17·09
16	− 28·63	− 14·95	− 27·97		− 25·71		− 28·63	− 14·95
17	+ 35·78	+ 19·23		+ 32·21		+ 28·45	+ 35·78	+ 19·23
18	− 28·63	− 17·09	− 28·63		− 25·77		− 28·63	− 17·09
19	+ 35·78	+ 21·36					+ 35·78	+ 21·36
20	− 35·78	− 19·23					− 35·78	− 19·23
21	+ 42·94	+ 23·50					+ 42·94	+ 23·50
22	− 35·78	− 21·36					− 35·78	− 21·36
23	+ 42·94	+ 25·63					+ 42·94	+ 25·63
24	− 42·94	− 23·50					− 42·94	− 23·50
25	+ 50·10	+ 27·77					+ 50·10	+ 27·77
26	− 42·94	− 25·63					− 42·94	− 25·63

Table LXXVIII.

Side Spans.—Table of Stresses in Chords or Booms in Tons.

Number of bay.	Case I. Span II. fully loaded.	Case II. Spans I. and II. fully loaded. Span I.	Case IIa. Spans I. and II. fully loaded. Span III.	Case III. Spans I. and III. fully loaded.	Maxima.	Minima.
25–27	+ 196·00	+ 246·23	+ 171·50	+ 164·15	+ 246·23	+ 164·15
26–28	− 196·00	− 246·23	− 171·50	− 164·15	− 246·23	− 164·15
27	+ 178·85	+ 219·55	+ 154·35	+ 139·53	+ 219·55	+ 139·53
28	− 180·08	− 221·60	− 156·80	− 141·58	− 221·60	− 141·58
29	+ 147·00	+ 170·31	+ 124·95	+ 94·39	+ 170·31	+ 94·39
30	− 148·23	− 172·36	− 124·95	− 96·44	− 172·36	− 96·44
31	+ 117·60	+ 125·06	+ 95·55	+ 53·35	+ 125·06	+ 53·35
32	− 118·83	− 127·22	− 98·00	− 55·40	− 127·22	− 55·40
33	+ 90·65	+ 84·13	+ 71·05	+ 16·42	+ 90·65	+ 16·42
34	− 91·88	− 86·18	− 71·05	− 18·47	− 91·88	− 18·47
				— — — —		
35	+ 66·15	+ 47·19	+ 46·55	− 16·42	+ 66·15	+ 16·42
36	− 67·38	− 49·25	− 49·00	+ 14·36	− 67·38	+ 14·38
37	+ 44·10	+ 14·36	+ 26·95	− 45·14	− 45·14	+ 44·10
38	− 45·33	− 16·42	− 26·95	+ 43·10	− 45·33	+ 43·10
		— — — —				
39	+ 24·50	− 14·36	+ 7·35	− 69·76	− 69·76	+ 24·50
40	− 25·73	+ 12·31	− 9·80	+ 67·71	+ 67·71	− 25·73
			— — — —			
41	+ 7·35	− 38·99	− 7·35	− 90·28	− 90·28	+ 7·35
42	− 8·58	+ 36·93	+ 7·35	+ 88·23	+ 88·23	− 8·58
	— — — —					
43	− 7·35	− 59·50	− 22·05	− 106·70	− 106·70	− 7·35
44	+ 6·13	+ 57·45	+ 19·60	+ 104·65	+ 104·65	+ 6·13
45	− 19·60	− 75·93	− 31·85	− 119·01	− 119·01	− 19·60
46	+ 18·38	+ 73·87	+ 31·85	+ 116·96	+ 116·96	+ 18·38
47	− 29·40	− 88·23	− 41·65	− 127·22	− 127·22	− 29·40
48	+ 28·18	+ 86·18	+ 39·20	+ 125·06	+ 125·06	+ 28·18
49	− 36·75	− 96·44	− 46·55	− 131·32	− 131·32	− 36·75
50	+ 35·53	+ 94·39	+ 46·55	+ 129·07	+ 129·07	+ 35·53
				— · — · —		
51	− 41·65	− 100·54	− 51·46	− 131·32	− 131·32	− 41·65
52	+ 40·43	+ 98·49	+ 49·00	+ 129·07	+ 129·07	+ 40·43
		— · — · —				
53	− 44·10	− 100·54	− 51·46	− 127·22	− 127·22	− 44·10
			— · — · —			
54	+ 42·88	+ 98·49	+ 51·46	+ 125·06	+ 125·06	+ 42·88
	— · — · —					
55	− 44·10	− 96·44	− 51·46	− 119·01	− 119·01	− 44·10
56	+ 42·88	+ 94·39	+ 49·00	+ 116·96	+ 116·96	+ 42·88
57	− 41·65	− 88·23	− 46·55	− 106·70	− 106·70	− 41·65
58	+ 40·43	+ 86·18	+ 46·55	+ 104·65	+ 104·65	+ 40·43
59	− 36·75	− 75·93	− 41·65	− 90·28	− 90·28	− 36·75
60	+ 35·53	+ 73·87	+ 39·20	+ 88·23	+ 88·23	+ 35·53
61	− 29·40	− 59·50	− 31·85	− 69·76	− 69·76	− 29·40
62	+ 28·18	+ 57·45	+ 31·85	+ 67·71	+ 67·71	+ 28·18
63	− 19·60	− 38·99	− 22·05	− 45·14	− 45·14	− 19·60
64	+ 18·38	+ 36·93	+ 19·60	+ 43·10	+ 43·10	+ 18·38
65	− 23·50	− 46·52	− 25·63	− 53·68	− 53·68	− 23·50
66	+ 13·48	+ 26·67	+ 14·70	+ 30·78	+ 30·78	+ 13·48
B. mt. at centre	44·60 foot-tons	105·00 foot-tons	48·40 foot-tons	132·7 foot-tons		
B. mt. at piers	1943·40 foot-tons	2334·00 foot-tons	1833·00 foot-tons	1608·00 foot-tons		

— — — — signifies point of contra-flexure.

— · — · — signifies centre between points of contra-flexure.

+ Tension. − Compression.

Table LXXIX.

Side Spans.—Summary of Stresses in Lattice Bars in Tons.

No. of bar.	Evenly distributed live and dead load stresses due to unequal loading of spans.		Stresses due to the dead load and the distributed moving load.		Stresses due to the dead load and the concentrated load of the traction engine.		Total final stresses.	
27	− 42·94	− 25·63					− 42·94	− 25·63
28	+ 46·52	+ 29·91					+ 46·52	+ 29·91
29	− 39·36	− 25·63					− 39·36	− 25·63
30	+ 42·94	+ 25·63					+ 42·94	+ 25·63
31	− 35·78	− 21·36					− 35·78	− 21·36
32	+ 39·36	+ 25·63					+ 39·36	+ 25·63
33	− 32·21	− 21·36					− 32·21	− 21·36
34	+ 35·78	+ 21·36					+ 35·78	+ 21·36
35	− 28·63	− 17·09	− 25·03		− 23·50		− 28·63	− 17·09
36	+ 32·31	+ 21·36		+ 28·61		+ 26·25	+ 32·21	+ 21·36
37	− 25·05	− 17·09	− 21·64		− 20·76		− 25·05	− 17·09
38	+ 28·63	+ 17·09		+ 25·03		+ 23·50	+ 28·63	+ 17·09
39	− 21·47	− 12·82	− 21·47		− 21·19		− 21·47	− 12·82
40	+ 25·05	+ 17·09		+ 25·05		+ 24·02	+ 25·05	+ 17·09
41	− 17·89	− 12·82	− 18·10		− 18·35		− 18·35	− 12·82
42	+ 21·47	+ 12·82		+ 21·47		+ 21·19	+ 21·47	+ 12·82
43	− 14·31	− 8·55	− 14·73		− 15·53		− 15·53	− 8·55
44	+ 17·89	+ 12·82		+ 18·10		+ 18·35	+ 18·35	+ 12·82
45	− 10·74	− 8·55	− 11·55		− 12·69		− 12·69	− 8·55
46	+ 14·31	+ 8·55		+ 14·73		+ 15·53	+ 15·53	+ 8·55
47	− 7·16	− 3·58	− 8·39	+ 0·02	− 9·85	+ 1·52	− 9·85	+ 1·52
48	+ 10·74	+ 7·16		+ 11·55		+ 12·69	+ 12·69	+ 7·16
49	− 4·27	− 0	− 5·44	+ 2·88	− 7·02	+ 4·27	− 7·02	+ 4·27
50	+ 7·16	+ 3·58	− 0·02	+ 8·39	− 1·52	+ 9·85	+ 9·85	− 1·52
51	+ 3·58	− 2·14	− 2·47	+ 5·74	− 4·19	+ 7·02	+ 7·02	− 4·19
52	+ 4·27	− 0	− 2·88	+ 5·44	− 4·27	+ 7·02	+ 7·02	− 4·27
53	+ 7·16	− 0		+ 8·77	− 1·35	+ 9·76	+ 9·76	− 1·35
54	− 3·58	+ 2·14	− 5·74	+ 2·47	− 7·02	+ 4·19	− 3·58	+ 2·14
55	+ 10·74	+ 2·14		+ 11·81		+ 12·51	+ 12·51	+ 2·14
56	− 7·16	− 3·58	− 8·77		− 9·76	+ 1·35	− 9·76	+ 1·35
57	+ 14·31	+ 4·27		+ 15·03		+ 15·25	+ 15·25	+ 4·27
58	− 10·78	− 2·14	− 11·81		− 12·51		− 12·51	− 2·14
59	+ 17·89	+ 6·41		+ 18·24		+ 18·00	+ 18·24	+ 6·41
60	− 14·31	− 4·27	− 15·03		− 15·25		− 15·25	− 4·27
61	+ 21·47	+ 8·55		+ 21·64		+ 20·76	+ 21·64	+ 8·55
62	− 17·89	− 6·41	− 18·24		− 18·00		− 18·24	− 6·41
63	+ 25·05	+ 10·68		+ 25·03		+ 23·50	+ 25·05	+ 10·68
64	− 21·47	− 8·55	− 21·64		− 20·76		− 21·64	− 8·55

TABLE LXXX.

CENTRAL SPAN.—TABLE OF SECTIONS REQUIRED AND PROVIDED IN LATTICE BARS.

No. of bars.	Final stresses.	Intensity of working stress in tons per square inch.	Effective areas in bars. Required sq. in.	Effective areas in bars. Provided sq. in.	Rivet areas. Required sq. in.	Rivet areas. Provided sq. in.	Dimension of section of bars.	No. of rivets.	Diameter of rivets in inches.
1	+ 9·44; − 1·66	2·43	3·90	6·00	2·36	4·81	2 channel bars $5\frac{3}{4}''$ × $2\frac{3}{8}''$ × $\frac{5}{16}''$ at $10\frac{1}{2}$ lbs. per foot	8	$\frac{3}{4}$
2	+ 4·60; − 4·34	1·39	3·30	6·00	1·15	4·81	Ditto	8	$\frac{3}{4}$
3	+ 10·01	5	2·00	5·00	2·5	4·81	2 bars 6″ × $\frac{1}{2}''$	8	$\frac{3}{4}$
4	− 9·44; + 1·66	2·43	3·90	6·00	2·36	4·81	same as (1)	8	$\frac{3}{4}$
5	+ 14·87	5	2·98	5·00	3·72	4·81	" (3)	8	$\frac{3}{4}$
6	− 10·01	2·8	3·57	6·00	2·50	4·81	" (1)	8	$\frac{3}{4}$
7	+ 15·69	5	3·14	5·00	3·92	4·81	" (3)	8	$\frac{3}{4}$
8	− 14·87	2·8	5·31	6·00	3·72	7·21	" (1)	12	$\frac{3}{4}$
9	+ 21·47	5	4·30	5·00	5·37	7·21	" (3)	12	$\frac{3}{4}$
10	− 15·69	2·8	5·60	6·00	3·92	7·21	" (1)	12	$\frac{7}{8}$
11	+ 22·32	5	4·46	5 00	5·58	7·21	" (3)	12	$\frac{7}{8}$
12	− 21·47	2·8	7·67	8·00	5·37	7·21	2 channel bars $5\frac{3}{4}''$ × $2\frac{9}{16}''$ × $\frac{1}{2}''$ at 13 lbs. per foot	12	$\frac{7}{8}$
13	+ 28·63	5	5·73	7·50	7·16	9·42	2 bars 6″ × $\frac{3}{4}''$	12	1
14	− 22·32	2·8	7·96	8·00	5·58	7·21	same as (12)	12	$\frac{7}{8}$
15	+ 29·32	5	5·86	7·50	7·33	9·42	" (13)	12	1
16	− 28·63	3·2	8·95	11·40	7·16	9·42	2 channel bars $6\frac{7}{8}''$ × $2\frac{9}{16}''$ × $\frac{13}{32}''$ at 19 lbs. per foot	12	1
17	+ 35·78	5	7·16	9·00	8·95	10·99	2 bars 7″ × $\frac{3}{4}''$	14	1
18	− 28·63	3·2	8·95	11·40	7·16	9·42	same as (16)	12	1
19	+ 35·78	5	7·16	9·00	8·95	10·99	" (17)	14	1
20	− 35·78	3·2	11·18	11·40	8·95	10·99	" (16)	14	1
21	+ 42·94	5	8·60	12·25	10·74	12·56	2 bars 8″ × $\frac{7}{8}''$	16	1
22	− 35·78	3·2	11·18	11·40	8·95	10·99	same as (16)	14	1
23	+ 42·94	5	8·60	12·25	10·74	12·56	" (21)	16	1
24	− 42·94	3·5	12·27	13·0	10·74	12·56	2 channel bars 8″ × $3\frac{1}{2}''$ × $\frac{3}{8}''$ at 22 lbs. per foot	16	1
25	+ 50·10	5	10·02	12·25	12·53	13·13	same as (21)	18	1
26	− 42·94	3·5	12·27	13·00	10·74	12·56	" (24)	16	1
27	− 42·94	3·5	12·27	13·00	10·74	12·56	" (24)	16	1

TABLE LXXXI.

SIDE SPANS.—TABLE OF SECTIONS REQUIRED AND PROVIDED IN LATTICE BARS.

No. of bars.	Final stresses.	Intensity of working stress in tons per square inch.	Effective areas in bars.		Rivet areas.		Dimensions of section of bars.	No. of rivets.	Diameter of rivets in inches.
			Required sq. in.	Provided sq. in.	Required sq. in.	Provided sq. in.			
28	+ 46·52	5·0	9·30	12·25	11·63	13·13	same as (21)	18	1
29	− 39·36	3·5	11·25	13·00	9·84	12·56	„ (21)	16	1
30	+ 42·94	5·0	8·60	12·25	10·74	12·56	„ (21)	16	1
31	− 35·78	3·2	11·18	11·40	8·95	10·99	„ (16)	14	1
32	+ 39·36	5·0	7·87	12·25	9·84	12·56	„ (21)	16	1
33	− 32·21	3·2	10·07	11·40	8·05	10·99	„ (16)	14	1
34	+ 35·78	5·0	7·16	9·00	8·95	10·99	„ (17)	14	1
35	− 28·63	3·2	8·95	11·40	7·16	9·42	„ (16)	12	1
36	+ 32·21	5·0	6·44	9·00	8·05	10·99	„ (17)	14	1
37	− 25·05	3·2	7·83	11·40	6·26	7·21	„ (16)	12	⅞
38	+ 28·63	5·0	5·73	7·50	7·16	9·42	„ (13)	12	1
39	− 21·47	2·8	7·67	8·00	5·37	7·21	„ (12)	12	⅞
40	+ 25·05	5·0	5·01	7·50	6·26	9·42	„ (13)	12	1
41	− 18·35	2·8	6·56	8·00	4·59	7·21	„ (12)	12	⅞
42	+ 21·47	5·0	4·29	5·00	5·37	7·21	„ (13)	12	⅞
43	− 15·53	2·8	5·55	6·00	3·88	4·81	„ (1)	8	⅞
44	+ 18·35	5·0	3·67	5·00	4·59	7·21	„ (3)	12	⅞
45	− 12·69	2·8	4·53	6·00	3·17	4·81	„ (1)	8	⅞
46	+ 15·53	5·0	3·11	5·00	3·88	4·81	„ (3)	8	⅞
47	− 9·85; + 1·52	—	4·10	6·00	2·46	4·81	„ (1)	8	⅞
48	+ 12·69	5·0	2·54	5·00	3·17	4·81	„ (3)	8	⅞
49	− 7·02; + 4·27	2·0	3·50	6·00	1·76	4·81	„ (1)	8	⅞
50	+ 9·85; − 1·52	2·4	4·10	6·00	2·46	4·81	„ (1)	8	⅞
51	+ 7·02; − 4·19	2·0	3·50	6·00	1·76	4·81	„ (1)	8	⅞
52	+ 7·02; − 4·27	2·0	3·50	6·00	1·76	4·81	„ (1)	8	⅞
53	+ 9·76; − 1·35	2·38	4·10	6·00	2·44	4·81	„ (1)	8	⅞
54	− 3·58; + 2·14	1·99	1·80	6·00	0·89	4·81	„ (1)	8	⅞
55	+ 12·51	5·0	2·50	5·00	3·13	4·81	„ (3)	8	⅞
56	− 9·76; + 1·35	2·38	4·10	6·00	2·44	4·81	„ (1)	8	⅞
57	+ 15·25	5·0	3·05	5·00	3·81	4·81	„ (3)	8	⅞
58				6·00		4·81	„ (1)	8	⅞
59				5·00		4·81	„ (3)	8	⅞
60				6·00		4·81	„ (1)	8	⅞
61				5·00		4·81	„ (3)	8	⅞
62				6·00		4·81	„ (1)	8	⅞
63				5·00		4·81	„ (3)	8	⅞
64				6·00		4·81	„ (1)	8	⅞

Pressure on Expansion Rollers.—The maximum pressure at abutments occurs when the spans I. and III. are completely loaded, and it equals 47 tons on each bearing.

Three rollers are provided, each 2 feet long by 2 feet deep and 4½ inches wide; the pressure per lineal inch is therefore—

$$\frac{47 \times 20}{72} = 13 \text{ cwt.}$$

The maximum pressure on the piers occurs when the spans I. and II. are completely loaded, and equals 155 tons on one bearing.

Seven rollers are provided, each 2 feet long by 2 feet deep and 4 inches wide; the pressure per lineal inch is therefore—

$$\frac{155 \times 20}{168} = 19 \text{ cwt.}$$

With such a small resistance the girders will expand most freely.

The main girders are fixed on pier I., and rest upon the expansion rollers at the other supports.

For a variation of ± 60° Fahr., the expansion may easily be shown to be 0·68 inch on No. 1 abutment, and 1·57 inch on No. 2 abutment.

The saddle pin on the expansion rollers over the piers is 2 feet 4 inches long, and 4¾ inches diameter; hence the pressure per lineal inch of pin is—

$$\frac{155}{28} = 5 \cdot 57 \text{ tons}$$

The pressure per square inch is—

$$\frac{5 \cdot 57}{4 \cdot 75} = 1 \cdot 21 \text{ ton}.$$

Pressure on Foundations.—River Piers.

Load on one cylinder when the superstructure is fully loaded	156	tons.
53 feet 1¼ inch of wrought-iron cylinder with concrete at 37·75 cwt. per foot run	100	„
Bracing taken at 2 cwt. per foot run	5	„
Total weight on bottom of wrought-iron cylinder ...	261	„

The pressure on bottom of wrought-iron cylinder 6 feet in diameter $= \frac{261}{28 \cdot 27} = 9 \cdot 23$ tons per square foot.

Load on cast-iron cylinder—

Consisting of the load on the bottom of wrought-iron cylinder	261 tons.
16 feet of cast-iron cylinder with concrete at 41·7 cwt. per foot run	33 „
Total weight on the bottom of cast-iron cylinder ...	294 „

Pressure on the bottom of the cast-iron cylinder 6 feet in diameter $= \frac{294}{28 \cdot 27} = 10 \cdot 4$ tons per square foot.

Land Piers or Abutments.

Load on one cylinder when the superstructure is fully loaded...	47 tons.
14 feet 1½ inches of wrought-iron cylinder with concrete at 21·6 cwt. per foot run	15 „
Bracing at 2 cwt. per foot run	1 „
52 feet of cast-iron cylinder with concrete at 24·8 cwt. per foot run	64 „
Total weight on bottom of cast-iron cylinder ...	127 „

Pressure on the bottom of the cast-iron cylinder 4 feet 6 inches in diameter $= \frac{127}{15 \cdot 9} =$ about 8 tons.

These pressures are not excessive, as the foundations are of sound rock.

The following table gives the quantities of materials used in the bridge which it would be most convenient to include in the contract for supply and delivery :—

TABLE LXXXII.

Description.	Quantities.			
	tons.	cwt.	qrs.	lbs.
Cast iron in cylinders, handrail brackets, ladder bracing ...	92	15	0	16
Cast iron in bed-plates, saddle-plates, rollers, caps, and blocks	14	4	3	2
Wrought iron in cylinders and bracing, including rivets ...	59	17	2	0
Wrought iron in main girders (including steel L covers), cross-girders, pockets, scuppers and wind bracing, and rivets	254	17	0	26
Wrought iron in rolled girders and straps of approach spans ...	18	10	2	26
Wrought iron in handrail standards and black bolts	6	14	2	8
Wrought iron in turned bolts and set screws	1	18	2	8
Steel in turned pins	0	11	3	8
Pig lead for lewis bolts	0	9	1	16
Sheet ditto	0	3	0	4
Gas-pipe handrails	4	19	3	9
Paint, as specified, two coats				
Lamp-posts, with lamps, complete				

The following table gives the quantities of materials which should be included in the erection of the bridge.

TABLE LXXXIII.

Description.	Quantities.
	tons. cwt. qrs. lbs.
Fixing and sinking cast-iron cylinders, complete	91 8 0 19
Excavating in cylinders, other than rock	142¼ cub. yards
Excavating in cylinders in rock	11½ " "
Concreting cylinders	355¾ " "
Granite or blue stone in bedstones, set	265 " feet.
	tons. cwt. qrs. lbs.
Fixing wrought iron in cylinders and bracing, complete ...	61 1 1 15
Fixing cast iron in bed-plates, saddle-plates, rollers, caps, and blocks	14 4 3 2
Fixing wrought iron and steel in main girders (including cast iron in ladder bracing), cross-girders, wind bracing, scupper and pockets, bolts, and set screws	256 9 0 20
Fixing wrought iron in rolled girders of approach spans ...	18 10 2 26
Fixing handrail, including wrought-iron and cast-iron brackets	7 2 0 3
Fixing lead	12½ cwts.
Sawn timber in longitudinal girders, timber packing pieces, cross-girders, and planking	4886 cub. feet
Hewn timber in kerbs and sills	520 " "
Tar, one coat	
" three coats	
Paint, as specified, two coats	
" " three coats, including stopping and filling ...	
Excavation for abutments other than rock, including unwatering	90½ cub yards
Concrete for abutments	44½ " "
Masonry "	136¼ " "
Hand-packed rubble, as specified	172½ " "
	tons. cwt. qrs. lbs.
Carriage of ironwork to site of bridge	449 8 2 21

CHAPTER XXII.

SWING BRIDGES.

AMONG the most important class of movable bridges are those known as swing bridges. These may be classified according to the method adopted for supporting their weight during the process of swinging, thus—

(*A*) Bridges entirely supported on rim-bearing turn-tables consisting of a circular roller path, with a ring of conical rollers, upon which rests a circular girder carrying the main girders of the bridge.

(*B*) Bridges supported on a centre-bearing turn-table, consisting of a central pivot, or a nest of one or more series of conical rollers, with or without a ring of live rollers, or two or more wheels for steadying the bridge during the process of opening. The weight may be proportioned in this class of bridge so as to partly rest on the central pivot and partly on the live rollers.

(*C*) Bridges which are partially or completely lifted by means of a central hydraulic press, and turn on a water centre or on a spherical pivot.

(*D*) Floating swing bridges, in which the load upon the ring of live rollers or central pivot is buoyed up by means of a hollow caisson which carries the bridge, so that only a portion of the load rests upon the pivot or rollers.

Class A.—This class of bridge is constructed with two cantilevers of equal or unequal lengths; in the former case one arm balances the other, in the latter the shorter arm must be loaded so as to balance the longer.

An example of this kind of bridge occurs at Goole over the river Ouse,[1] in which the two arms are each 100 feet in length.

[1] Vol. lvii. *Proc. Inst. C.E.*, paper by J. Price, M.Inst.C.E.

Other examples exist at the entrance from Alfred Dock to East Float, Birkenhead, the entrance to Alfred Dock,[1] the river Hull south bridge,[1] and in numerous highway bridges over navigable rivers. The bridges mentioned are moved by hydraulic power, but the special design of this class of bridge has very little to do with the motive power used for swinging it, whether hydraulic, steam, or hand power. It frequently happens that hand-power is the only means available for rotating the bridge.

It is important in this class of bridge that the weight should be equally distributed over the live rollers, so that they may work with the minimum friction. The main girders generally rest upon the circular girder at four points, which arrangement does not distribute the weight with sufficient equality over the rollers. It is much better to divide the weight equally over six equidistant points. This matter will be further illustrated.

Class B.—This class includes a large number of important swing bridges. The central pivot may be arranged to carry the whole of the weight, but the more usual proportion is from two-thirds to three-quarters.

The bridge over the Raritan[1] erected by the Keystone Bridge Company, Philadelphia, from the designs of Mr. J. H. Linville, is 472 feet long, with two passages each 216 feet.

The total dead load is 4500 lbs., and the live load 2500 lbs. per foot run. The load upon the central pivot may be regulated to any desired extent by means of the suspension bolts. Before being swung, the bridge is lifted by four hydraulic rams to a height of 4 inches clear of the abutments, and while being closed it is lowered directly on to the abutments, so that wedges or other modes of setting up are unnecessary.

The bearings are adjusted to such a height that each carries half the weight of the bridge; the lower chords are parted in the centre, and a slotted link connection is used in the top chord, so that the bridge acts as two cantilevers while being swung, and as two independent girders when resting on the abutments. In this way the disadvantages of a long continuous girder are overcome, and the end does not rise when a heavy train approaches from the other end of the bridge, producing a hammer-like action of the main girder on the abutments. To overcome

[1] Vol. lvii. *Proc. Inst. C.E.*, paper by J. Price, M.Inst.C.E.

this difficulty, latching apparatus is sometimes provided for fixing the ends when the bridge is closed.

The Kansas city bridge consists of two openings each 160 feet, and is set up on the abutments by means of sliding wedges. Both these bridges are moved by means of steam power. The Hawarden bridge over the river Dee, England, includes a swing span 284¾ feet long, forming openings 87 and 140 feet long respectively.[1] The main girders are of steel 32 feet deep over the centre of the swing pier, 9½ feet deep at the ends. The girders are lattice, divided into panels 17 feet long, with single triangulated vertical and diagonal members, except over the pivot, where the members cross each other. The top booms are curved, and the bottom booms horizontal, of trough-shaped section. The cross-girders are spaced 17 feet centres, and are 29 inches deep over the angle irons. The weight per square foot of the deck is 38 lbs., including brackets and gangway plates, but exclusive of the permanent way. The main girders were built without camber, and provision was made in the lifting apparatus at the tail end for taking out the deflection which occurs when the longer end is unsupported, so that the stresses in the girder due to its own weight are removed as much as possible.

The weight of the tail end is made 15 tons heavier than the longer end by loading it, and the load upon the central pivot is regulated by means of suspension bolts acting on the pivot cross-bearing girder which supports the bridge over the pivot pier, so that the whole or any portion of the total load of the swing span may be thrown upon the central pivot. The bridge has a slight bascule movement for the purpose of lifting the fore ends out of their bearing sockets, which is effected by lowering the tail ends by means of two rams, each 25 inches in diameter and 8¾ inches stroke, capable of supporting and lifting 300 tons. Four rollers of cast steel 3 feet in diameter are provided, two under each end of the main bearing girder, and directly under the main girders on the pivot pier. The rollers revolve in wrought-iron carriages upon fixed radial axles. Two cast-steel rollers, also 3 feet in diameter, are arranged at the tail end, one under each main girder, which carry the preponderating load only, viz. 15 tons.

A solid bearing pivot was adopted in preference to a hydraulic ram or water-bearing, as, the necessary vertical move-

[1] Vol. cviii. *Proc. Inst. C.E.*, paper by F. Fox, M.Inst.C.E.

ment of the bridge being provided for, a ram was not necessary. The cup-and-ball pivot is 30 inches in diameter, of forged wrought iron, case hardened, which provides for both rotating and bascule movement. The lubricant used is heavy cylinder oil, which is injected into the bearing about once for every three swings. The weight resting on the pivot is 732 tons while the bridge is being turned, and the average pressure per square inch about 1 ton; the maximum pressure at the centre is, however, much greater, as the radius of the cup is ¼ of an inch larger than that of the ball.

The bridge is moved by two single-acting hydraulic rams securely attached to the solid brickwork of the main pier. The slewing drum is 32 feet in diameter, and forms a part of the main bearing girder. The tail-end rollers are fitted with a winch gear and brake, in order that the bridge may be opened and closed by hand. The bascule movement of the bridge is utilized in mechanically locking it in position when closed, and for closing the space which would otherwise occur between the ends of the rails on the moving and fixed portions of the bridge. The important moving parts of the bridge are interlocked with the signals on the railway. The opening of the bridge requires the following operations: (1) Lifting the tail end of the bridge ⅛ inch by the tail-end rams; (2) withdrawing the sliding bearing blocks from under the tail ends; (3) lowering the tail-end rollers; and (4) setting in motion the rotating rams.

Class C.—This class of bridge, which is made to turn entirely on a long central pivot of conical form, is used largely in Holland. It is necessary for the roadway to be sufficiently high above the water-level to allow of the necessary length of the pivot, and also for the main girders to be arranged below the roadway, keeping the centre of gravity below the point of support. The foundations must be good, as any settlement might cause the bridge to jam so that it could not be turned.

Class D.—This class of bridge is used largely in connection with the entrances to docks. The bridge is arranged so as to be out of balance, and is supported in the centre upon an hydraulic press and at the tail end upon wheels. When the centre is raised, the bridge tilts over until the wheels on the tail end come in contact with the tram plate. The tail end may tilt upwards or downwards; in the former case the tram plate is inverted. The central pivots are made spherical, but the bridge actually turns on a water-centre. Examples of this class of bridge exist at the Albert Dock, Hull; the West India Docks, London; the Leith bridge, England; and the Marseilles bridge, France.

Another example of this class of bridge is the swing bridge of Le Pollet (Dieppe).[1] This bridge was necessitated by the New Dock Works, Dieppe, and crosses a waterway 131 feet wide. The longer arm of the bridge, when swinging, is 154 feet, and the shorter arm 77 feet. The main girders are 30 feet deep over the pivot, with horizontal bottom booms and curved top booms of trough-shaped section. The panel length is 16 feet, with vertical posts and diagonals, in both directions. The total weight of the bridge swinging is 800 tons, and the ballast weighs 234½ tons, or 20 tons more than is necessary to counterbalance the bridge. The deflection of the long end in swinging is 4½ inches. The bridge, when in use, rests on three sets of bearings, viz. at the tail end and on the passage walls, being clear of the pivot. When turning it is carried on a hydraulic plunger, and the excess weight at the tail end rests by two pairs of rollers on the rail track. The opening of the bridge requires the following operations: (1) The cylinder under the pivot is opened to the pressure water, and the plunger-head brought in contact with the bridge. The bridge does not yet lift, as the area of the plunger is only proportioned to 624 tons. (2) Pressure is opened in two hydraulic cylinders placed one under each main girder near the tail end of the bridge. The bridge is now lifted from its bearings, and is water-borne on three cylinders. (3) The supports under the tail end are withdrawn by means of a small hydraulic cylinder operating through a crank and connecting rod. (4) Pressure is withdrawn from the two lifting cylinders at the tail end. The bridge now falls again, and, owing to the overbalance, tilts backwards until the rollers at the tail-end rest on the rail, the central plunger sinking to the bottom of its cylinder, where it rests upon a steel ring let into the bottom of the casting. (5) The bridge is now swung by the usual hydraulic arrangements. The weight is thus divided in turning between the water pressure, which carries 624 tons, the

[1] *Annales des Ponts et Chaussées*, December, 1891, p. 584; abstract, vol. cix., p. 416, *Proc. Inst. C.E.*

steel ring in the cylinder carrying 156 tons, and the rail track at the tail carrying 20 tons. In closing the bridge these operations are reversed. The opening or closing takes usually from two to three minutes, although it can be done in one minute and a half.

The large swing bridge over the Tyne, at Newcastle, cannot be included strictly in any of the foregoing classes. This bridge is supported and turned on a water-centre at a constant level, acting as a relieving, not a lifting, pivot. It is in every respect a most satisfactory structure.

In designing a swing bridge, whether for road or railway traffic, it is necessary to reduce the dead weight of the moving portion of the bridge, in order to reduce the frictional resistances to turning, which are proportional to this weight.

Steel is obviously the best material to use for the moving portion of a swing bridge, and the tensile working stresses in the various members of the structure when closed should be determined in the manner explained in Chapter I. for ordinary bridges. The compressive working stresses may be determined by the formulæ given in Chapter XII. When the bridge is being swung, the stresses in the various members of the structure are due to the dead load only; but, since in starting and stopping there is a certain amount of impact, it is desirable to adopt the rule given by Mr. T. Cooper in his specifications, viz.—

"For swing bridges and other movable structures, the dead-load unit stresses during motion must not exceed three-fourths of the unit stresses for dead load on stationary structures."

In the Hawarden bridge the working stress in tension was 6½ tons, and in compression 5 tons.

As to whether the main girders should be made continuous or discontinuous over the swing pier will depend upon the width of the openings provided, and the live load upon the bridge, as well as the method proposed for fixing the ends. In an ordinary highway bridge with two openings, each 60 feet, there is no reason why the girders should not be made continuous, and in railway bridges of greater length the girders may be made continuous, provided that the ends are securely fixed in position when the bridge is closed. If the ends of a long railway swing bridge merely rests upon the supports at the ends, the tendency of a heavy engine entering the bridge at one end is to raise the other end, and it is necessary to make proper provision at the ends to resist this tendency, or the girders should be made discontinuous when closed by hinging them over the swing pier, and connecting the girders to a vertical post over the hinge by means of tension bars, which are brought into action when the bridge is moving, or the same method may be adopted as that described for the Raritan bridge.

It is very important that provision should be made in all swing bridges, which are not lifted mechanically before turning, for taking out the droop of the girders when turning, such as a toggle joint hydraulic lifting arrangement, as in the Ouse bridge, or a cam worked by hand-gear in ordinary hand-power bridges, or the methods adopted in the Hawarden and the Le Pollet bridges may be used.

If the end drop or deflection is not taken out by some means, the stresses due to the deflection must be added to those produced by the live load, which is unsatisfactory in every way.

The frictional resistances during turning in bridges where the whole of the load is carried on a ring of live rollers, will depend to a large extent upon the equal distribution of the load over these rollers, and distributing girders should be arranged between the underside of the main girders and the circular girder immediately above the live rollers, in order to distribute the load equally over six equidistant points of the circular girder.

On Plate IV. are shown details of a design for a swing bridge, with a roadway 18 feet wide and two openings each of 60 feet in the clear. The main girders are of the lattice type 21 feet 4 inches between centres, and 152 feet over all, the depth at centre being 7 feet, and at the ends 5 feet effective. The boom sections are formed of steel T bars and flange plates 2 feet wide of wrought iron, and the web consists of wrought-iron lattice bars and ⊔ irons braced to form struts, the central 14 feet 6 inches being plated and stiffened over points of support and at the centre. At each lower apex is a wrought-iron cross-girder, resting on a saddle secured to main girder, and carrying seven longitudinal timber stringers, to which is spiked the 4-inch diagonal planking forming the roadway.

At the centre the main girders are secured each to two wrought-iron stools 12 feet 4 inches apart, while the ends, when the bridge is closed, are raised and supported on cast-iron cams.

The conditions for calculation are therefore those of a continuous girder, with two side spans of 70 feet and a central span of 12 feet 4 inches, the side spans acting as cantilevers when the bridge is swinging. The girders are designed for a distributed live load of 84 lbs. per square foot, and a concentrated rolling load of 16 tons on a 10-feet wheel base. To minimize the deflection at ends when the bridge is open, an excess of area is provided throughout.

In accordance with the most recent practice, particular attention has been given to the method of distributing the weight of the superstructure evenly on to the rollers. These are of cast iron, thirty-two in number, 18 inches diameter, and 6 inches face, revolving round a central pivot, and carrying a wrought-iron annular girder 21 feet 4 inches diameter, to which the weight of the superstructure is delivered at six equidistant points. As shown on diagrams, Plates IV., IV*a*, IV*b*, and IV*c*, each main girder rests on two stools, secured each to a longitudinal distributing girder, one end of which is carried by a stool on the annular girder, and the other by the transverse distributing girder. The position of the stools under the main girders is such that one-third of the load is delivered direct from each to the annular girder, and two-thirds to the transverse distributing girders, and by them to the annular girder. By this means, if W be the weight on each stool, two-thirds of W is taken at each of the six points. When the bridge is swinging, the total load on rollers due to superstructure, including the weight of distributing arrangement, etc., is 173 tons; so that the pressure on rollers is 18 cwt. per lineal inch of face. Secured to the upper annual girder and to a casting which revolves round the pivot are six radial distance girders of light section. The rollers are conical, with axes terminating at the centre of the pivot at a point in the same horizontal plane as the upper surface of rollers. The roller tracks are of cast iron in twelve segments, and special arrangements have been made for obtaining a perfectly level surface, when erecting the lower track, by means of steel adjusting wedges and iron cement. For horizontal adjustment a fine circular grove is cut in the track, and a single roller rotated until its inner edge coincides with the grove throughout. To allow of the rollers being run in or out, each roller rod is fitted with an adjustable gun-metal bush on which the roller revolves. The joints in tracks are made diagonal in order to distribute the pressure over two segments.

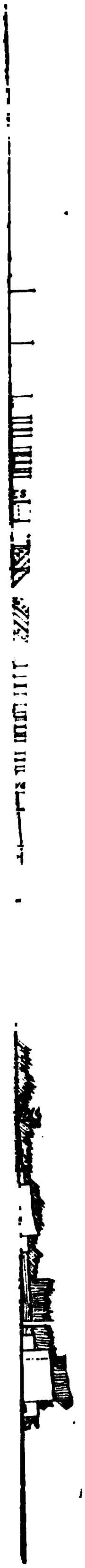
The machinery for operating the swing span is carried on a platform on the outside of the downstream main girder, and is worked from the deck of the bridge, both lifting and turning gear being driven from the same handle. The machinery for lifting the ends has been designed for a deflection of 2 inches, and consists of bevel wheels and pinions operating a shaft which runs the whole length of the bridge, carried by brackets on the main girder. At each end of this shaft is a steel worm gearing into a phosphor-bronze wheel attached to a transverse shaft, with two cams of a 1-inch throw at each end working in cast-iron cam rollers, on which the bridge rests. These in turn are supported on cast-iron chairs over piers, provided with cast-iron bearing blocks fitted with wedges to allow of adjustment when erecting. To prevent over-winding, the second motion shaft is screwed for a portion of its length, and provided with two gun-metal stops and a gun-metal nut, with arms working in cast-iron guides bolted to the outer web of the main girder.

By means of a clutch on the first motion shaft worked by a hand-wheel on the inside of main girder, the machinery for either lifting or turning the bridge may be thrown into gear as desired. The latter consists of gearing which operates a pinion E on a vertical shaft working into a rack secured to the lower annular girder. The rack is of cast iron in five segments, and allows for a movement through 150°.

It is calculated that two men will be able to swing the bridge in four minutes, the operation of lifting occupying the same time. A tell-tale near the operating handle indicates the position of the bridge during opening and closing.

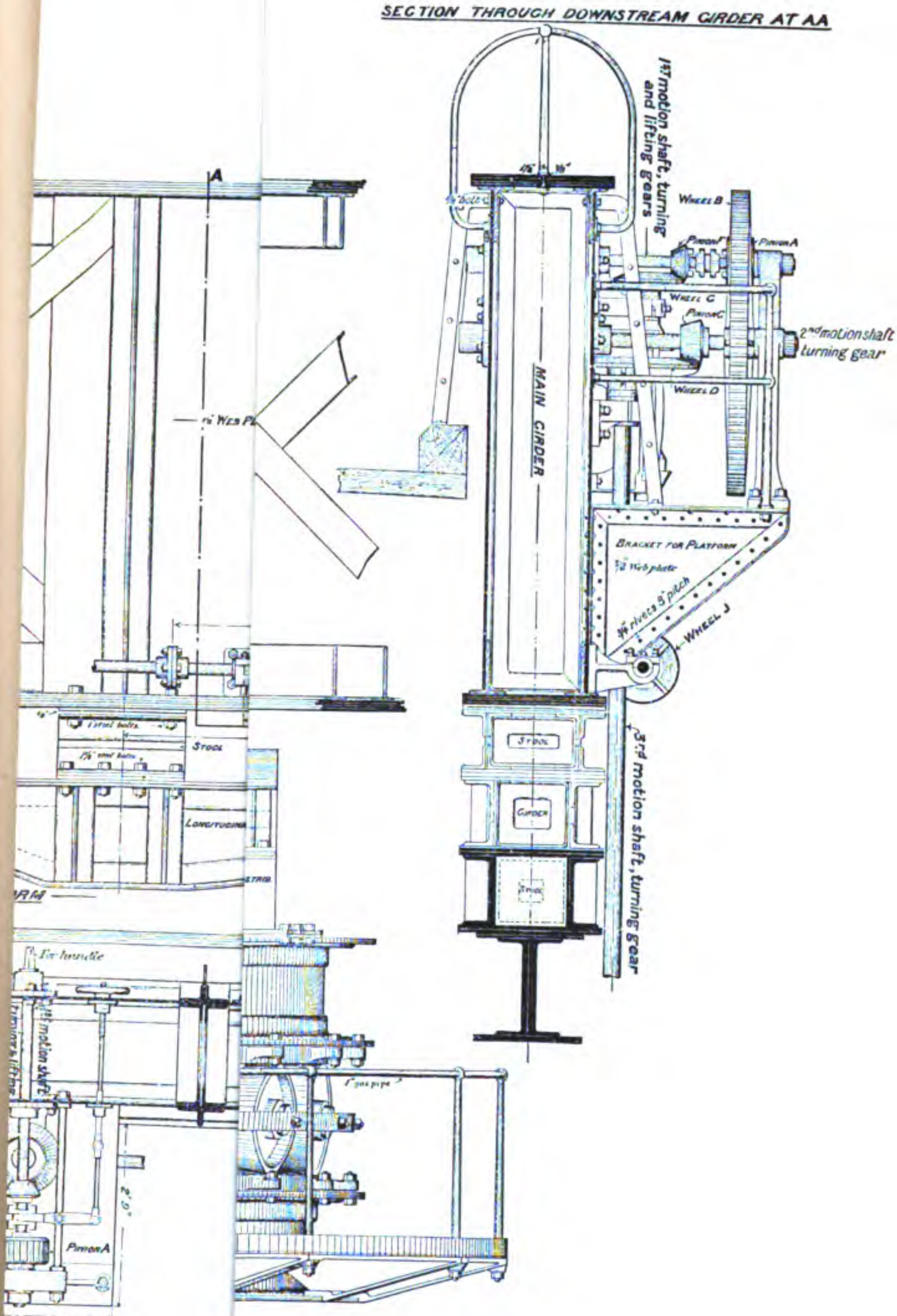
The swing pier consists of six cylinders, 3 feet 6 inches diameter, spaced equidistantly round a circle of 21 feet 1½ inch diameter, and securely braced to each other and to a central cylinder carrying the pivot.

The piers at ends of span are formed each of two cylinders 6 feet in diameter, with diaphragm bracing. All the cylinders are of cast iron to high-water mark, above which they are of wrought iron, and they are filled with concrete. The bridge illustrated is suitable for light traffic which does not include vessels of large size, but the piers would require to be protected by means of dolphins and guide piles, or a wrought-iron caisson filled with concrete might be substituted for the six cylinders with advantage. The foundations are good in the case illustrated, but, if there was any likelihood of settlement, a much more substantial central pier would have been necessary. The dimensions of the girders may be verified by the student, the calculations being similar to those already illustrated.



DE

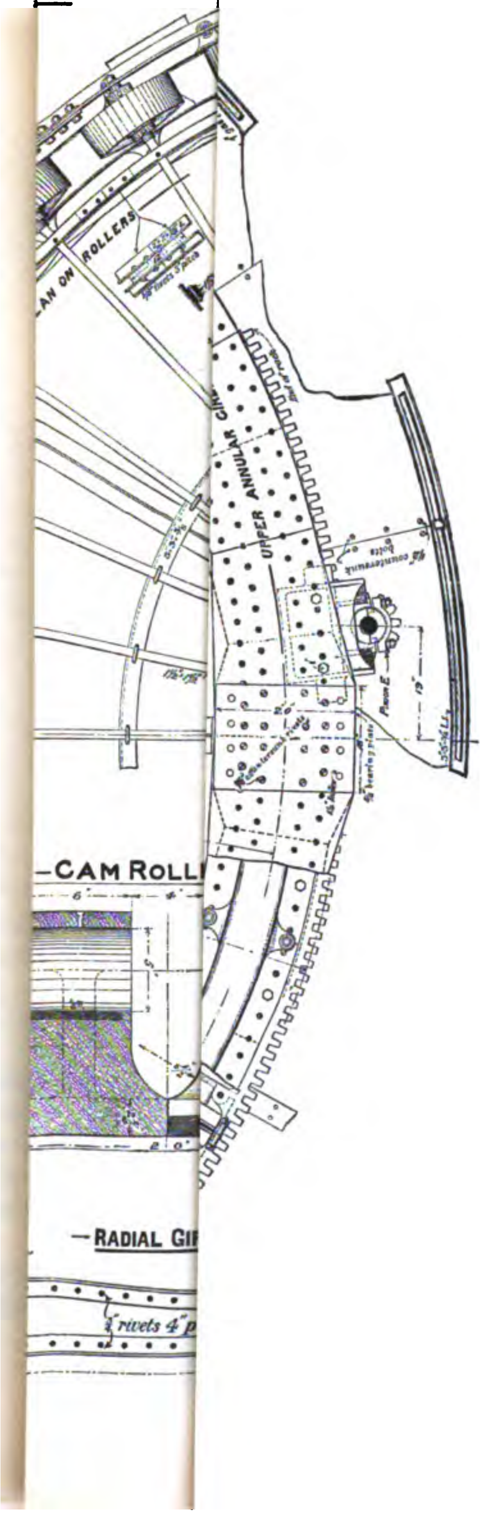
PLATE IVb.


SECTION THROUGH DOWNSTREAM GIRDER AT AA
1st motion shaft, turning and lifting gears
Wheel B
Pinion A
Wheel C
Pinion C
2nd motion shaft turning gear
Wheel D
MAIN GIRDER
Bracket for Platform
Wheel J
Stool
Girder
3rd motion shaft, turning gear
Stool
Longitudinal
Pinion A
2'0"

VING BRID

PLATE IVc.

S.



CHAPTER XXIII.

ARCHED AND SUSPENSION BRIDGES.

THE arch is more economical than the most economical truss or girder, considered merely with regard to the ratio of its carrying capacity to its own weight. This is seen when the weight of a bowstring girder is compared with an arched rib. In the bowstring girder, the curved member must be designed to resist the direct stresses developed in it, and the horizontal member the constant tensile stress. We have seen that for uniform loading, or for equal loads concentrated at each panel-point, there is no stress in the web bracing, but for partial loading produced by a passing train the web bracing will be stressed. The piers supporting the truss must be designed to carry the weight of the structure and the maximum load carried by it. If the ends of the curved member are made to abut against the piers or abutments the horizontal tie may be dispensed with, but the abutments or piers (if there is more than one span) must be made larger, in order that they may be capable of resisting the thrust produced by the end reactions of the curved rib.

If the extra cost of the abutments is less than the cost of the tie, then the arch is a cheaper structure than the truss.

In long spans the saving effected by dispensing with the tie and the bracing bars of the web is considerable, which is due not merely to their extra cost above the extra cost of the abutments and the lighter bracing of the arch, but to the fact that in the arched rib the load sustained is less by the extra weight of material necessary in the bowstring truss.

The suspension bridge is the inverted form of the arched bridge, in which the tension in the suspended rib or cables corresponds with the compression in the arched rib. In either case, if the rib is loaded in any way whatever, the correct form, in order that it may sustain the load without bending, is the equilibrium polygon for the load in question.

Let W_1, W_2, W_3 . . . W_6 (Fig. 352) denote a series of loads acting on a straight beam *amh*, and let *abc* . . . *h* denote the equilibrium polygon for these loads obtained by drawing lines parallel to the strings in the force polygon, Fig. 353. Draw

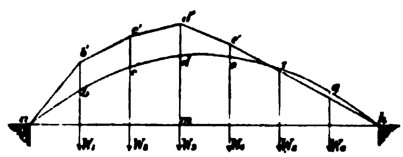


FIG. 352.

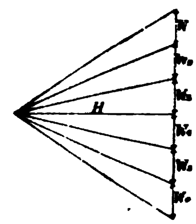


FIG. 353.

the line H perpendicular to the load line, Fig. 353, representing to the scale of the force polygon the horizontal component of the stress in *ah*.

Then, by Culman's principle, the bending moment at any point in the beam, such as *m*, is equal to—

$$dm \times \text{H}$$

If we denote the bending moment at any point by $\Sigma \text{W}x$, and the corresponding ordinate of the equilibrium polygon by V, then—

$$\Sigma \text{W}x = \text{HV, and V} = \frac{\Sigma \text{W}x}{\text{H}}$$

From this equation we observe that the height of the equilibrium polygon is inversely proportional to H.

It may be shown that the form of the equilibrium polygon for loads distributed equally along the horizontal is a parabola. for loads equally distributed along the length of the rib it is a catenary, and for uniform pressure acting normally at any point in a curve it is a circle.

If the form of the arched rib corresponds with the polygon *abc* . . . *h*, there will be no bending stress upon it; but if the system of loading produces an equilibrium polygon *ab'c'd'e'h*, the arched rib will be subjected to bending stress.

Let V′ denote the ordinate of the polygon, and V that of the rib, and M the bending moment at the point *m*; then—

$$M = \Sigma Wx - HV'$$

but since $\Sigma Wx = HV$

$$M = -H(V' - V)$$

The bending moment at any point of an arched rib for any system of loading is proportional to the difference between the ordinates of the rib and of the equilibrium polygon for that system of loading.

This is perfectly general, and applies to arches with fixed and free ends.

The loads upon an arched rib produce at any section—

(1) A normal thrust which is uniformly distributed over the area of the rib at the section, its horizontal component being H.

(2) A shearing stress at the section.

(3) A bending moment $H(V' - V)$ producing tension and compression on opposite sides of the neutral axis of the section.

The direct compression, combined with bending, produces a distribution of stress over the section similar to that which has been shown to exist in short columns. The methods adopted in the foregoing investigations of the two-hinged arched rib, also the arched rib without hinges, is partly taken from Professor Burr and Professor Eddy.[1]

Let S = the shearing stress at any section.
W = any load upon the rib.
M = the bending moment at any section.
V = the vertical deflection of the rib at any section.
V_h = the horizontal deflection.
a = the strain, extension, or compression in any fibre situated at a perpendicular distance unity above or below the neutral axis and parallel to it.
n = the length of the parts into which the arched rib is divided; the smaller the value of n the more accurate the results.

Then the following approximate equations may be used:—

$$S = \Sigma W \qquad (1)$$

$$M = \Sigma Wx \qquad (2)$$

[1] "Stresses in Bridge and Roof Trusses," by W. H. Burr; "Researches in Graphical Statics," by H. T. Eddy.

$$a = \frac{M}{EI} \qquad (3)$$

$$\Sigma na = \Sigma \frac{nM}{EI} \qquad (4)$$

$$V = \Sigma nax = \Sigma \frac{nM}{EI} x \qquad (5)$$

$$V_h = \Sigma nay = \Sigma \frac{nM}{EI} y \qquad (6)$$

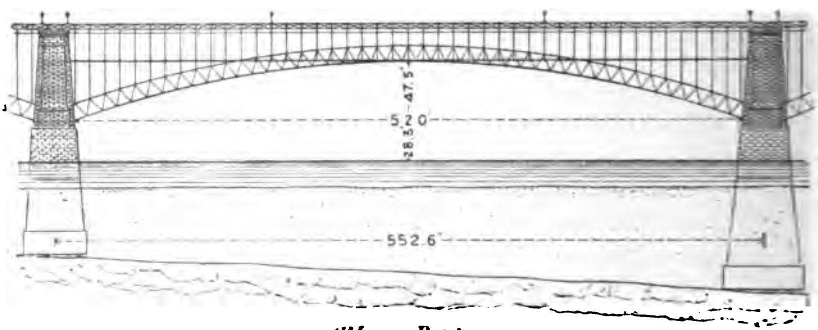
From (2) and (5) we have—

$$M : V :: \Sigma Wx : \Sigma \frac{nM}{EI} x$$

$$:: \Sigma W : \Sigma \frac{nM}{EI}$$

Hence in any equilibrium polygon in which $\frac{nM}{EI}$ is used instead of W, the ordinates will represent the deflections.

In an arched rib with fixed or free ends, the depth should be made sufficient to enable it to retain its form under the partial distribution of the maximum live load. This is most conveniently done by dividing the material of the rib into two portions, placed one below the other, forming two arches united



St. Louis Bridge.

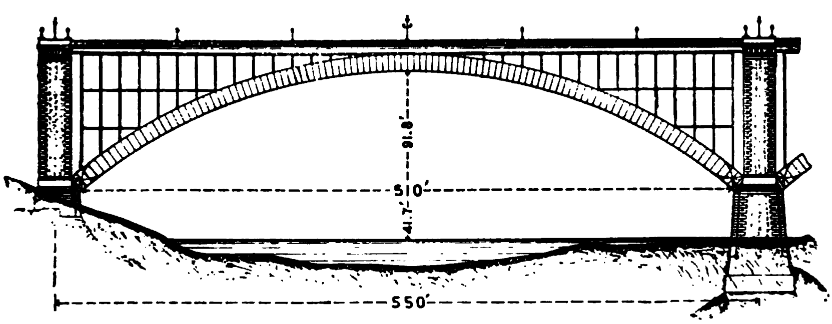
FIG. 354.

together with suitable bracing. The finest example of the rigid arch occurs in the bridge over the Mississippi at St. Louis (see Fig. 354), which consists of three spans, 502, 520, and 502 feet. Each span consists of four ribs, and each rib is constructed with two steel tubes spaced vertically 12 feet apart centre to centre and united with bracing. The dead load per lineal foot is 1 ton on each rib, and the live load produced with a crowd of people on a roadway and footpaths 52 feet wide, and two lines of rails underneath loaded with engines, is 0·8 ton per foot run on each rib.

The total weight of iron and steel in one rib of the central span is 218 tons. The total weight of iron and steel in the four ribs, including vertical struts to roadway, and railways, wind bracing, etc., is about 1600 tons. The total cost of the entire bridge was £1,361,873.

Four bowstring trusses of the same central span as the St. Louis bridge would weigh at least 500 tons more. The temperature stresses were calculated for an extreme variation of ± 80° Fahr. The maximum working stress allowed was 13·4 tons in compression, and 8·9 tons in tension.

As an example of an arched bridge hinged at the springing, we may select the Harlem River bridge (Fig. 355), which consists of two steel arches, each 510 feet span, with masonry piers and abutments. Each span is constructed with six steel



Harlem Bridge.

FIG. 355.

ribs spaced 14 feet centres under the roadway, which is 80 feet wide, with two footpaths. The effective depth of the arched ribs is 12 feet. The bridge was designed for the following loads.

Floor of Bridge.—A dead load 225 lbs. per square foot.
A live load 100 „ „ „
Or a 20-ton steam roller.

Arched Ribs.—A dead load of each rib 1·1 ton per foot run.
A dead load of floors and paving 1·35 ton per foot run.
A live load of 0·6 ton per foot run.

A wind pressure of 5·4 tons per foot distributed in proportion to the exposed surfaces.

The modulus of elasticity of the iron was assumed to be 26,000,000 lbs. per square inch. Working stress in ribs, tension or compression, on gross area, 6·7 tons per square inch;

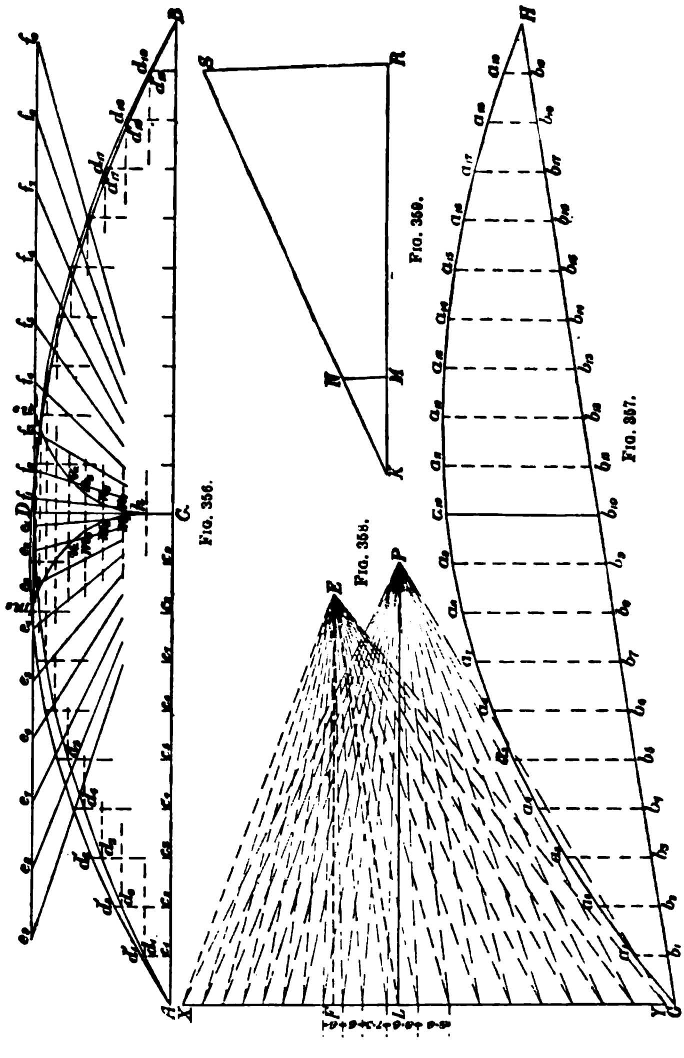


Fig. 356. Fig. 357. Fig. 358. Fig. 359.

on net sections, 8 tons per square inch. Extreme variation of temperature, ± 75° Fahr.

Arched Rib hinged at the Springing.—The equilibrium polygon for any loads acting on an arched rib hinged at the springing and continuous at the centre may be determined in the following manner.

Let $Ad_1, d_2, \ldots$ B, Fig. 356, denote the neutral axis of the rib, which is 147 feet span, with a central rise of 20·5 feet.

Let the centre line of the rib be divided into a number of equal parts, the length of which may be denoted by n, representing the points of application of the loads upon the rib; the smaller the distance between these divisions the more nearly will they represent a uniform load, and if the points of application are taken sufficiently near to each other, the distance n may be considered to have the same value whether measured horizontally or along the curve of the rib. We may consider the rib as one of a pair, spaced 16 feet apart centre to centre, in a bridge carrying a roadway 18 feet wide between the kerbs. The deck may be supposed to be secured to two longitudinal girders supported at the panel-points of the rib by means of vertical columns. Let the dead load concentrated at the panel-points of the ribs, 7·35 feet apart, be 6 tons, and the live load equivalent to 80 lbs. per square foot of roadway be 2·6 tons. Only one case of loading will be considered, viz. that in which each panel-point on the left half of the rib is loaded with the live and dead load, or 8·6 tons, and each panel-point on the right half with 6 tons; the centre panel-point is loaded with $\frac{6 + 8{\cdot}6}{2} = 7{\cdot}3$ tons. As a matter of fact, the dead load is not uniformly distributed horizontally, being greater towards the abutments. Draw the equilibrium polygon for this distribution of loads, $Ga_1a_2a_3 \ldots$ H, Fig. 357, with an assumed polar distance EF, Fig. 358, setting off the loads on the vertical line XLY, Fig. 358, and draw the closing line of the polygon GH, Fig. 357. Now, since the extremities of the neutral axis of the rib A and B remain unchanged in position at the springing, whatever loads, producing deflections, act upon the rib between these points, it follows that the sum of all these deflections, both horizontally and vertically, between these points A and B must equal zero; hence—

$$\Sigma_B^A \frac{nMx}{EI} = \Sigma_B^A \frac{nMy}{EI} = 0$$

$$\therefore \Sigma_B^A Mx = \Sigma_B^A My = 0$$

In consequence of the hinges, the ends of the rib may rise or fall vertically without affecting the conditions of bending, so that it is only necessary to consider the sum of the horizontal deflections—

$$\Sigma_B^A My = 0$$

But since the neutral axis of the rib $Ad_1d_2d_3$ . . . B may be considered as an equilibrium polygon for its proper load, denote its ordinates by M_d, and the ordinates of the equilibrium polygon $Ga_1a_2a_3$ . . . H by M_a. Then we have seen that—

$$M = -H(V^1 - V)$$

$$\therefore \Sigma_B^A M_a y - \Sigma_B^A M_d y = \Sigma_B^A My = 0$$

$$\Sigma_B^A M_a y = \Sigma_B^A M_d y$$

To draw the horizontal deflection polygon for the arched rib subjected to the bending moments denoted by M_a, we may treat it as a vertical beam fixed at C and loaded at points at a distance from C equal to the vertical heights of the points d_1d_2 . . . above AB, using, however, the ordinates M_a in the same manner as loads in an ordinary equilibrium polygon.

Assume C as the pole, and set off the ordinates M_a along a line Df_9, parallel to AB, to a convenient scale, thus—make $Df_1 = a_1b_1 + a_{19}b_{19}$, $f_1f_2 = \frac{a_2b_2 + a_{18}b_{18}}{4}$, and so on.

Draw the lines from $f_1f_2f_3$ . . . f_9 to C, also the lines from $d_1d_2d_3$, etc., parallel to AB cutting CD. The intersection of the line from d_1d_{19} with CD will be taken as one point in the deflection polygon, since the moments at A and B are zero. Draw hn_1 parallel to Cf_1 intersecting d_2d_{18} in n_1, and n_1n_2 parallel to Cf_2 intersecting d_3d_{17} in n_2, and so on, thus obtaining the deflection polygon hn_1n_2 . . . n_9. In a similar manner we obtain the deflection polygon hm_1m_2 . . . m_9, using the ordinates M_d instead of M_a and setting them off on the left of CD. The ordinates have been taken in pairs in each case, since each member of the pair acts at the same distance y from C. Since

Dn_9 represents the maximum deflection for bending moments M_a, and Dm_9 for M_d, it follows that, since $\Sigma_B^A M_a y = \Sigma_B^A M_d y$, Dn_9 should equal Dm_9; but Dm_9 is less than Dn_9, hence the ordinates M_a are too large or the polar distance EF is too small, since by Culman's principle the ordinates multiplied by the polar distance equals the bending moment. The moments represented by the polygon M_a must therefore be decreased by multiplying them by the ratio $\frac{Dm_9}{Dn_9}$. Set off a line KM $= Dm_9$, Fig. 359, and draw MN at right angles to KM. With K as centre and Dn_9 as radius, cut MN in N. Then the polar distance must be increased in the ratio KN to KM. Make KR equal to the assumed polar distance, and draw RS parallel to MN; then KS is the true polar distance for the equilibrium polygon M_a. To find the true pole, draw EL parallel to GH, then draw LP parallel to EF and equal to KS. The vertical reactions are LX and LY at B and A, or 63·5 and 75·2 tons respectively. We may find these reactions independently by calculation. The horizontal thrust found by measuring LP is 132·5 tons. The horizontal thrust may be calculated by the following formula, which is due to Professor Winkler.

Let H = the horizontal thrust.

S = the span, in this case 147 feet.

W = a load concentrated at any point of the rib.

x = horizontal distance of the load W from the crown of the rib.

r = the rise of the rib = 20 feet.

Then—

$$H = \frac{5W(5s^2 - 4x^2)(s^2 - 4x^2)}{128rs^3}$$

The only variable quantities are W and x, therefore substitute for W the loads concentrated at the panel-points of the rib, and put $x = 0, \frac{s}{20}, \frac{2s}{20}, \frac{3s}{20}$, etc.; thus let $x = \frac{s}{20}$, and W = 8·6;

$$H = \frac{5 \times 8{\cdot}6 \times 147\{[5 - 4(\frac{1}{20})^2][1 - 4(\frac{1}{20})^2]\}}{128 \times 20} = 12{\cdot}19 \text{ tons}$$

Repeating this calculation for each of the loads and adding the results together, we obtain for the horizontal thrust 134 tons, which checks very well with the value 132·5 tons found by graphical construction.

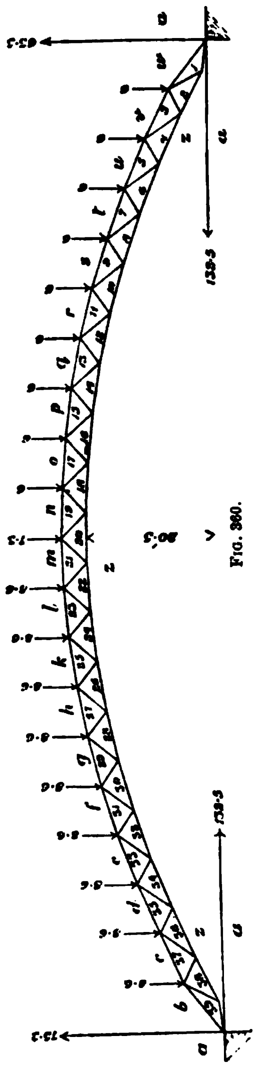


Fig. 360.

Having obtained the horizontal thrust, we may find the stresses in the arched rib by the method of moments, or we may join the pole P with the points of the force polygon XLY, and construct the equilibrium polygon $Ad_1'd_2'B$. The intercepts between the polygon last drawn and the rib multiplied by the polar distance or horizontal thrust LP equals the bending moments at the points d_1, d_2, d_3, etc. The line Ad_1d_2B denotes the neutral axis of the arched rib, and may be considered as the line passing through the centres of gravity of the cross-section of the rib.

The depth of the rib should be made from $\frac{1}{40}$ to $\frac{1}{50}$ of the span in order to give sufficient stiffness under partial loading; in the examples selected the depth is made 3 feet 6 inches between the centres of gravity of the top and bottom flanges. The system of bracing adopted between the flanges is sufficiently illustrated in Fig. 360.

The stresses in the various members of the arch are shown by means of the stress diagram, Fig. 361, which needs no explanation. As a check on the accuracy of the work, a few of the members should be checked by moments; thus C_{37}, by measurement from the stress diagrams, is found to be 86·5 tons; by moments it is 87 tons. Bar 21–22 is subjected to a compressive stress of 12·5 tons, d_{36} to a compressive stress of 93·5 tons, Z_{10} to a compressive stress of 127 tons.

The foregoing method illustrates how the stresses in an arched rib hinged at the springing may be determined. In any actual case it is necessary to determine the horizontal thrust and the various stresses produced by it for the various positions of the live load. The stresses should then be tabulated, and the rib designed for the maximum stress in the various bars.

In consequence of the continuity of the rib at the crown, it will be subject to stresses, in addition to the foregoing, from changes in temperature. Since the ends of the rib are hinged, any variation in temperature will cause the length to vary, and the crown to rise or fall. The effect of a rise in temperature

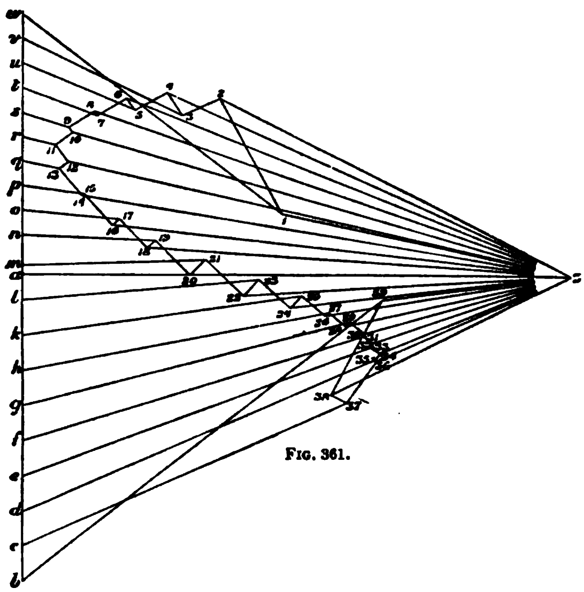


FIG. 361.

above the mean temperature for which there is no stress, is to cause a horizontal thrust at the ends. The deflection in the centre of the rib caused by a variation of temperature of ± 50 Fahr. will be 1 inch approximately, or $\frac{1}{12}$ of a foot. The following approximate formulæ may be used to calculate the deflection of a parabolic arched rib due to its expansion or contraction from variations of temperature.

Let s = the length of half the arched rib.

r = the rise, or central deflection.

y = the half-span.

Then—

$$s = \sqrt{y^2 + \tfrac{4}{3}r^2}$$

To find the deflection from the length of the curve and the span, let r' = the deflection, and s_1 = the altered length; then—

$$r^1 = \sqrt{(\tfrac{3}{4}s_1^2 - y^2)}$$

The horizontal thrust due to this deflection may be considered as acting along the line AB.

Comparing the following equations :—

$$\Sigma Wx = Hy$$
$$\Sigma nMx = \Sigma M'x = EIV$$

where H = the horizontal thrust required.

$$M' = nM, \text{ and } M' = nHy, \text{ also } H = \frac{EIV}{\Sigma nyx}$$

From these equations it is clear that if M′ be taken as vertical loading, and EI as the polar distance, the ordinates of the corresponding equilibrium polygon will represent the deflections to the same scale as x; also that M′ and EI are of the same denomination, and hence must be measured to the same scale. It is necessary now to consider the value EI, to do which we must assume a cross-section which can be easily designed approximately by means of the stresses obtained disregarding temperature. The section should be dimensioned in feet for finding the moment of inertia, and E may be taken as 26,000,000 lbs. per square inch, or 1,670,400 tons per square foot. It will then be found that the product EI is about 1,600,000 foot-tons with the assumed section of rib shown in Fig. 362.

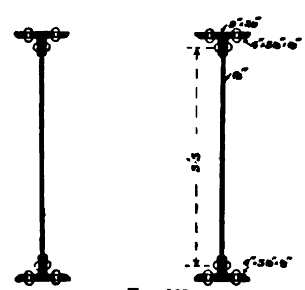


Fig. 362.

In Fig. 364 draw LP parallel to AB, Fig. 363, and equal to EI, or 1,600,000 foot-tons to scale. If LP is made 16 inches, then 1 inch will represent 100,000 foot-tons.

Set off the ordinates a_1b_1, a_2b_2, a_3b_3, etc., . . . DC on the load line LM, making each one-half of its length in the polygon Aa_1a_2 . . . B, in order to avoid an unnecessarily large figure. Draw the polygon Fc_1c_2 . . . E, with P as the pole representing the actual deflections of the rib to an exaggerated scale; thus

LF represents the deflection of the point A, which we have assumed to be 1 inch, which will measure 3·1 inches if the

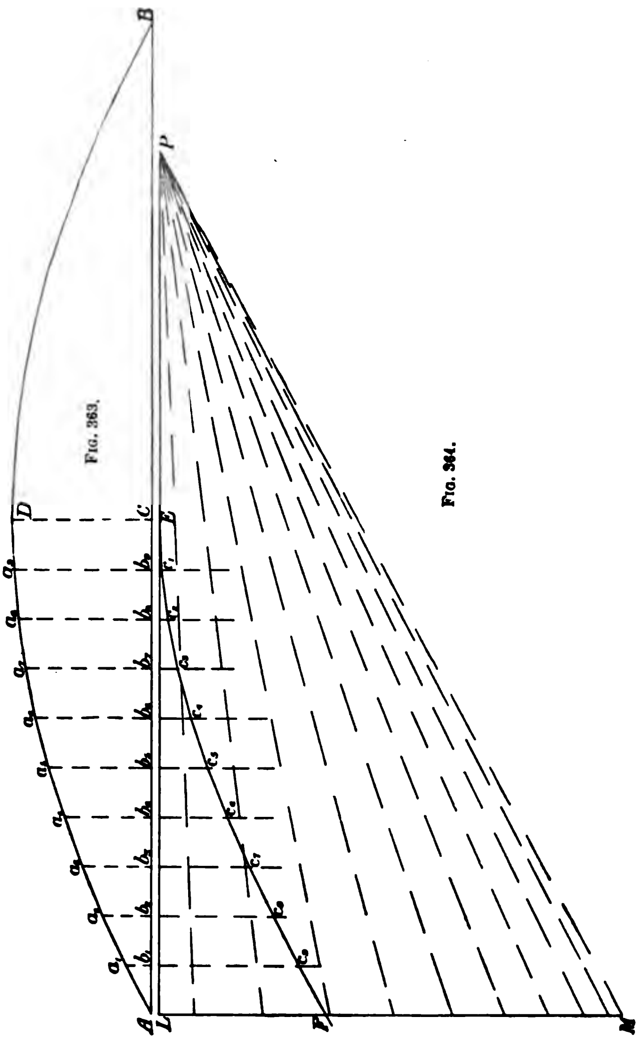


FIG. 363.

FIG. 364.

rib is drawn to a scale of 8 feet to an inch, corresponding with $8 \times 3{\cdot}1 = 24{\cdot}8$ feet; the bending moments must be reduced in the ratio of 24·8 to $\frac{1}{12}$, or $\frac{1}{297{\cdot}6}$.

The bending moment at the point A is $\dfrac{\dfrac{2{\cdot}56}{2} \times 100000}{297{\cdot}6}$,

and the horizontal thrust is $H = \dfrac{M_1}{ny} = \dfrac{1{\cdot}28 \times 100000}{297{\cdot}6 \times 7{\cdot}35 \times 20{\cdot}5}$
$= 2{\cdot}8$ tons.

Since $H = \dfrac{EIV}{\Sigma nyx}$, we might calculate the horizontal thrust by this equation, but it would be necessary to measure the quantities yx, and take their sum.

Professor Winkler's formula is as follows:—

$$H = \frac{15EIaet}{8ar^2 + 15I}$$

where a = the area of the rib = 48 square inches.
e = the coefficient of expansion = 0·0000068.
t = the temperature = 50°.

$$\therefore H = \frac{15 \times 1600000 \times \frac{48}{144} \times 0{\cdot}0000068 \times 50}{8 \times \frac{48}{144} \times 20{\cdot}5^2 + 15 \times 0{\cdot}9} = 2{\cdot}4 \text{ tons}$$

This agrees very well with result found by means of the equilibrium polygon.

In the Harlem bridge the range of temperature allowed was 75° Fahr. above and below the mean, and the increase in the stresses amounted to 1 ton per square inch.

Arched Rib with Fixed Ends.—The equilibrium polygon for

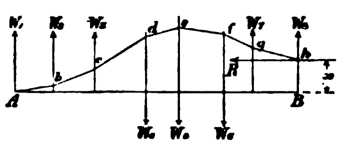


Fig. 365.

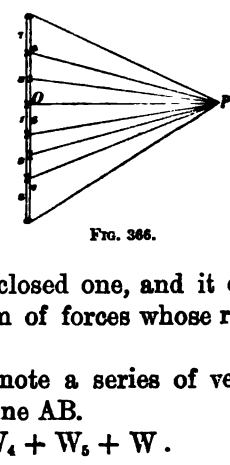


Fig. 366.

an arched rib with fixed ends is not a closed one, and it corresponds with that obtained from a system of forces whose resultant is a couple.

Let W_1, W_2, . . . W_8, Fig. 365, denote a series of vertical forces acting at points of a horizontal line AB.

Let $W_1 + W_2 + W_3 + W_7 + W_8 = W_4 + W_5 + W$.

Set off the forces on a double line, Fig. 366; draw a horizontal line through O, and on it take any point P as a pole, so that the line PO represents the polar distance. Join P with the angular points of the force polygon, and draw the polygon A*bcdefgh*, Fig. 365, the sides of which are parallel to the lines converging at P. This polygon will not close, showing that the resultant is a couple whose moment is the polar distance PO multiplied by the lower arm x, or Rx, Fig. 365.

In the rib with fixed ends, as with free ends, we have—

$$M = -H(V^1 - V),$$

for to fix the rib at the ends we have only to apply the proper couple, whose moment is m, remembering that the vertical dimensions of the equilibrium polygon will be increased over the free-ended rib by a constant quantity, which is H.

Take moments about any point of the neutral axis of rib, and there results—

$$M = \Sigma Wx + m - H\left(V' + \frac{m}{H}\right) = \Sigma Wx - HV'$$

and since $\Sigma Wx = HV$, $M = -H(V^1 - V)$

Let ADB, Fig. 367, denote the neutral axis of an arched rib fixed at ends of the same span, as the one last considered, and subjected to the same loading. The rib is supposed to be divided into twenty equal parts, each equal to 7·35 feet, which are the panel lengths. As before, we assume that n is equal to the horizontal distance between the panels approximately.

Since the ends of the rib are fixed, the sum of the strains at any given distance from the neutral axis of the rib considered as a girder, and extending between the points A and B, must equal zero.

$$\Sigma_B^A na = \Sigma_B^A \frac{nM}{EI} = \frac{n}{EI}\Sigma_B^A M = 0$$

$$\therefore \Sigma_B^A M = 0$$

but $M = H(V' - V)$, where V denotes the ordinates of the polygon, and V′ those of the rib.

The above equation therefore shows that the sum of the intercepts $(V' - V)$ on one side of the rib is equal to the sum on the other.

The sum of the horizontal and vertical deflections must equal zero, as in the free-ended rib, so that the three conditions may be expressed thus—

$$\sum_B^A M = 0$$

$$\sum_B^A Mx = 0$$

$$\sum_B^A My = 0$$

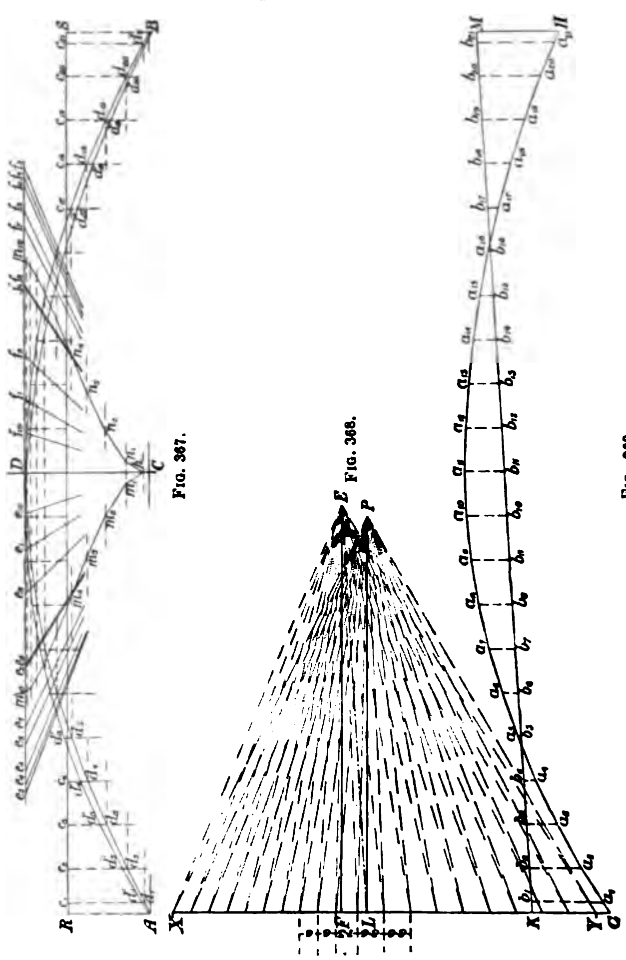


Fig. 367. Fig. 368. Fig. 369.

Choose E as the assumed pole, and set off the loads on the line XLY, as in the free-ended rib, and draw the polygon

$Ga_1a_2 \ldots H$. The line GH is not the closing line, which must be located so that the equations $\Sigma M = 0$ and $\Sigma Mx = 0$ will be satisfied. Let KM be located by trial, then the algebraic sum of its ordinates must equal zero, and the algebraic sum of the products of each ordinate into its abscissa must equal zero. This line can generally be found by one or two trials. In the figure ordinates are drawn through b_1 and b_{21} at distances of one-fourth the panel length from A and B, also K and M respectively, representing loads concentrated at the middle of each half-panel at the ends; these ordinates may be added to the others in the summation, if we divide them first by two; thus we have—

$$\Sigma_B^A M_a = \frac{a_1b_1}{2} + a_2b_2 + a_3b_3 + \ldots + a_{20}b_{20} + \frac{a_{21}b_{21}}{2} = 0$$

The closing line RS of the rib, considered as an equilibrium polygon, can be easily located, because it is parallel to AB, and if the first condition, viz. $\Sigma_B^A M_d = 0$, the second condition, $\Sigma_B^A M_d x$, must equal zero, since the curve is symmetrical with reference to D.

The third condition requires that $\Sigma_B^A M_a y = \Sigma_B^A M_d y = 0$. Treating M_a and M_d as loads as in the rib with free ends applied at distances y from the assumed origin, the ordinates of the equilibrium polygons will represent the horizontal deflections of the rib.

Set off, therefore, $Df_1, f_1f_2, \ldots f_9f_{10}$ on the left of DC equal to the ordinates of the polygon $Ga_1a_2 \ldots a_{21}H$ taken in pairs, and join the points f_1f_2, etc., with the assumed pole C, and draw the deflection polygon $hn_1n_2 \ldots n_{10}$, as in the case of the rib with free ends.

Draw also the deflection polygon $hm_1m_2 \ldots m_{10}$, using the ordinates of the rib as loads in precisely the same manner as before.

It will be found by measurement that Dm_{10} is greater than Dn_{10}, showing that the quantities Ma are too small, or the polar distances too large. Draw EL parallel to the closing line KM, and LP parallel to AB, make the new polar distance $PL = \frac{Dn_{10}}{Dm_{10}} \times EF$, and draw a new set of lines radiating from

P, from which the true equilibrium polygon $Ad_1'd_2'$ . . . B is obtained. Measure LP to the same scale as that used for the loads XLY, and it will be found to be 132 tons. This force acts along the line RS. The bending moment at any point of the arched rib equals the intercept between the rib and the true equilibrium polygon multiplied by the horizontal thrust, viz. 132 tons.

Winkler gives the following formula for locating the line RS:—

$$l = \frac{r(as^2 + 24I)}{3as^2}$$

where l = the height above AB, the other quantities having the same significance as in the free-ended rib. Using the values found for the free-ended rib, we have—

$$l = \frac{20 \cdot 5\{\frac{48}{144} \times \overline{147}^2 + 24(0 \cdot 9)\}}{3 \times \frac{48}{144} \times \overline{147}^2} = 6 \cdot 8 \text{ feet}$$

It scales 6·9 feet on the diagram.

The reactions at A and B are not the same as in the free-ended rib, being less at A and greater at B; for the fixed rib we measure the segments of the load line LY and LX, Fig. 368, but in the free-ended rib, we measure the segments formed by drawing a line from E parallel to the line joining GH.

The temperature stresses in the rib fixed at each end are much greater than in the free-ended rib, and the increase in the horizontal thrust due to the maximum range of temperature may be found by either of the methods explained for the free-ended rib.

By Winkler's formula we have—

$$H = \frac{45EIaet}{4ar^2 + 45I}$$

Inserting the quantities used in the free-ended rib, we obtain—

$$H = \frac{45 \times 1600000 \times \frac{48}{144} \times 0 \cdot 0000068 \times 50^\circ}{4 \times \frac{48}{144} \times 20 \cdot 5^2 + 45(0 \cdot 9)} = 14 \cdot 5 \text{ tons}$$

This increase is due to ± 50°, or 50° on either side of the mean temperature at which there is no stress.

Having found the horizontal thrust for the various positions of the live load, the stresses in the various members of the rib should be found either by moments or by means of a diagram as in Figs. 360 and 361. The stresses obtained should then be tabulated, and the maximum stress on each member obtained.

Three-hinged Arched and Suspension Rib.—Let ABC, Fig. 370, denote the neutral axis of an arched rib, hinged at A,B, and C, and let W denote a load applied to the rib at a horizontal distance AF from A. The resultant force acting at the crown B for the unloaded half must pass through the hinges B and C,

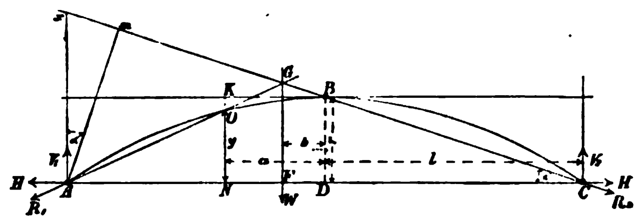


FIG. 370.

and the resultant for the loaded half must pass through G and A. AG and GC are therefore the directions of the resultant pressures at A and C, the magnitude of which may be found by resolving W along these lines, and may be denoted by R_1 and R_2. R_1 and R_2 may each be resolved horizontally and vertically into HV_1 and HV_2. H denotes the horizontal thrust, and V_1 and V_2 the vertical reactions at A and C.

Since the load W produces pressures along the lines GA and GC, take moments about A, and draw Am perpendicular to CB produced. Draw also Ax perpendicular to AB; then the angle ACB equals the angle mAx, and—

$$W \times AF = R_2 \,.\, Am \quad \therefore R_2 = W\frac{AF}{Am}$$

$$R_2 \sin a = V_2 \quad \therefore V_2 = W \,.\, \frac{AF}{Am} \times \frac{Am}{AC} = W \,.\, \frac{AF}{AC} = \frac{l-b}{2l}W$$

$$R_2 \cos a = H \quad \therefore H = R_2 \,.\, \frac{Am}{Ax} = W\frac{AF}{2DB} = \frac{l-b}{2r} \,.\, W$$

$$\text{also } V_1 = \frac{l+b}{2l}W$$

Having found the horizontal thrust and vertical reactions, the values of which are independent of the shape of the rib, the stresses in the various members of the arch may be determined by moments, or by means of a stress diagram.

The stresses may be tabulated for each load which acts upon the arch, and the maximum stresses determined in the usual way.

Variations in temperature do not produce stresses in this form of arch, as the hinged connections allow the crown to rise or fall freely.

One of the most economical forms of the three-hinged arch is shown in Fig. 371. It was proposed and thoroughly investigated by Mr. Claxton Fidler, M.I.C.E., and published in *Engineering* in 1875. Inverted, it becomes an equally economical

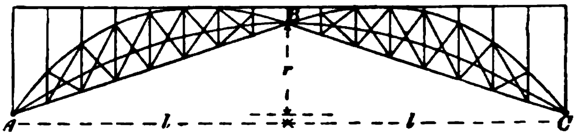


FIG. 371.

suspension bridge. The neutral axis ABC is assumed to be a parabolic curve, and each half-rib, AB and BC, is a parabolic girder, the central depth of which is slightly greater than $\frac{r}{2}$. The straight portion AB is a tangent to the parabolic curve of the half-rib BC.

Captain Ead's arch is somewhat similar to Fidler's, but with curved lower members, similar to the upper members of the semi-ribs. Fidler gives the following values of H for his bridge.

For a uniformly distributed load of w per unit of length over the whole span, we have the horizontal stress at any point of the upper or lower member—

$$Hw = -\frac{wl^2}{4r}$$

For a live load, w_1 distributed over the half-span l, or the whole span $2l$—

$$Hw_1 = -\frac{w_1l^2}{4r}$$

There is no stress in the diagonal bracing for either of these cases.

$$\text{The maximum stress H} = -(w + w_1)\frac{l^2}{4r}$$

$$\text{The minimum stress H} = -\frac{wl^2}{4r}$$

The greatest flange stress in the semi-ribs occurs when the live load extends from B to a point D, Fig. 371, such that $FD = b = \frac{al}{l + a}$, which becomes—

$$\text{Maximum H}w_1 = -\frac{w_1 l^2}{4r}\left(1 + \frac{a}{l + a}\right)$$

If the load extends from A to F—

$$\text{Minimum H}w_1 = -\frac{w_1 l^2}{4r}\left(\frac{a}{l + a}\right)$$

The maximum or minimum value of the tensile stress in the suspension rib, or compressive stress in the arch, is found from the following values of H:—

$$\text{Max. H} = \text{H}w + \text{max. H}w_1 = -\frac{l^2}{4r}\left\{w + w_1\left(1 + \frac{a}{l + a}\right)\right\}$$

$$\text{Min. H} = \text{H}w + \text{min. H}w_1 = -\frac{l^2}{4r}\left\{w - w_1\left(\frac{a}{l + a}\right)\right\}$$

The greatest stress in any diagonal is the same as in a parabolic girder of span l and depth equal to that of the semi-rib.

In any hinged rib, the neutral axis of which lies in a parabolic curve, it can be proved that if a denote the distance of any point O from B, Fig. 370, and b the extremity of the live load, which we will call the load boundary, then—

$$b = \frac{al}{2l + a}$$

This may be proved in the following manner: For any load between N and D, Fig. 370, the bending moment is found thus—

$$M = V(l - a) - Hy$$

M will be positive or negative according to the position of the load. If the load is attached at the point O, the bending moment is zero, and—

$$V(l - a) = Hy$$

$$\therefore y = \frac{V(l - a)}{H}$$

substituting for V and H the values already found :—

$$y = \frac{r(l + b)(l - a)}{l(l - b)}$$

but since the curve is parabolic—

$$OK : r :: a^2 : l^2 \quad r - y : r :: a^2 : l^2$$

$$\therefore y = r\left(1 - \frac{a^2}{l^2}\right)$$

$$\therefore 1 - \frac{a^2}{l^2} = \frac{(l + b)(l - a)}{l(l - b)}$$

$$\therefore b = \frac{al}{2l + a}$$

The maximum bending moment in the semi-rib for the live load may be positive or negative, and has the following values:—

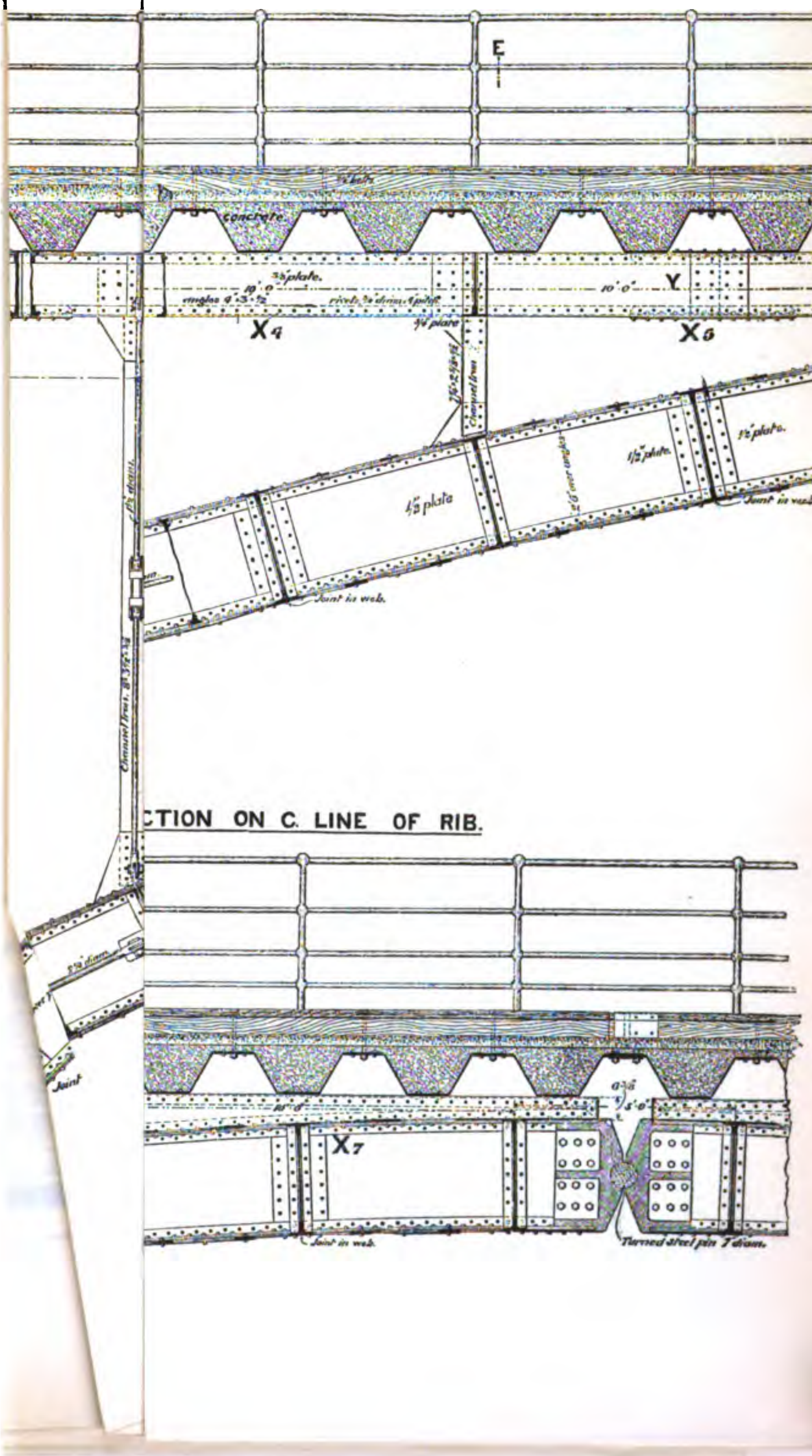
$$M = \pm \frac{w_1 a(l - a)}{4}\left(1 + \frac{a}{2l + a}\right)$$

If H denote the horizontal thrust running through the arch or suspension rib, and d = the depth of the girder at a, then the horizontal component of the flange stress in the semi-rib is—

$$H_{\text{flange stress}} = \frac{H}{2} \pm \frac{M}{d}$$

The following example of a three-hinged parabolic arched rib will be completely worked out, and the bridge partly designed to carry a roadway 18 feet wide. The bridge, illustrated in Plates V., V*a*, and V*b*, consists of two parabolic ribs, 147 feet 2 inches span, and 20 feet 7½ inches rise between centres of hinges. The deck is carried by two longitudinal girders supported by vertical columns, which stand upon the top flanges of the arched ribs. Both the longitudinal girders and the arched

TH PLATE V.



concrete
¾" plate.
angles 4' 3" ½
X4
X5
Y
10' 0"
½" plate
½" plate
Joint in web.
CTION ON C. LINE OF RIB.
X7
Joint in web.
Turned steel pin 7" diam.
Joint

ribs are spaced 16 feet apart, centre to centre, and the vertical columns divide the arch into bays of 10 feet, excepting the two central bays, which are each 12 feet 6 inches.

The deck is formed of wrought-iron troughing laid transversely on top of the longitudinal girders (Chapter XIV., Fig. 319), and filled in with coke concrete to a level of 2 inches over the upper surface of the trough. Above the concrete is spread tarred basalt metal to a depth of 5 inches at the centre and 3 inches at the sides. The ends of the troughing are protected by facia plates secured to it by angle irons.

The proportions of the coke concrete are—four of coke broken to a gauge of one inch, two of sand, and one of Portland cement. The weight per cubic foot of the coke concrete is 90 lbs., whereas stone concrete would weigh 128 lbs., or 30 tons more on the deck of the bridge.

The strength of the troughing may be approximately calculated as follows:—

Dead load, including concrete, metal, and troughing 3 feet wide between longitudinals = 2024 lbs. per foot run

$$\text{Bending moment} = \frac{Wl}{8} = \frac{2024 \times 3 \times 16}{2240 \times 8} = 5{\cdot}42 \text{ foot-tons}$$

Live load consisting of traction-engine shown in Figs. 322 and 323

Bending moment $= 4{\cdot}75(8 - 2{\cdot}5) = 26{\cdot}13$ foot-tons

Total bending moment on one trough $= 31{\cdot}55$ foot-tons $= 378$ inch-tons

Treating each trough as a girder, with each flange consisting of the horizontal portion plus one-half the sloping portion above or below the neutral axis, we have—

Area of one flange $= (28{\cdot}5 - 4 \times \frac{3}{4})\frac{5}{16} = \frac{5}{16} \times 25{\cdot}5$ square inches

Distance between centres of gravity $= 11{\cdot}375$ inches

$$\left.\begin{matrix}\text{Moment of}\\ \text{resistance}\end{matrix}\right\} = \frac{25{\cdot}5 \times 5 \times 11{\cdot}375 \times 20}{16} = 1812{\cdot}9 \text{ inch-tons}$$

$$\text{Hence the factor of safety is } \frac{1812{\cdot}9}{378} = 4{\cdot}8$$

Longitudinal girders consist of two plate web girders spaced 16 feet centres across the bridge, each 18 inches deep, tapering to 6⅝ inches near centre of bridge. They are stiffened over columns with diaphragms, and the bottom flanges are laced with $3 \times \frac{3}{8}$ bars. The ends of longitudinals are continued over abutments, and have gun-metal bearing-plates secured on their lower flanges sliding on cast-iron bed-plates secured to bed-stones.

Each longitudinal girder is calculated as a continuous girder of five equal spans, the two central bays being supported throughout their length by the rib.

The loads on the longitudinal girder are as follows :—

Live load, taken as 84 lbs. per square foot of roadway = 0·35 tons per lineal foot on each girder.

Dead load = 0·7 tons per lineal foot on each girder.

The maximum bending moments for these loads occur when the bays x_1, x_3, and x_5 are fully loaded, and bays x_2 and x_4 are loaded with the dead load only.

The bending moments at centres of bays x_1, x_3, and x_5, Plate V., may be found, by the methods already explained, to be 13 foot-tons, and at centres of bays x_2 and x_4 to be 8·7 foot-tons.

The traction engine produces the greatest bending moment when the load on one wheel is carried entirely by one girder and $\frac{11{\cdot}33}{16}$ of the other also.

The total load on one girder due to the driving wheels $= 4{\cdot}75\left(1 + \frac{11{\cdot}33}{16}\right) = 8{\cdot}11$ tons, and the load due to the trailing wheels is $3{\cdot}25\left(1 + \frac{11{\cdot}33}{16}\right) = 5{\cdot}55$ tons.

Bending moment at centre of bay due to driving wheels
= 20·3 foot-tons
Bending moment at centre of bay due to trailing wheels
= 13·0 foot-tons
Bending moment at centre of bay due to dead load
= 8·75 foot-tons

The results are summarized as follows, the working stresses being taken as 4 and 5 tons compression and tension respectively :—

TABLE LXXXIV.

Flange.	Effective area.			
	Driving wheels over bay x_1 trailing over abutment.		Driving wheels over centre of bay x_2 trailing over x_1	
	Required.	Provided.	Required.	Provided.
	sq. in.	sq. in.	sq. in.	sq. in.
Top	$\frac{20}{1{\cdot}42 \times 4} = 3{\cdot}5$	Two angles $4 \times 3 \times \frac{1}{2}$ = 6·5	$\frac{21}{1{\cdot}42 \times 5} = 3{\cdot}0$	Two angles as before less rivets = 5·75
Bottom	$\frac{20}{1{\cdot}42 \times 5} = 2{\cdot}8$	Do. less rivets = 5·75	$\frac{21}{1{\cdot}42 \times 4} = 3{\cdot}7$	Do. gross = 6·5

Maximum shear due to dead and live load = 11·61 tons.

The web plate is made $\frac{3}{8}$ thick, and the rivets $\frac{3}{4}$ inch diameter, 4 inches pitch, which may easily be shown to be safe.

The columns are calculated by the formula—

$$f = \frac{19}{1 + \frac{1}{900}\left(\frac{l}{r}\right)^2}$$

r being in this case the breadth of the channels. They are made larger than necessary for appearance.

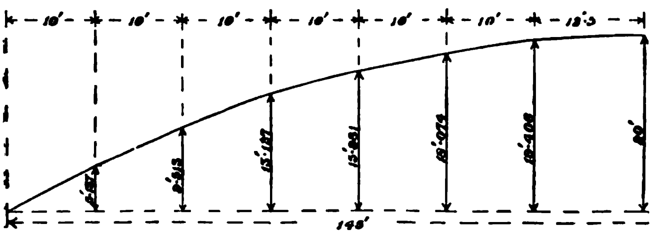


FIG. 372.

Column 1	requires by above formula	4·6 sq. in.	Area provided is	13·2 sq. in.
" 2	" "	5·3 "	" "	13·2 "
" 3	" "	4·1 "	" "	10·2 "
" 4	" "	3·6 "	" "	10·2 "

Arched Ribs.—The load resting upon the arched ribs may now be ascertained.

TABLE LXXXV.

Point number.	Dead load. Deck and longitudinal girder.	Column and bracing of rib.	Total.	Distributed live load equals dead load $\times \frac{0.35}{0.7} = \frac{1}{2}$
	tons.			
1	0·7 × 10 × [illegible] = 2·8	2·1	4·9	1·4
2	× [illegible] = 8·0	2·8	10·8	4·0
3	× [illegible] = 6·8	2·3	9·1	3·4
4	× [illegible] = 6·8	2·0	8·8	3·4
5	× [illegible] = 8·0	1·8	9·8	4·0
6	× [illegible] + 0·7 × 0·5 = 6·3	1·6	7·9	3·2
7	0·7 × 11·25 = 7·9	1·7	9·6	3·9
8	0·7 × 6·25 = 4·4	1·2	5·6	2·2
		Totals	66·5	25·5

The bending moments in the arched rib considered as 145 feet span, with a rise of 20 feet (Fig. 372) for live and dead load, may now be calculated, using the formula $b = \frac{al}{2l + a}$ to find the distribution of loads which will give the maximum; thus for point 2—

$$b = \frac{62·5 \times 72·5}{2 \times 72·5 + 62·5} = 21·8 \text{ feet}$$

For Maxima Bending Moment. Positive (see Fig. 373)—

$$0 = 72·5V - 20H + 10·8 \times 10 + 9·1 \times 20 + 8·8 \times 30 + 9·8 \times 40 + 7·9 \times 50 + 13·5 \times 60 + 7·8 \times 72·5$$

$$0 = 72·5V + 20H - 14·8 \times 10 - 12·5 \times 20 - 12·2 \times 30 - 13·8 \times 40 - 11·1 \times 50 - 13·5 \times 60 - 7·8 \times 72·5$$

$$\therefore H = 149·1, \text{ and } V = 3·65$$

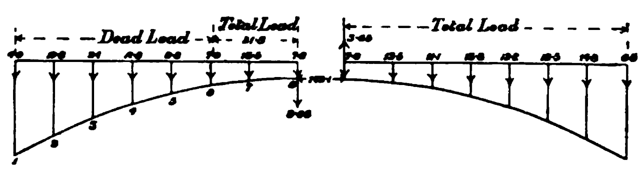


FIG. 373.

Using these values, and taking moments about the neutral axis of the rib at 2, we have—

$M_2 = 3{\cdot}65 \times 62{\cdot}5 - 149{\cdot}1 \times 14{\cdot}86 + 7{\cdot}8 \times 62{\cdot}5 + 13{\cdot}5 \times 50 + 7{\cdot}9 \times 40 + 9{\cdot}8 \times 30 + 8{\cdot}8 \times 20 + 9{\cdot}1 \times 10 = + 52{\cdot}01$

Negative (see Fig. 374)—

$$0 = -72{\cdot}5V - 20H + 14{\cdot}8 \times 10 + 12{\cdot}5 \times 20 + 12{\cdot}2 \times 30 + 13{\cdot}8 \times 40 + 11{\cdot}1 \times 50 + 9{\cdot}6 \times 60 + 5{\cdot}6 \times 72{\cdot}5$$

$$0 = -72{\cdot}5V + 20H - 10{\cdot}8 \times 10 - 9{\cdot}1 \times 20 - 8{\cdot}8 \times 30 - 9{\cdot}8 \times 40 - 7{\cdot}9 \times 50 - 9{\cdot}6 \times 60 - 5{\cdot}6 \times 72{\cdot}5$$

$\therefore H = 129{\cdot}4$, and $V = 3{\cdot}65$

$$M_2 = -3{\cdot}65 \times 62{\cdot}5 - 129{\cdot}4 \times 14{\cdot}86 + 5{\cdot}6 \times 62{\cdot}5 + 9{\cdot}6 \times 50 + 11{\cdot}1 \times 40 + 13{\cdot}8 \times 30 + 12{\cdot}2 \times 20 + 12{\cdot}5 \times 10 = -94$$

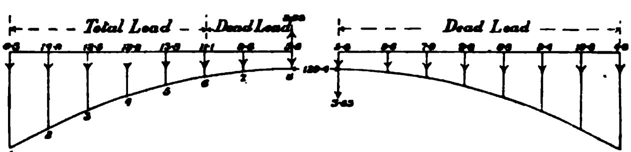


FIG. 374.

In the same manner it may be shown that for the point 3 the load boundary = 19·3 feet. For maximum positive bending moments $H = 149{\cdot}1$, and $V = 3{\cdot}65$; $M_3 = +98{\cdot}07$. For maximum negative bending moments $H = 129{\cdot}4$, $V = 3{\cdot}65$, and $M_3 = -140{\cdot}03$.

For the point 4 the load boundary is 16·3 feet, and we obtain the same values of H and V as for points 2 and 3. $M_4 = +123{\cdot}31$ and $-158{\cdot}10$.

For point 5 the load boundary is 12·3 feet; $H = 143{\cdot}23$, $V = 5{\cdot}21$, and $M_5 = +118{\cdot}04$; also $H = 135{\cdot}25$, $V = 5{\cdot}21$, and $M_5 = -153{\cdot}03$.

For point 6 the load boundary is 8·8 feet; the values of H and V are the same as for point 5; and $M_6 = +112{\cdot}29$ and $-117{\cdot}25$.

For point 7 the load boundary is 4·7 feet, the values of H and V are the same as for points 5 and 6; and $M_7 = +78{\cdot}12$ and $-74{\cdot}92$.

Stresses due to Dead Load and Traction Engine.—The maximum loads carried by one rib due to a traction-engine rolling over the bridge will be the same as in the longitudinal girder, viz. 8·11 tons concentrated on the driving and 5·55 tons on the trailing wheels.

Let the driving wheels be over column 2, Fig. 375.

Then—

$$0 = -72{\cdot}5V - 20H + 18{\cdot}91 \times 10 + 9{\cdot}1 \times 20 + 8{\cdot}8 \times 30 + 9{\cdot}8 \times 40 + 7{\cdot}9 \times 50 + 9{\cdot}6 \times 60 + 5{\cdot}6 \times 72{\cdot}5$$

$$0 = -72{\cdot}5V + 20H - 10{\cdot}8 \times 10 - 9{\cdot}1 \times 20 - 8{\cdot}8 \times 30 - 9{\cdot}8 \times 40 - 7{\cdot}9 \times 50 - 9{\cdot}6 \times 60 - 5{\cdot}6 \times 72{\cdot}5$$

$$\therefore H = 118{\cdot}18, \text{ and } V = 0{\cdot}56$$

$$M_2 = -0{\cdot}56 \times 62{\cdot}5 - 118{\cdot}18 \times 14{\cdot}86 \times 5{\cdot}6 + 62{\cdot}5 + 9{\cdot}6 \times 50 + 7{\cdot}9 \times 40 + 9{\cdot}8 \times 30 + 8{\cdot}8 \times 20 + 9{\cdot}1 \times 10$$

$$= -84{\cdot}15$$

The maximum negative bending moment at point 2 is therefore greater for the traction engine driving wheel over the point

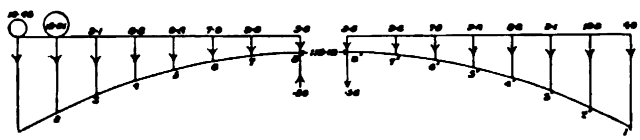


FIG. 375.

2 than for the dead and distributed live loads when the latter occupies the worst position, and it will be found that the maxima negative moments occur at the various points in nearly every case when the driving wheel of the traction engine is over these points respectively. So far as this example is concerned, the negative bending moments for the dead and distributed live loads need not have been considered, since the traction engine always produces the greater effect.

The results are given in Table LXXXVI.

The bending moments recorded in the following table, which have been calculated with reference to the neutral axis of the rib, must be divided by the effective depth of the rib in order to find the stresses in the flanges due to bending. The positive sign denotes the upward bending moments, and the negative sign the downward bending moments, so that the former produces tension in the extrados and compression in the intrados, while the latter produces compression in the extrados and tension in the intrados.

The horizontal thrust and the vertical reactions and loads produce compressive stresses in the rib which are independent of the depth, and these must be combined with the stresses due to bending in order to find the resultant stresses.

TABLE LXXXVI.

Number of point.	Maxima moments due to the dead and partially distributed live load.		Maxima moments due to the dead load and the traction engine. Rolling to the right. Driving wheels over 2	3	4	5	6	7	8	Rolling to the left. Driving wheels over 2_1	3_1	4_1	5_1	6_1	7_1	8_1	Maxima moments due to any manner of loading.	
2	+ 52·01	− 94·00	− 84·15	− 112·63	− 85·30	− 58·89	− 31·86	− 4·69	+ 28·33	+ 14·15	+ 6·23	− 2·00	− 10·56	− 18·48	− 26·56	− 36·83	+ 52·01	− 112·63
3	+ 98·07	− 140·03	− 70·10	− 155·33	− 159·25	− 108·40	− 77·03	− 5·55	+ 57·65	+ 11·31	− 2·17	− 15·85	− 29·85	− 43·33	− 56·91	− 73·95	+ 98·07	− 159·25
4	+ 123·31	− 158·10	− 53·70	− 117·14	− 180·45	− 163·26	− 90·07	− 16·84	+ 73·37	+ 6·10	− 10·36	− 26·95	− 43·83	− 60·29	− 76·81	− 97·47	+ 123·31	− 180·45
5	+ 118·04	− 153·03	− 40·28	− 84·58	− 128·80	− 173·42	− 136·60	− 44·28	+ 69·69	+ 3·88	− 12·92	− 29·80	− 46·93	− 63·73	− 80·57	− 101·59	+ 118·04	− 173·42
6	+ 112·29	− 117·25	− 18·69	− 46·44	− 74·15	− 102·12	− 129·88	− 76·51	+ 58·00	− 5·51	− 21·06	− 35·65	− 50·54	− 64·93	− 79·55	− 97·70	+ 112·29	− 129·88
7	+ 78·12	− 74·92	− 6·73	− 20·49	− 34·25	− 48·14	− 51·91	− 75·67	+ 20·71	− 7·3	− 17·01	− 26·75	− 36·61	− 46·24	− 56·08	− 68·21	+ 78·13	− 75·67

Let F = the sum of the vertical forces on the right side of the neutral axis of the rib, including the vertical reaction.

a = the angle which the tangent to the neutral axis makes with the horizon.

H = the horizontal thrust due to the vertical forces.

Then the compressive stress in the extrados and intrados of the rib is—

$$\frac{F \sin a + H \cos a}{2}$$

The shearing stress is—

$$F \cos a - H \sin a$$

If we calculate the values of sin a and cos a at the various points in the neutral axis corresponding with the points about which the bending moments were taken, we can find the values of the compressive and shearing stresses and combine these with the stresses due to bending.

Remembering that the neutral axis of the rib is parabolic, and that the subtangent is twice the horizontal distance from the point at which the tangent is drawn to the centre of the hinge at the springing, we can tabulate the values of the sine and cosine as in the following table. The effective depth of the web is 2·5 feet.

The maximum stress due to bending is 72·18 tons, therefore the sectional area of the top and bottom flanges is—

$$\frac{72{\cdot}18}{4} = 18{\cdot}045 \text{ square inches}$$

The area provided (Plate V.) is—

Four angles 4 × 3 × ½	13·00	square inches.
Two plates 9 × ¾	6·75	" "
Total area of one flange	19·75	" "
Total area of both flanges	39·50	" "

The maximum resultant is 138 tons compression, which is distributed over about 28 inches of the depth of the rib, as a stress uniformly varying from 0 to a maximum. If we take half the depth of the web as effective, and add the area to the area of one flange, we have for the two web plates, taken as ½ inch thick,

TABLE LXXXVII.

Point number.	Bending moments +.	Stresses due to bending moments.		Sin a.	Cos a.	Sum of vertical forces. F.	Horizontal thrust. H.	$\frac{F \sin a + H \cos a}{2}$		Resultant Stresses.		Shearing stresses $F \cos a - H \sin a$.
		Extrados tension.	Intrados compression.					Extrados compression.	Intrados compression.	Extrados compression.	Intrados tension.	
2	52·01	20·80	20·80	0·249	0·969	71·35	149·10	81·12	81·12	60·32	109·92	32·01
3	98·07	39·23	39·23	0·231	0·973	60·55	149·10	79·53	79·53	40·30	118·76	23·08
4	123·31	49·32	49·32	0·214	0·977	51·45	149·10	78·34	78·34	29·02	127·66	18·36
5	118·04	47·22	47·22	0·196	0·981	40·31	143·23	74·21	74·21	26·99	121·43	11·47
6	112·29	44·92	44·92	0·178	0·984	30·51	143·23	73·18	73·18	28·26	118·10	4·53
7	78·12	31·25	31·25	0·160	0·987	22·61	143·23	72·49	72·49	41·24	103·74	− 0·60

TABLE LXXXVIII.

Point number.	Bending moments −.	Stresses due to bending moments.		Sin a.	Cos a.	Sum of vertical forces. F.	Horizontal thrust. H.	$\frac{F \sin a + H \cos a}{2}$		Resultant stresses.		Shearing stresses $F \cos a - H \sin a$
		Extrados compression.	Intrados tension.					Extrados compression.	Intrados compression.	Extrados compression.	Intrados compression and tension.	
2	112·63	45·05	45·05	0·249	0·969	73·68	121·54	68·06	68·06	113·11	+ 23·01	41·13
3	159·25	63 70	63·70	0·231	0·973	62 08	125·01	67·99	67·99	131·69	+ 4·29	31·53
4	180·45	72·18	72·18	0·214	0·977	47·43	125·01	66·14	66·14	138·32	− 6·04	19·59
5	173·42	69·37	69·37	0·196	0·981	37·63	128·41	66·67	66·67	136·04	− 2·70	11·75
6	129·88	51·95	51·95	0·178	0·984	26·88	131·84	67·26	67·26	119·21	+ 15·31	2·98
7	75·67	30·27	30·27	0·160	0·987	18·04	135·25	68·19	68·19	98·46	+ 37·92	− 3·83

15 square inches, or a total area of 34·5 square inches; hence the intensity of stress is about—

$$\frac{138}{34 \cdot 5} = 4 \text{ tons per square inch}$$

The maximum shearing stress is 41·13 tons, or $\frac{41 \cdot 13}{2 \cdot 5} = 16 \cdot 4$ tons per foot on the two web plates, or 8·2 tons on each web plate.

Assume that the rivets are $\frac{7}{8}$ inch diameter and 4 inches pitch, then the intensity of shearing stress on the rivets is—

$$3 \times 0 \cdot 6 \times 2 \times f = 8 \cdot 2$$
$$\therefore f = 2 \cdot 3 \text{ tons per square inch}$$

The intensity of pressure on the bearing area is—

$$P \times 3 \times \tfrac{1}{2} \times \tfrac{7}{8} = 8 \cdot 2$$
$$\therefore p = 6 \cdot 2 \text{ tons per square inch}$$

The intensity of shearing stress on the web is—

$$\frac{8 \cdot 2}{(30 - 2 \times \frac{7}{8})\frac{1}{2}} = 0 \cdot 58 \text{ tons per square inch}$$

Hence there is no danger of buckling, and the sizes of rivets, pitch, and thickness of web may be adopted.

The section of the web is made uniform throughout, and for convenience of carriage the joints occur in the centre of each bay.

Hinges.—The hinges at the centre and springing are formed each of two castings working on a steel pin 7 inches in diameter, and 2 feet $4\frac{1}{4}$ inches long. The maximum pressure on the hinges occurs when the bridge is fully loaded. In this case H = 162·3 tons, and V = 0.

At the centre the pressure is 162·3 tons, and the intensity of pressure is—

$$\frac{162 \cdot 3}{7 \times 28 \cdot 25} = 0 \cdot 82 \text{ tons per square inch}$$

At the springing—

$$F \sin a + H \cos a = 92 \times 0 \cdot 481 + 162 \cdot 3 \times 0 \cdot 875 = 186 \cdot 26 \text{ tons}$$

The intensity of pressure per square inch is—

$$\frac{186 \cdot 26}{7 \times 28 \cdot 25} = 0 \cdot 94 \text{ tons}$$

The intensity of compressive stress on the rib at the springing when the bridge is fully loaded is—

$$\frac{186 \cdot 26}{69 \cdot 5} = 2 \cdot 7 \text{ tons nearly}$$

The details of the bridge are sufficiently illustrated in Plates V., V*a*, and V*b*.

The calculations for wind pressure present no difficulty; the sections adopted for the wind braces are shown on the Plate V*a*.

Abutments.—The abutments are composed of sandstone ashlar masonry, set in cement mortar, with wing walls and pilasters; inequalities in the rock face may be filled with cement concrete. The bedstones or skewbacks should be made of granite or other suitable stone.

Erection of the Ribs.—The ribs may be riveted on a staging, so that the longitudinal girders may have, when fixed in position, an even fall of 3 inches from the central hinge to the end girder when the temperature in the sun is 70° Fahr. Should the ribs be erected on a warmer or colder day, the necessary allowance for temperature must be made in wedging up the end hinges.

The foregoing method of calculation may be used for either a plate web or lattice web; if the web is designed with a single system of triangulation as in Fig. 360, the stresses may be obtained graphically as shown in Fig. 361, or by passing a section through any three members of the rib and taking moments in the usual way.

Suspension Bridges consisting of Steel Wire Cables stiffened near the Towers by means of Inclined Stays.—This is not a satisfactory construction as ordinarily carried out, as at best the stays leave the centre portion of the span unsupported, and the lengths of the inclined stays cannot be adjusted so that they will act efficiently under variations of temperature; the only useful purpose they serve is to check the vibrations and oscillations produced by the rolling load.

Suspension Bridges consisting of Steel Cables with a Stiffening Girder.—The steel cables may be combined with a horizontal stiffening girder, in which case the cables must sustain the whole live and dead load, the girder merely acting when the bridge is partially loaded, and the effect is to distribute the load over the cables.

The cables themselves tend to adapt themselves to the form of the equilibrium curve for the load they sustain. If the load is equally distributed along the horizontal, the curve of equilibrium may be proved to be a parabola.

Ordinary suspension bridges approximate closely to this curve, and it is assumed that the curve is a parabola in the following calculations. Since the steel cables carry the whole of the load, and the stiffening girders merely distribute the load without relieving the stress in the cables—in fact, adding to it by their own weight—this form of construction is by no means as satisfactory as the suspended rib proposed by Fidler, in which there is no increase in the flange areas of the ribs for stiffening the bridge when it is partially loaded; the form secures rigidity, and therefore requires no stiffening.

The use of the stiffening girder for cable suspension bridges has been very general, and is still in use; steel-wire ropes may be obtained at a moderate cost which will bear a tensile stress of 100 tons per square inch of sectional area, and the cost of these bridges is certainly not great where the site is suitable. The equation of maximum bending moments in the stiffening girder hinged at the centre, where the quantities have the same meaning as in the arched rib (Fig. 370), is, if the cables are parabolic and the girder rigid—

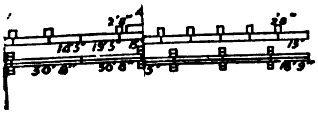
$$M = \pm w_1 \frac{a(l - a)}{4}\left(1 + \frac{a}{2l + a}\right)$$

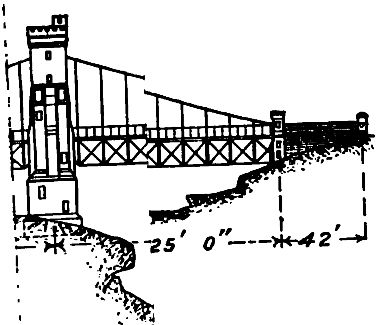
For a uniformly distributed load over the whole bridge, there is no stress on the girder; for a load distributed uniformly over one-half of the bridge, the maximum flange stress in the stiffening girder is $\pm \frac{w_1 l^2}{16d}$.

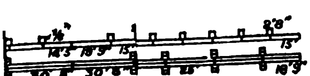
The stresses in the flanges alternate between tension and an equal compression, so that a low value must be taken for the working stress. The stiffening girder is generally hinged at the centre to avoid temperature stresses, and it should be secured at one end by bolting it to one of the piers or abutments, so as to fix it horizontally, and it should be allowed to expand freely over the other supports.

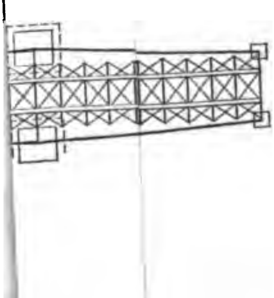
The hinge at the centre renders the calculations an easy matter, but if the girder were made strong enough to withstand the temperature stresses without a hinge, and securely fixed to

PLATE VI.









the abutments and piers, it would be much more rigid than the hinged bridge.

The following example of a stiffened suspension bridge[1] will illustrate the method of determining the most important of the stresses in this class of structure.

The bridge, illustrated in Plates VI., VI*a*, and VI*b*, is 775 feet long, in three spans of 150, 500, and 125 feet respectively, and 28 feet wide in the clear. It consists of steel-wire cables suspended from masonry towers and supporting steel lattice stiffening girders and steel cross-girders.

The cables, which are six in number, three on each side of the bridge, consist each of seven ropes formed with plough-shear steel wire. The central deflection of the cables is 38 feet 6 inches, and they are continued over the main towers on each side of the central span, and pass below the ground, through inclined shafts excavated in the rock, to the anchorages.

The sectional area of steel wire in each rope is 2 square inches, and the total sectional area in the six cables is 84 square inches. The arrangements for anchoring the cables to the rock are shown in Plate VI*b*., and consist of six hollow cast-steel beams, 15 inches in diameter, and 2½ inches thick, arranged three in each of two chambers excavated in the rock. An inclined rectangular shaft gives access to each of the chambers referred to from the ground level, and the cables pass down these to cast-iron thimbles, around which the ropes are coiled, and the ends of the wires are spread out in conical holes, where they are firmly secured by driving in round steel taper pins. The width between the cables over the piers is 48 feet centre to centre, and in the centre of the bridge 32 feet centre to centre. The towers are 112 feet high, and the cables discharge their weight upon expansion rollers 3 feet in diameter, which rest upon granite bed-stones, from which the pressure is distributed uniformly over the towers.

The stiffening girders, which are designed to prevent the main cables altering their curvature when the bridge is traversed by a moving load, are hinged at the centres, and spaced 15 feet centre to centre across the bridge.

The main girders are 12 feet 6 inches effective depth, and the cross-girders are 2 feet deep. The method of connecting the cross-girders to the main girders, and the cables, and

[1] North Sydney Suspension Bridge. Engineers, J. E. F. Coyle and the author.

the sections of these girders are sufficiently illustrated in Plates VI*a*, VI*b*.

Calculations.—The total dead load, including cables, girders, deck, bracing, handrails, etc., on central span is 0·93 ton per lineal foot.

The live load for a crowd of people, at 40 lbs. per square foot of deck, is 0·50 ton per lineal foot.

The live load for a tram motor and three loaded cars is equivalent to a load of say 0·5 ton per foot run over 150 feet.

The cables must be strong enough to carry the total live load of 0·93 + 0·5 = 1·43 ton per foot lineal; say 1·5 ton.

$$\text{Stress in centre} = \frac{\overline{500}^2 \times 1{\cdot}5}{8 \times 38{\cdot}5} = 1218 \text{ tons}$$

$$\text{Stress at towers} = \sqrt{375^2 + 1218^2} = 1275 \text{ tons}$$

Hence the maximum intensity of working stress in the cables is—

$$\frac{1275}{84} = 15{\cdot}2 \text{ tons per square inch}$$

$$\text{or a factor of safety of } \frac{100}{15{\cdot}2} = 6{\cdot}58$$

This factor of safety will be slightly reduced by a fall in temperature, which diminishes the central deflection, and increased by a rise in temperature.

It can easily be shown, by equating the moment of resistance of the hollow steel girders at the anchorages with the bending moment produced by the maximum pull on the cables, that the sizes mentioned are sufficient for their purpose.

Again, the stability of the towers against a wind pressure of 30 lbs. per foot can easily be shown to be ample.

The stiffening girders receive their maximum stresses during the passage of a tram motor with three loaded cars, and, as the tram line is shown on one side, so as to leave room for a carriage way on the other, the stiffening girder nearest the centre of the tram line will receive twice as great a load as the other.

Let a = the distance from the central hinge to the point where the bending moment is a maximum.

l = the half-span.

Let w_1 = the equivalent distributed live load per foot run, which will produce the maximum stresses in the stiffening girder.

Then the greatest bending moment producing stresses in the booms of the stiffening girder which alternate between tension and an equal compression is—

$$M = \pm w_1 \frac{a(l - a)}{4}\left(1 + \frac{a}{2l + a}\right)$$

The stresses and areas may be tabulated thus—

TABLE LXXXIX.

Distance *a* from central hinge in feet.	M. Foot-tons.	Stress in booms. Tons.	Total area required in booms in square inches.	Area required in booms in square inches.	
				Girder A.	Girder B.
0	0	0	0·0	17	14·25
25	739	59	19·6	28	14·25
50	1363	101	33·7	42	21·00
75	1856	149	49·6	42	21·00
100	2188	175	58·5	42	21·00
125	2344	188	62·7	42	21·00
150	2308	185	61·6	42	21·00
175	2066	157	52·3	42	21·00
200	1607	129	43·0	42	21·00
225	922	74	24·6	28	14·25
250	0	0	0·0	17	14·25

The working stress is taken as 3 tons, because the stresses alternate between tension and an equal compression in consequence of the maximum bending moment acting upwards and downwards.

Shearing Stresses.—Let a single weight W act at any point situated at a distance y from the left support. Let x denote any distance from the left support less than y. The reaction at the left support is—

$$\frac{W(2l - y)}{2l}$$

In consequence of the assumed rigidity of the truss and the parabolic curve of the cables, the upward pull upon the girders is $\frac{Wy}{l^2}$ per unit of the length.

The shearing stress at any point between the left-hand support and W is—

$$\frac{W(2l - y)}{2l} - \frac{Wyx}{l^2} = S$$

If w_1 denote the load per unit of length distributed over a distance y from the left support, then the equation of shearing stress becomes—

$$\frac{w_1 y}{2l}\left(2l - \frac{y}{2}\right) - \frac{w_1 y^2 x}{2l^2} = S$$

Let the live load of half a ton per foot run be distributed over 150 feet of the bridge, extending uniformly from the left abutment, the remainder of the bridge being unloaded with the live load. Then the maximum shearing stress will occur at the first bay from the left tower—

$$\frac{150}{2 \times 500}(500 - 75) = 63{\cdot}75 \text{ tons}$$

The equation of shearing stress becomes—

$$S = 63{\cdot}75 - 0{\cdot}09x.$$

The maximum stress in the end diagonals is therefore—

$$63{\cdot}75 \times 1{\cdot}414 = 90{\cdot}14 \text{ tons}$$

Hence, if the working stress is 6 tons per square inch, the area required is $\frac{90{\cdot}14}{6} = 15{\cdot}02$ square inches.

The stresses in the diagonals are always tensile, one diagonal acting when the right-hand girder is loaded, the other diagonal acting when the left-hand girder is loaded.

It is clear, from the above equation, that the shearing stress diminishes slowly from the left support; consequently the diagonals should not be reduced towards the centre to the same extent as in ordinary lattice or truss girders.

Again, for a moving load extending over a given distance the maximum stress will be produced in the left end panels, when the load extends from the left. If the load moves towards the centre, the shearing stress becomes smaller.

For a moving load of half a ton per foot, distributed over the left 250 feet, the remainder of the bridge being unloaded, the equation of shearing stress becomes—

$$S = \frac{3}{8} \times 250 - \frac{x}{4} = 93{\cdot}75 - \frac{x}{4}$$

The maximum stress in the end diagonals is—

$$93{\cdot}75 \times 1{\cdot}414 = 131{\cdot}6 \text{ tons.}$$

And the area required is—

$$\frac{131{\cdot}6}{6} = 21{\cdot}9 \text{ square inches}$$

The vertical compression members consist of four angles arranged in a rectangular form.

The maximum stress in the end columns is—

$$93{\cdot}75 - \frac{12{\cdot}5}{4} = 90{\cdot}63 \text{ tons}$$

And since the stress is compressive, the working stress should be 4 tons, allowing for the slight tendency to buckle laterally.

The area required is therefore—

$$\frac{90{\cdot}63}{4} = 22{\cdot}66 \text{ square inches}$$

The actual sections provided in the bridge are sufficiently illustrated in Plates VI., VI*a*, and VI*b*.

Stresses in the Cross-Girders.—The longitudinal stiffening girders distribute the loads from the driving wheels of the tram motor over two or more cross-girders, and equalize the tensile stresses on the suspension rods, so that the live load upon the cross-girders must be considered in regard to the load upon the tram line estimated at half a ton per foot run, and the crowd of people estimated at 40 lbs. per square foot of the deck not occupied by the tram. The dead load consists of the weight of the deck floor, handrailing, longitudinal timber beams, portion of stiffening girders, and cross-girder.

It may be shown that the maximum vertical pull on any suspension rod is about 11 tons, and the stress in the rod about 11½ tons. The suspension rods are 2 inches in diameter, and the effective area of the two bolts passing through the clips on the cables should be equal to that of the suspension rods.

It may also be shown that the maximum intensity of working stress in the tension flange of the cross-girder shown in Plate VIb. is 6 tons per square inch.

Wind Bracing.—The maximum wind pressure over the whole bridge at 30 lbs. per square foot acts horizontally over a surface equivalent to 500 feet × 15 feet = 7500 square feet in the centre span, and 2250 and 1875 square feet in the side spans. The total force in the central span is 7500 × 30 = 225,000 lbs., or say 100 tons. Since there are forty bays, the apex loads are 2½ tons.

The wind pressure is resisted partly by the cables and deck, and partly by the wind bracing between the cross-girders and main girders. If 1½ ton is supposed to act at points along the top booms of the stiffening girder corresponding with the positions of the cross-girders, it will be found that the maximum stress in the end diagonals is 30 tons. The remaining stresses may be found in the same manner as in an ordinary lattice girder.

The cross-girders act as struts, and the double system of diagonal bracing will never act at the same time; one system will always be slack.

The remaining 1-ton apex load may well be left for the cables and the continuous deck of tallow-wood timber. This bridge is designed for light traffic, such as a horse tramway, which may be replaced by an electric tramway, and the live loads assumed in the foregoing calculations are greatly in excess of what is expected to pass over it. It is certainly not strong enough for ordinary town traffic. It should be regarded as a bridge of the minimum strength necessary to carry light traffic, such as that referred to, over a long span.

A three-hinged or two-hinged arched bridge would have been equally suitable for the site, and would have worked out quite as economically.

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